HIGHER LEVEL

LEAVING CERT MATHS

BY DUBLIN MATHS

"The Essential LC HL Study Guide"

INTRODUCTION

The aim of this revision book is to help you enhance your grade in your Leaving Certificate. It does this by breaking exam questions down by subtopic, in a way that is easy to understand, helping the student to recognise what the question is *really* asking. This book is most effective when the questions are answered in order.
At the start of each section, there is a link to a collection of similar questions and solutions, which can be used for extra study and practice. Each chapter of this book is covered in more detail during our weekly free group grind that takes place on www.dublinmaths.ie.
We strongly encourage you to attend these sessions.
Recordings are also available for playback.
The Leaving Cert curriculum is broad, and daunting. Don't be discouraged by a challenging question. As in the actual exam, difficult questions can sometimes begin with one or two simple parts. You should answer as much as you can.
We hope that this book offers even a small beacon of hope as you prepare for the big day.

Thank you for trusting us. We hope it pays off in spades!

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Chapter 1

ALGEBRA

- Basic Factorising
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Basic Factorising

- Highest Common Factor
- Grouping
- Quadratic
- Difference of 2 Squares
- Difference/Sum of 2 Cubes

New !

Basic Solving

- Linear
- Quadratic
- Fractional
- Simultaneous Equations

Inequalities

1) Linear

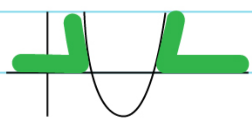
Tip If you divide by a negative number, the inequality sign flips.

Example :

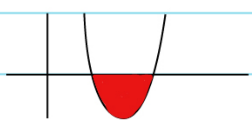
$$4(3-x) < 20$$
$$12-4x < 20$$
$$-4x < 8$$
$$x > -2$$

2) Quadratic

$> 0 \rightarrow$ flying



$< 0 \rightarrow$ drowning



3) Discriminants

Remember,

a = number before x^2
b = number before x
c = constant

$b^2 - 4ac > 0$ Real roots

$b^2 - 4ac = 0$ Equal roots
One real solution

$b^2 - 4ac < 0$ Imaginary roots
Non real roots
No solutions
Doesn't cross x-axis

4) Fractional

Tip Multiply both sides by denominator squared.

Example :

$$\frac{x+3}{2x-1} \leq 3$$

Continue

$$(2x-1)^2 \left(\frac{x+3}{2x-1}\right) \leq 3(2x-1)^2$$

5) Proofs

- $(R)^2 \geq 0$
- $-(R)^2 \leq 0$
- $a^2 - 2ab + b^2 \rightarrow (a-b)^2$

Memorise this

Factor Theory

- If $(2x-1)$ is a factor of $2x^3+x^2-13x+6$, this means :

→ $(2x-1)$ will divide in evenly
→ $X = 1/2$ is a solution

If you sub a solution into an equation, the equation should $= 0$

Example :

$$2(1/2)^3+(1/2)^2-13(1/2)+6=0$$

if $x = 3$ is a solution of $2x^3+x^2-11x-30$, this means :
→ $(x-3)$ is a factor

Absolute Value

$|-3| = 3$ and $|3| = 3$

Means distance from 0.
Always positive

- Tip Square both sides of the equation

Example :

$$|x+3|=3$$

$$(x+3)^2=3^2$$

continue

Square Roots

Square both sides of the equation Tip

Binomial Theorem

$$\binom{n}{r}(x)^{n-r}(Y)^r$$

Used for expanding brackets

Example :

$$(1-x)^7$$

↓

$$\binom{7}{0}(1)^7(-x)^0+\binom{7}{1}(1)^6(-x)^1+\binom{7}{2}(1)^5(-x)^2$$

continue

$$\binom{7}{r}(1)^{7-r}(-x)^7$$

General Form

Factorise fully $6cx - 3bc + 4dx - 2bd$

Factorise fully $6ax - 3by + 2ay - 9bx$

Factorise fully $5n - 2am - 2an - 5m$

Factorise fully $x^2 - 121$

Factorise fully $2x^2 - 98$

Factorise fully $4y^2 - 36x^2$

Factorise fully $x^2 + 13x + 42$

Factorise fully $x^2 + 7x - 30$

Factorise fully $x^2 - 3x - 18$

Factorise fully $x^2 - 11x + 30$

Factorise fully $2x^2 - 3x - 9$

Factorise fully $6x^2 - 19x + 15$

Factorise fully $\frac{2n^2+n-15}{n^2-9}$

Factorise fully $16x^4 - 686x$

Factorise fully $27y^3 - 64$

Factorise fully $8x^3 + 81$

Write the following as a single fraction $\frac{2x-1}{4} - \frac{3x+2}{5}$

Write the following as a single fraction $\frac{3}{x+4} - \frac{-2-x}{2x-1}$

Basic solving

Solve the equation: $3(2x - 4) + 2 = 3x - 7$

Solve the equation $x^2 - 4x - 8 = 0$

Give each answer in the form $a \pm a\sqrt{b}$, where $a, b \in N$

Solve the equation, where $x \in R$:

$$\frac{4x+2}{5} - \frac{6-x}{3} = -5$$

Solve the equation, where $x \in R$:

$$\frac{1}{2x-2} - \frac{-3}{x+4} = 2$$

Solve the equation, where $x \in R$:

$$\frac{1}{2x-3} - \frac{1}{2x+3} = \frac{6}{7}$$

Solve the simultaneous equations:

$$x + 5y + 5z = -2$$

$$4x - 5y + 4z = 19$$

$$x + 5y - z = -20$$

where $x, y, z \in Z$

Solve the simultaneous equations:

$$3x + 4y + z = 1$$

$$x - 5y - 3z = -4$$

$$\frac{1}{4}x + \frac{2}{3}y + z = 5.5$$

where $x, y, z \in Z$

Solve the simultaneous equations:

$$3x + 2y - 4z = 12$$

$$\frac{3x}{2} + \frac{5y}{3} + 2z = \frac{2}{3}$$

$$4y - x + 5z + 16 = 0$$

where $x, y, z \in Z$

Solve the simultaneous equations:

$$x + 2y = 143$$

$$y + 3w = -74$$

$$4x + 5w = 4$$

where $x, y, w \in Z$

Solve the simultaneous equations:

$$x + 3y = -5$$

$$2x^2 + y^2 = 41$$

where $x, y, \in Z$

Solve the simultaneous equations:

$$x^2 + xy + 2y^2 = 4$$

$$2x + 3y = -1$$

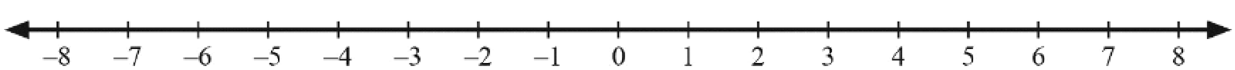
where $x, y, \in Z$

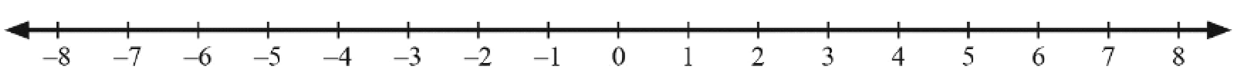
List the elements of the solution set of the inequality:

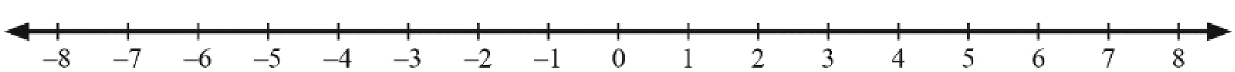
$$-4 < 3x + 2 \leq 11, \ x \in Z$$

Solve the inequality: $-5 < \frac{x}{3} - 4 \leq -2$

and graph the solutions on a N, Z and R number line.







Solve the following inequality: $x^2 + 7x + 10 > 0$

Solve the following inequality: $2x^2 + 7x - 4 \leq 0$

Solve the following inequality: $-x^2 + 9x + 22 > 0$

Solve the following inequality, for $x \in R$, and $x \neq 1$:

$$\frac{3x+1}{x-1} \leq 6$$

Solve the following inequality for $x \in R$:

$$-4 \leq \frac{3x+5}{2x-3}$$

Find the two values of $m \in Z$ for which the following equation in x has exactly **one** solution:

$$3x^2 - mx + 3 = 0$$

Explain why the following equation in x has no real solutions:

$$(2x + 3)^2 + 7 = 0$$

The equation $x^2 + kx + (k + 3) = 0$, where k is a constant, has two distinct real roots.

Show that $k^2 - 4k - 12 > 0$, and hence, find the possible values for k

If $t \geq 0$, find the range of value of t for which $(5t + 1)x^2 - 8tx + 3t$ has real roots.

If $(4q - 1)x^2 + 5qx + 2q$ has imaginary roots, find the range of values of q.

Show that $a^2 + 25 + b^2 \geq 2ab$ for all values of $a, b \in R$.

Show that for all real numbers $a \geq 2\sqrt{ab} - b$.

Show that for all real numbers $a^{2} - 2ab > - 5b^{2}$

Show that for any real values of a, b and h, the quadratic equation $(x - a)(x - b) - h^{2}$ has real roots.

$2x - 1$ is a factor of $2x^3 + x^2 + kx + 6$.

Find the value of , and hence find the 3 roots of $2x^3 + x^2 + kx + 6$.

$f(x) = 2x^3 - x^2 + 2x - 16$ is a cubic function.

Show that $(x - 2)$ is a factor of $f(x)$, and find the other two factors.

Show that $x = 10$ is a root of the following equation, where $x \in R$, and hence find the other two roots in the form $p \pm \sqrt{q}$, where $p, q \in Z$:

$$x^3 - 101x + 10 = 0$$

Solve the equation $x^3 - 3x^2 - 9x + 11 = 0$.

Write any irrational solution(s) in the form $a + b\sqrt{c}$, where $a, b, c \in Z$.

Given that $(x - 2)$ and $(2x - 1)$ are factors of $ax^3 + x^2 + bx + 6$, find the value of a and the value of b. Hence, find the third factor.

$x^2 - px + 1$ is a factor of $x^3 - 2x - 3r$, where $p, r \in R$ and $p < 0$.

Find the value of p and r.

Given that $x^2 + x - 6$ is a factor of $2x^3 - px^2 + qx - 6$, find the value of p and q.

Show that $2x - 3$ is a factor of $4x^2 - 2(1 + \sqrt{3})x + \sqrt{3}$ and find the other factor.

Using the Binomial theorem, or otherwise, expand $(1 - 2x)^5$

Write out the general form of the binomial expansion $(x^2 - \frac{1}{x})^{15}$

Find the value of the term that is independent of x in the expansion of $(x^2 - \frac{1}{x})^{15}$.

The terms of the binomial expansion of

$$(x + \frac{k}{x})^{10}$$

are written in ascending powers of x, where $x, k \in R$ and k is a constant. Find, in terms of k and x, the third term in this expansion.

The coefficient of x^4 in this expansion is 7680. Find the value of k.

$$(x + \frac{k}{x})^{10}$$

Find the two values of $m \in R$ for which $|5 + 3m| = 11$.

Given that $x = -3$ is a solution to $|x + p| = 5$, find the two values of p, where $p \in Z$

Find the range of values of x for which $|2x + 5| - 1 < 0$, where $x \in R$

Solve the equation $x = \sqrt{7x - 6} + 2$, where $x \in R$

Solve the equation $\sqrt{3x - 5} - 3 = \sqrt{x - 6}$, where $x \in R$

Simplify $(x^2 + \sqrt{2} + \frac{1}{x^2})(x^2 - \sqrt{2} + \frac{1}{x^2})$ and express your answer in the form $x^n + \frac{1}{x^n}$

Given that $4x^2 + 8x + 3 = a(x + b)^2 + c$

Find the values of the constants a, b and c.

Simplify fully:

$$\frac{25x^2-16}{3x^2+20x-7} \div \frac{5x+4}{9x^4+60x^3-21x^2}$$

For the real number h, j and express k in terms of h and j:

$$\frac{1}{h} = \frac{k}{j+k}$$

Chapter 2

LOGS AND INDICES

$\log_3 x$ $\quad (2^3)^3 \quad e^{3t}$

Indices

- These rules work both ways. You must be able to work from left to right, and vice versa.

1) Rules + Examples

$2^3 \times 2^5 = 2^8$

$\sqrt{2^3} = 2^{3/2}$

$\frac{2^6}{2^2} = 2^4$

$\left(\frac{2^7}{2^3}\right)^2 = \left(\frac{(2)^{7^2}}{(2)^{3^2}}\right)$

$(2^3)^2 = 2^6$

$2^0 = 1$

$2^{1/2} = \sqrt{2}$

$2^{1/3} = \sqrt[3]{2}$

$2^{-1} = \frac{1}{2}$, $2^{-3} = \frac{1}{2^3}$

$5(2^{-4}) = \frac{5}{2^4}$

2) Solving

(i) **Matching Bases,** all terms can be written with the same base number.

Example:

$$8^{3x} = \frac{32^{x+1}}{4^{2x}}$$

$$= (2^3)^{3x} = \frac{(2^5)^{x+1}}{(2^2)^{2x}}$$

$$= 2^{9x} = \frac{2^{5x+5}}{2^{4x}}$$

$$= 2^{9x} = 2^{x+5}$$

continue $\quad 9x = x + 5$

(ii) **Different Bases,** terms cannot be written with the same base number.

Example: $2^{2x+1} - 5(2^x) - 12 = 0$

Let $Y = 2^x$, $\quad (2^x)^2 x2 - 5(2^x) - 12 = 0$

$(Y)^2 x\, 2 - 5(Y) - 12 = 0$

continue $\quad 2Y^2 - 5Y - 12 = 0$

Logs

- These rules work both ways. You must be able to work from left to right, and vice versa.

1) Rules + Examples

- $\log_2 2x \rightarrow \log_2 2 + \log_2 x$

  $= 1 + \log_2 x$

- $\log_2\left(\frac{x}{2}\right) \rightarrow \log_2 x - \log_2 2$

  $= \log_2 x - 1$

- $\log_2 x^3 \rightarrow 3\log_2 x$ Important

- $\log_{\text{anything}} 1 = 0$

- $\log_5 5x^2$ vs $\log_5 (5x)^2$

$= \log_5 5 + \log_5 x^2$	$= 2\log_5 5x$
$= 1 + 2\log_5 x$	$= 2[\log_5 5 + \log_5 x]$
	$2 + 2\log_5 x$

2) Solving Log Equations

(i) Matching Bases

Example:

$\log_2 x = \log_2 7$

$x = 7$

Example: $\log_4(4x-7) = \log_2 2x$

- We don't have matching bases?

New Rule → $\log_b x = \dfrac{\log_a x}{\log_a b}$ Used to change the base in logs

$a = 2$
$x = 4x-7$
$b = 4$

Calculator

$$\frac{\log_2(4x-7)}{\log_2 4} = \log_2 x$$

$$\frac{\log_2(4x-7)}{2} = \log_2 x$$

$$\log_2(4x-7) = 2\log_2 x$$

continue

$$\log_2(4x-7) = \log_2 x^2$$

(ii) Left $^{\text{Right}}$ = Middle

Example: $\log_3 x = 2$ (L = 3, M = x, R = 2)

$3^2 = x$

- Use this rule when the log equation has a non log component (constant)

Solving exponentials

- Used when our variable is an index.
- Get the variable as isolated as possible, then take the log of both sides.

Example :

$$450e^{-3t} + 25 = 100$$

$$450e^{-3t} = 75$$

$$e^{-3t} = \frac{75}{450}$$

ln (e) is $\log_e e$

$$\ln\left(e^{-3t}\right) = \ln\left(\frac{75}{450}\right)$$

$$-3t \, \ln(e) = \ln\left(\frac{75}{450}\right)$$

$$t = \frac{\ln\left(\frac{75}{450}\right)}{-3}$$

Example :

$$27(32^{5x}) = 150{,}000$$

$$32^{5x} = \frac{150{,}000}{27}$$

$$\log_{32} 32^{5x} = \log_{32} \frac{150{,}000}{27}$$

$$5x\,(\log_{32}(32)) = \log_{32} \frac{150{,}000}{27}$$

$$x = \frac{\log_{32}\left(\frac{150{,}000}{27}\right)}{5}$$

Rewrite the following fraction in the form 2^n without a calculator, where $n \in Z$

$$\frac{(8^{\frac{2}{3}})(32^{\frac{4}{5}})}{2\sqrt{16}}$$

Rewrite the following expression in the form x^n, where $n \in Q$

$$\frac{x^3(x^{\frac{2}{3}})^2}{\sqrt[3]{x}}$$

Rewrite the following expression in the form e^{n}, without a calculator where $n \in Q$

$$\left(\frac{e^{-3}}{\sqrt{e}}\right)^{\frac{5}{2}}$$

Solve the following equation:

$$3^{3x+1} = \frac{243}{\sqrt{3}}$$

Solve the following equation:

$$\sqrt{\frac{5^{3x-1}}{5^{x+1}}} = 125$$

Solve the following equation:

$$2^{x} + 16(2^{-x}) - 10 = 0$$

Solve the following equation:

$$2^{2x+1} - 5(2^{x}) - 12 = 0$$

Given that $x = 5^y$, show that the equation $p5^y + 5^{-y} = 5$ can be written in the form $px^2 - 5x + 1 = 0$.

Hence, find the value of p for which the equation has equal roots

Solve for x:

$$2^x + 2^{1-x} = 3$$

Given that $p = log_c x$, express $log_c \sqrt{x} + log_c(cx)$ in terms of p.

Given that $log_a 2 = p$ and $log_a 3 = q$, express the following in terms of p and q:

$log_a \frac{9a^2}{16}$

If $A = B(t + 1)^c$, express c in terms of $log_{10}A$, $log_{10}B$ and $log_{10}(t + 1)$

Given that $s = log_a x$ and $t = log_a y^2$, express:

$log_a \frac{\sqrt{ax}}{y}$ in terms of s and t.

Log equations (Matching bases)

Solve $2log_a x = log_a(5x - 4)$

Solve $log_9(3x + 2) = log_3(x + 1)$

Solve $log_4 x - log_2(x - 2) = 0$

Solve the following equation:

$$2log_9 x = \frac{1}{2} + log_9(5x + 18)$$

Solve the following equation:

$$log_3 x + log_x 729 = 7$$

Solve the following equation:

$$log_3 t + log_3 9 + log_3 27 + log_3 81 = 10$$

The proportion of information recollected by a student after t hours is given by the function:

$$r(t) = 0.82 - 0.12ln(t + 1)$$

After how many hours, would exactly 55% of information be recalled correctly?

The magnitude of an earthquake on the Richter Scale is given by:

$$M = log_{10} [\frac{I}{I_0}]$$

Where l is the quake intensity, and I_0 is 10^{-3}. What is the quake intensity of an earthquake that measures 8 on the Richter scale

A heated metal ball is dropped into liquid. As it cools, its temperature in degrees celsius after t minutes is given by:

$$T = 400e^{-0.05t} + 25$$

Find the time it takes for the ball to reach a temperature of 300 degrees.

Each injection of a particular drug has 15 mg in it. The amount of drug left in a patient's system t days after receiving a single injection is given by:

$$15(0.6)^{t}$$

After how many days will there be exactly 1 mg of the drug left in a patient's body?

Solve the following correct to two decimal places:

$$7.4e^{-0.5t} = 3.7e^{0.07t}$$

Chapter 3

CO-ORDINATE GEOMETRY

The Line

- To find the equation of a line you need (i) a point, (ii) the slope

- $ax + by + c = 0$, slope $= \frac{-a}{b}$
- $y = mx + c$, slope $= m$

- A line crosses y-axis when $x = 0$
- A line crosses x-axis when $y = 0$

If l and J are perpendicular, and the slope of l is $\frac{2}{3}$ → the slope of J is $\frac{-3}{2}$
[Flip the fraction, change the sign]

- Area of a triangle? → translate one point to (0,0)

- Point of intersection ?
  → Simultaneous Equations

- Can we sub anything in for x and y?

The circle

- To find the equation of a circle you need (i) the centre, (ii) the radius

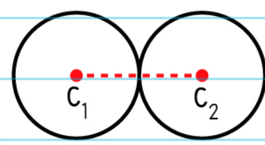
- $(x-h)^2 + (y-k)^2 = r^2$

with centre $= (h,k)$ and radius $= r$

- $x^2 + y^2 + 2gx + 2fy + c = 0$

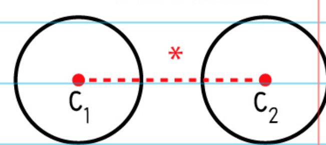
with centre $= (-g,-f)$ and radius $= \sqrt{g^2 + f^2 - c}$

Touch Externally

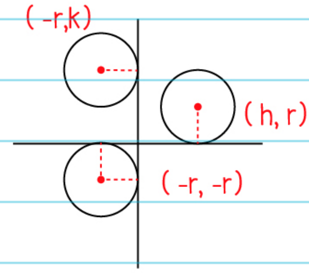


$|c_1c_2| = |r_1| + |r_2|$

Don't Touch



$|c_1c_2| > |r_1| + |r_2|$



- Tangents are always perpendicular to the line connecting the tangent point and the centre

- A tanget is a line that touches a circle or curve at a point
- Can we sub anything in for x and y?
- Can we sub (h,k) or (-g,-f) into anything?

$A(-1, k)$ and $B(5, l)$ are two points, where $k, l \in Q$.

Show that the midpoint of $[AB]$ is $(2, \frac{k+l}{2})$.

Hence, given that the perpendicular bisector of $[AB]$ is $3x + 2y - 14 = 0$, find l and k.

The line l has equation $x - 2y + 8 = 0$. The point P has coordinates $(k, \frac{k+8}{2})$.

Show that for all values of k the point p lies on l.

The line l has equation $3x - 6y + 2 = 0$. The point P has coordinates $(k, \frac{2k+2}{3})$.

The point P lies on l. Find the value of k.

A line passes through the point P $(k, \frac{k+8}{2})$ and the point $A(-1, 1)$. Find the slope of AP in terms of k.

$A(4, -1)$ and $B(7, t)$ are the endpoints of a line segment that is perpendicular to $3x - 4y - 12 = 0$. Find the value of t.

Line 1	$3x = -2y + 6$
Line 2	$4y = x + 2$
Line 3	$y = \frac{2}{3}x + 7$
Line 4	$y = \frac{1}{4}x - 7$
Line 5	$-x - 4y = 3$

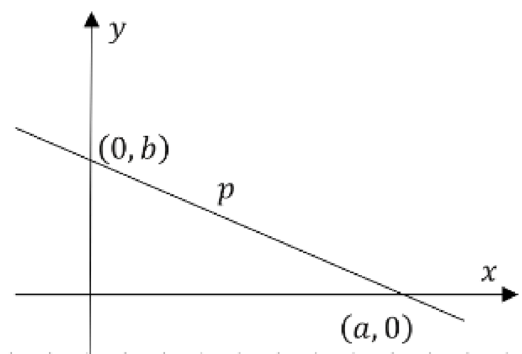
Identify 2 lines that are perpendicular, and identify two lines that are parallel

The line segment l has endpoints $A(-\ 2, -\ 2)$ and $B(4, 7,)$ Find the point of intersection between l and m: $2x\ +\ 5y\ =\ 24$.

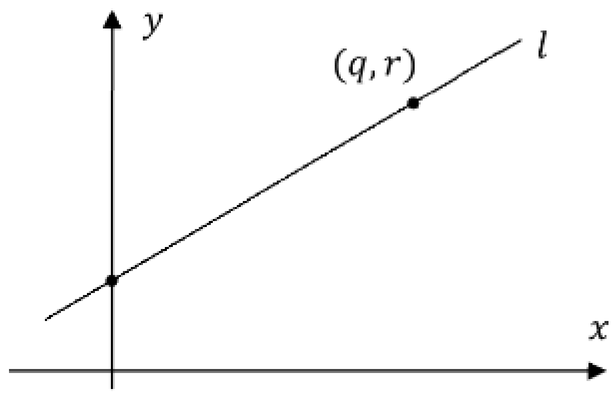
Hence, determine the ratio for which m divides l

$A(4, 6)$ and $B(13, 12)$ are two points. The point C divides the line $[AB]$ internally in the ratio $3: 2$. Find the coordinates of the point C.

Show that the equation of p can be given as $\frac{x}{a} + \frac{y}{b} = 1$



The line l has a slope of m and contains the point (q, r). Find the coordinates of the y intercept in terms of m, q, r.



The equation of the line S can be given by $px + (p - 1)y + 1 = 0$. Find the slope of S in terms of p.

Calculate the shortest distance between the line $6x + 7y - 10 = 0$ and the point $(3, 6)$.

There are two lines parallel to $2x - y - 3 = 0$, each a distance of $2\sqrt{5}$ away. Find the equations of these two lines.

The line k has equation $mx - y + m - 1 = 0$. If the distance from k to the point $(7, -5)$ is 8 units, find the value(s) of m.

Find the acute angle between $x + \sqrt{3}y - 10 = 0$ and $\sqrt{3}x + y - 10 = 0$

Find the equations of the lines through the point $(-4, -2)$ which make an angle of $45°$ with the line $x + 2y = 7$

Two lines make an angle of $tan^{-1}(\frac{1}{4})$ with the the line $2x + y = 3$. Find the slopes of these lines.

Area of a triangle

Find the area of the triangle with vertices $(-1, 4)$, $(3, 2)$, $(5, 3)$

The area of the triangle with vertices $(3, 0)$, $(-2, 5)$, $(2, p)$ is 5. Find the two values of p.

Find the centre and radius of these two circles:

c: $x^2 + y^2 + 6x - 8y + 12 = 0$ and s: $x^2 + y^2 - 2x + 4y - 8 = 0$

Circle s has centre $C_1 = (1, 3)$ and radius length 5. Find the equation of the circle s.

A circle has equation $x^2 + y^2 + px + qy + 43 = 0$. Given that the points $(-4, 7)$ and $(-2, 5)$ lie on the circle, find the values of p and q, and hence find the centre and radius.

A circle has endpoints of a diameter $A(-\ 3, 2k)$ and $B(5, k)$, with a centre of $(1, 3)$. Find the equation of this circle.

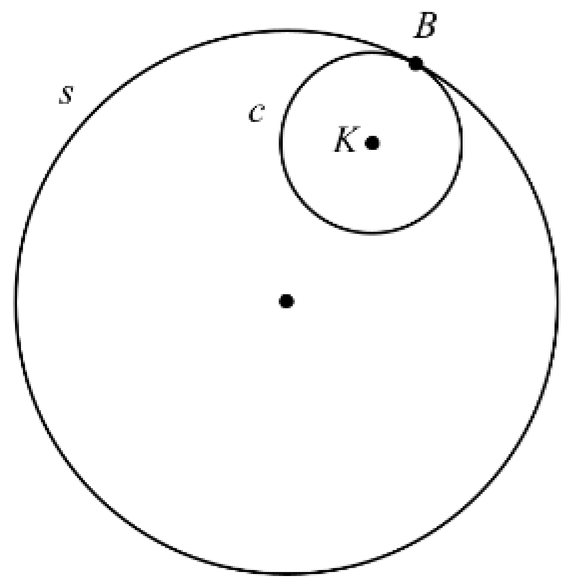
Prove that the circles c: $x^2 + y^2 + 6x - 8y + 12 = 0$ and $x^2 + y^2 - 2x + 4y - 8 = 0$ touch externally, and find the point of contact.

Two circles s and c touch internally at B as shown.

The equation of circle s is $(x - 1)^2 + (y + 6)^2 = 360$.

$B(7, 12)$.

The radius of c is one third the radius of s. Find the equation of circle c.



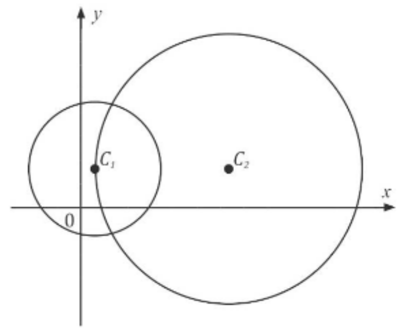
The equation of a given circle is $x^2 + y^2 = 20$. Find the equation of the tangent to the circle at the point $(9, 11)$.

A straight line passes through the point $A(4, 5)$ and is a tangent to $x^2 + y^2 + 6x - 12y + 43 = 0$ at the point B. Find $|AB|$

Find the length of the tangents from $P(-4, 0)$ to the circle $x^2 + y^2 - 4x - 8y - 30 = 0$, and hence show that one of the tangents is $x + y + 4 = 0$.

Find the equation of the tangent to the circle q at the point $(-4, 1)$

Circle s has $C_1 = (1, 3)$ and radius length of 5. A larger circle k has a diameter that is two and a half times that of s. The line joining C_1C_2 is parallel to the x-axis. Find the equation of the tangent to s at the point $(-2, 7)$, and hence investigate if this tangent is also a tangent of circle k.



The line $5x - 4y + k = 0$ is a tangent to $x^2 + y^2 - 6x - 4y - 28 = 0$. Find two possible values for k.

Circles with centres that lie on a given line

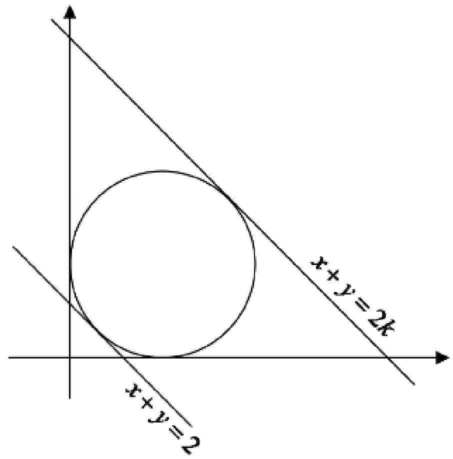
Find the equation of a circle which contains the points $(-3, 5)$ and $(7, 11)$ and whose centre lies on the line $3x - 5y = -34$.

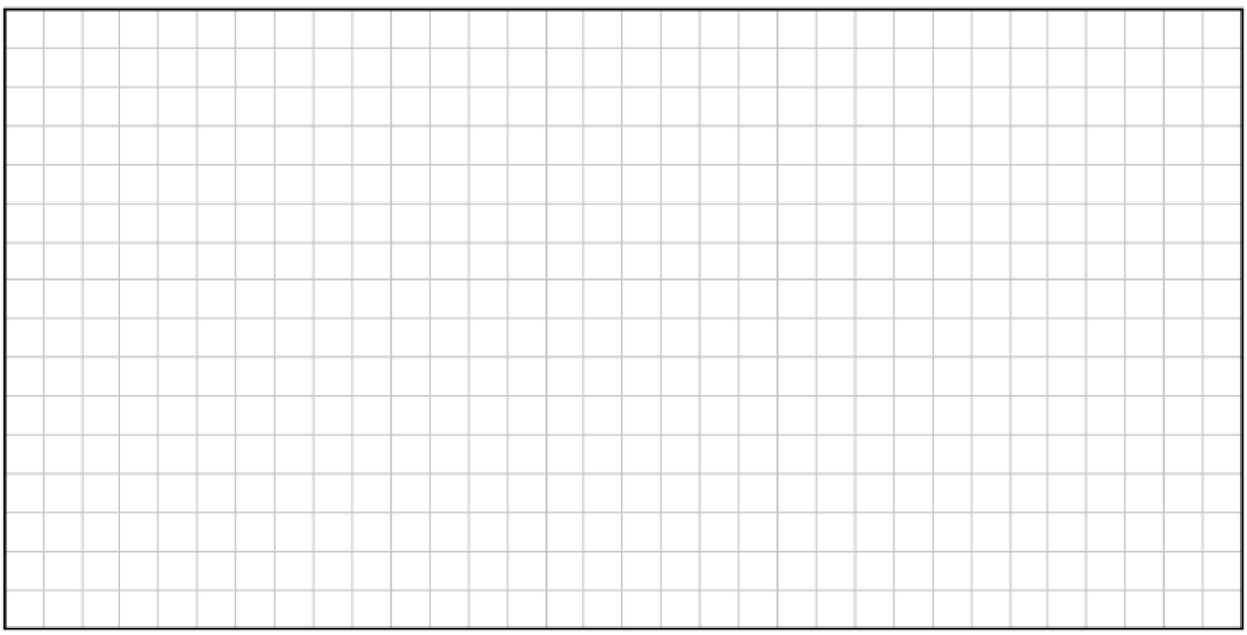
A circle of radius length 5 contains the point $(7, 8)$. Its centre lies on the line $-2x + y = -4$. Find the equations of two circles that satisfy these conditions.

x-axis and/or y-axis as tangents.

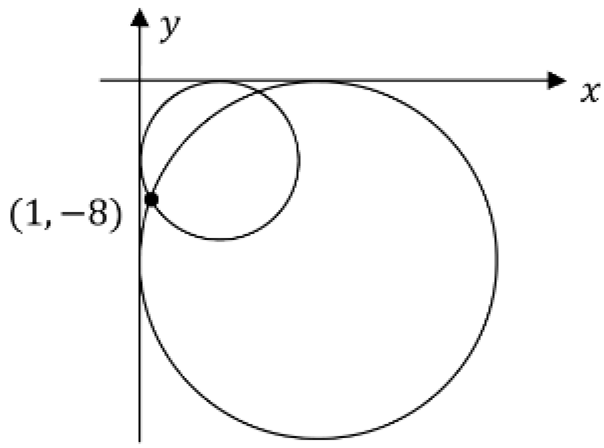
The centre of a circle lies on the line $x - 2y - 1 = 0$. The x-axis and the line $y = 6$ are tangents to the circle. Find the equation of this circle.

The circle shown in the diagram has, as tangents, the x-axis, y-axis, $x + y = 2k$ and $x + y = 2$. Find the value of k.





Two circles each have both the x-axis, and y-axis as tangents, and both contain the point $(1, -8)$. Find the equations of both of these circles.



Chapter 4

TRIGONOMETRY

S O/H C A/H T O/A

- Triangles
- Equations
- Functions
- Identities

Triangles

Is it right angled?

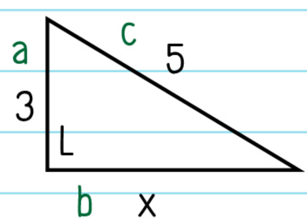
YES

- **Pythagoras**
  - $\underline{\underline{c}}^2 = a^2 + b^2$
  - Always the hypotenuse
  - Use if given 2 sides, and asked to find 3rd side
- **$S\frac{O}{H}$ $C\frac{A}{H}$ $T\frac{O}{A}$**
  - Use otherwise

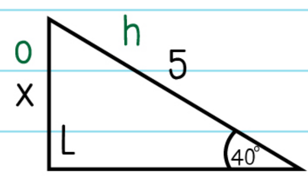
NO

- **Cosine**
  - $\underline{\underline{a}}^2 = b^2 + c^2 - 2bc\,Cos\underline{\underline{A}}$
  - a or A can be unkown
  - Use if given 2 sides that sandwich an angle.
- **Sine**
  - $\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$
  - Use otherwise

Examples :

① 

$$3^2 + x^2 = 5^2$$
$$9 + x^2 = 25$$
$$x^2 = 25-9$$
$$x^2 = 16 \; ,$$
$$x = 4$$

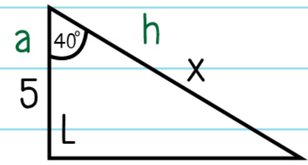
② 

$$Sin(40) = \frac{x}{5}$$
$$5Sin(40) = x$$
$$x = 3.21$$

Big letters = Angles ,
Small letters = Sides

Sin(angle) = opp/hyp
Cos(angle) = adj/hyp
Tan(angle) = opp/adj

③

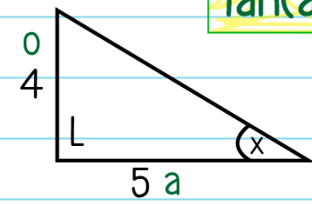


$$\cos(40) = \frac{5}{x}$$

$$x\cos(40) = 5 \rightarrow x = \frac{5}{\cos(40)}$$

$x = 6.53$

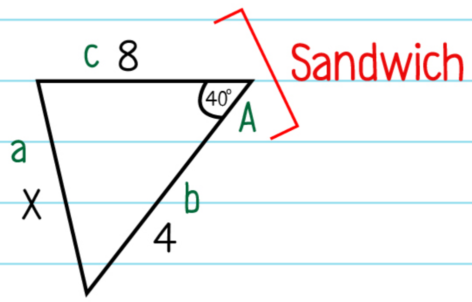
④



$$\tan(x) = \frac{4}{5}$$

$$x = \tan^{-1}\frac{4}{5} \rightarrow$$ **$x = 38.7°$**

⑤

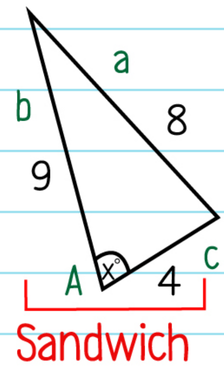


$$x^2 = 4^2 + 8^2 - 2(4)(8)(\cos(40))$$

$$x^2 = 31$$

$$x = \sqrt{31} \rightarrow$$ **$x = 5.57$**

⑥



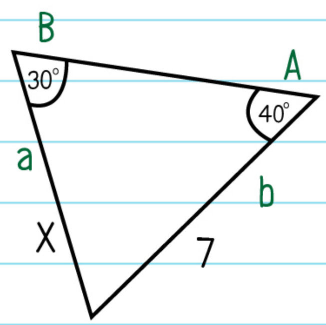
$$8^2 = 9^2 + 4^2 - 2(9)(4)(\cos(x))$$

$$8^2 - 9^2 - 4^2 = -72\cos(x)$$

$$\frac{-33}{-72} = \cos(x) \rightarrow x = \cos^{-1}(33/72)$$

$x = 62.7°$

⑦



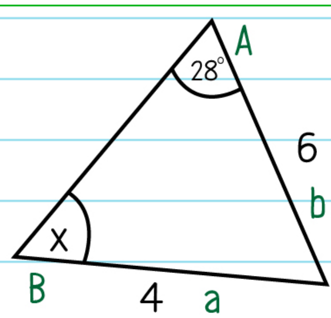
$$\frac{x}{\sin(40)} = \frac{7}{\sin(30)}$$

$$X\sin(30) = 7\sin(40)$$

$$X = \frac{7\sin(40)}{\sin(30)}$$

$x = 9$

⑧



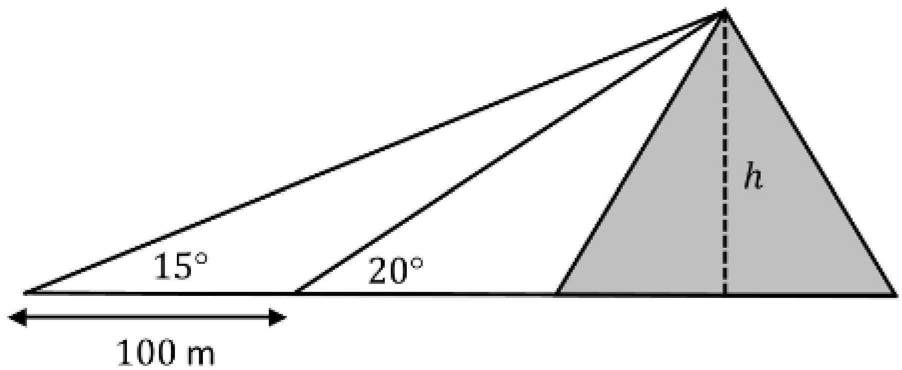
$$\frac{4}{\sin(28)} = \frac{6}{\sin(x)}$$

$$4\sin(x) = 6\sin(28)$$

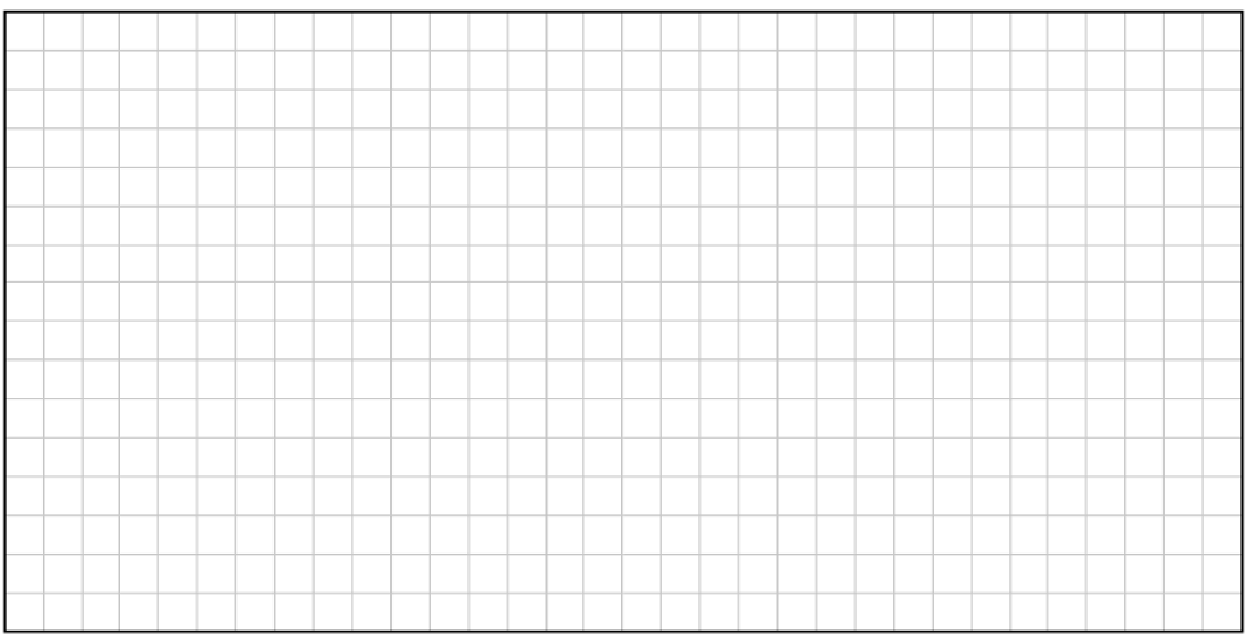
$$\sin(x) = \frac{6\sin(28)}{4}$$

$$x = \sin^{-1}\left(\frac{6\sin(28)}{4}\right)$$

$x = 69.9°$



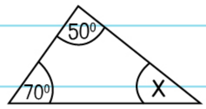
Calculate h, correct to one decimal place.

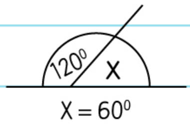


•) Extra information :

Area → If right angled : $\frac{1}{2}$ base x height

If not : $\frac{1}{2}$ ab Sin C [Sandwich]

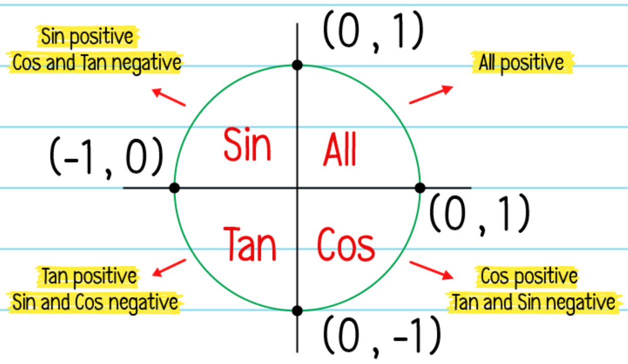
- All 3 angles in a triangle add to 180^0 e.g. 

 $x=60^0$
- A straight line adds to 180^0 e.g 

Equations and Functions

1) Equations

Tip If the question mentions $0^\circ \leq 0^\circ \leq 360^\circ$, or something similar, the question will likely use trig equations.



- Radians → degrees

$$\times \frac{180}{\pi}$$

- Degrees → radians

$$\times \frac{\pi}{180}$$

Example :

$$180^\circ \times \frac{\pi}{180} = \pi \text{ radians}$$

$$2\pi \times \frac{180}{\pi} = 360^\circ$$

2) Functions

Crosses Y-axis @ Max/Min

↑

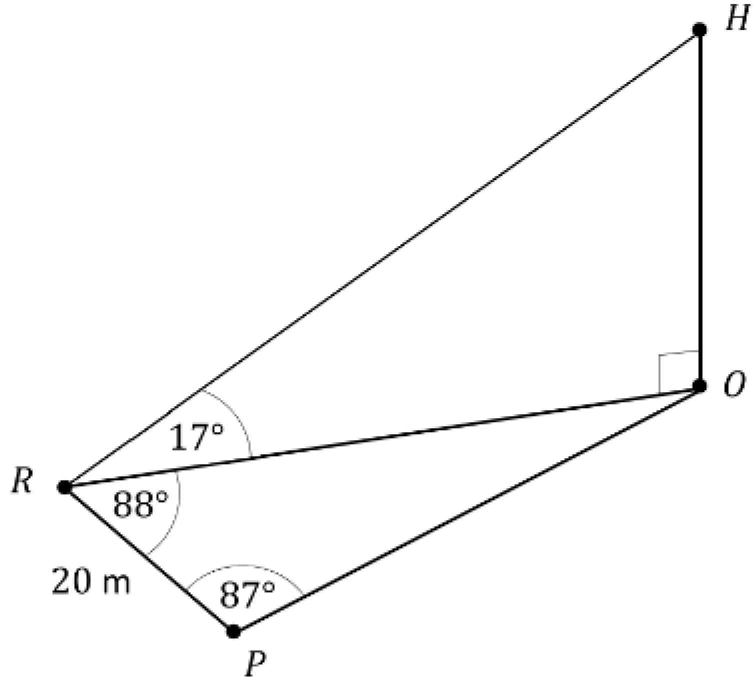
a + bCos(cx)

a + bSin(cx)

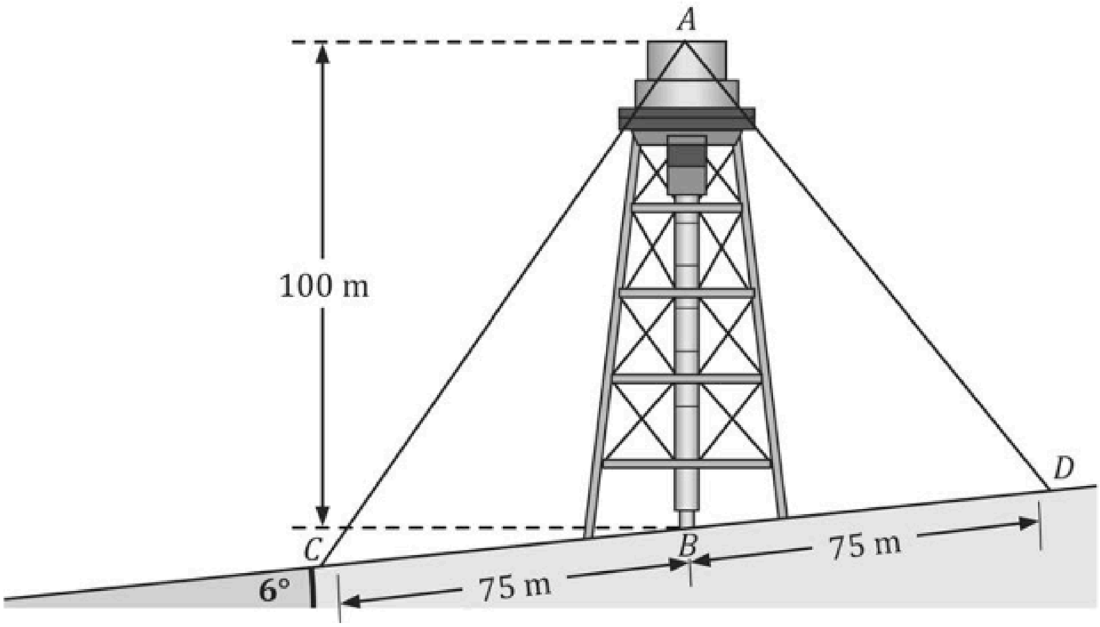
↓

Crosses Y-axis @ Midway

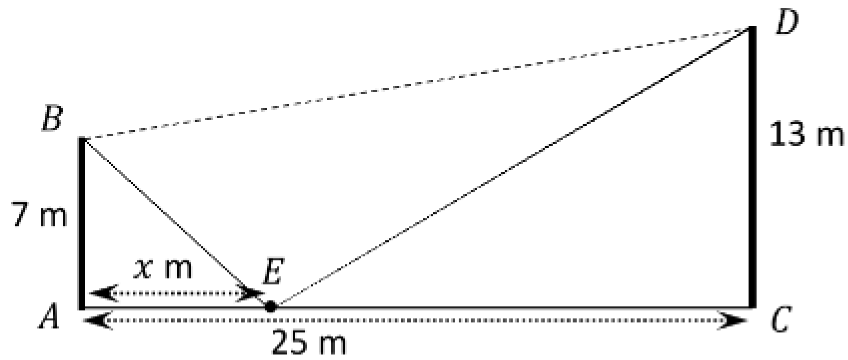
- a = Midway line
- b = amplitude → distance from middle to top
- $\frac{2\pi}{c}$ = period → one full cycle
- [a - b, a + b] = range
- Frequency = $\frac{1}{\text{period}}$



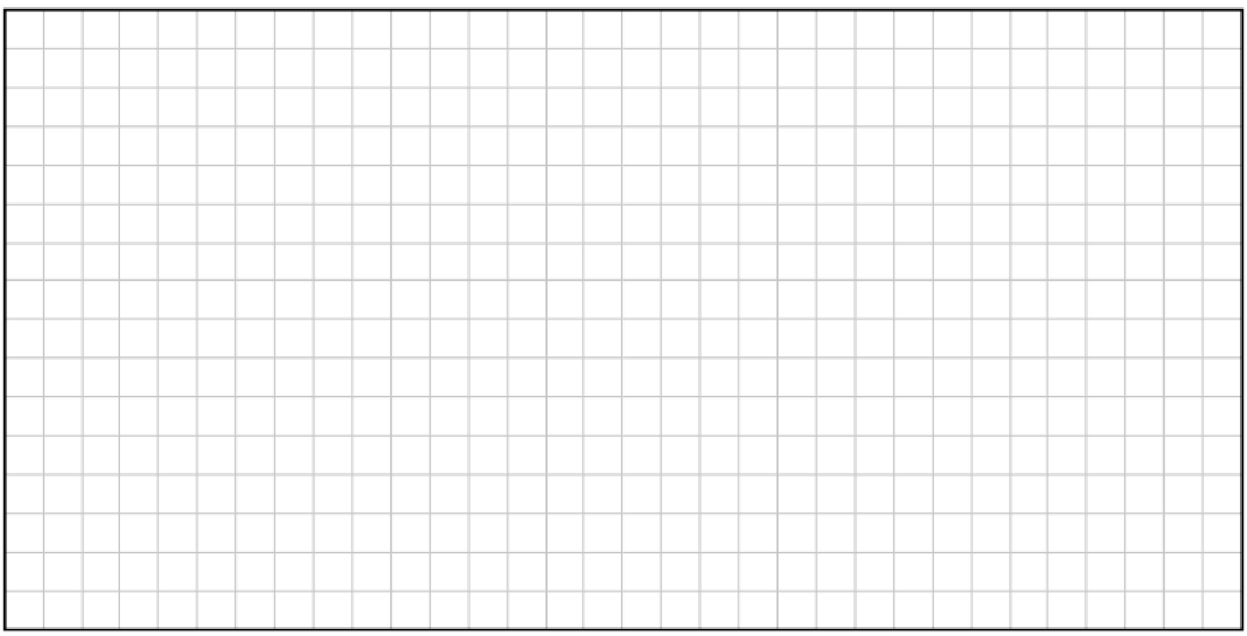
Calculate $|OH|$ correct to two decimal places.

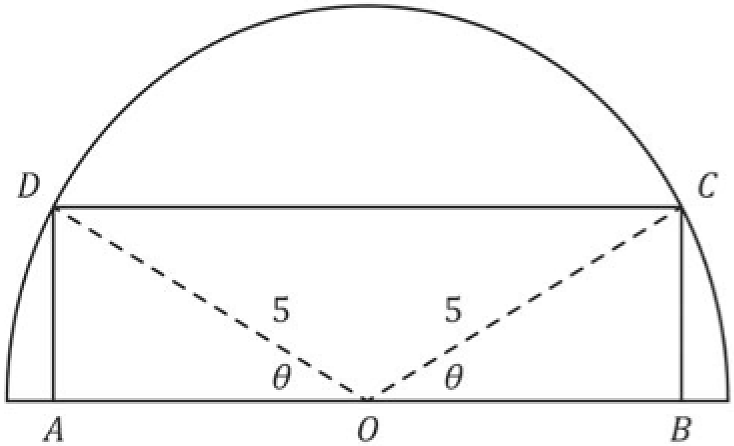


Calculate $|AC|$ and $|AD|$ correct to two decimal places.

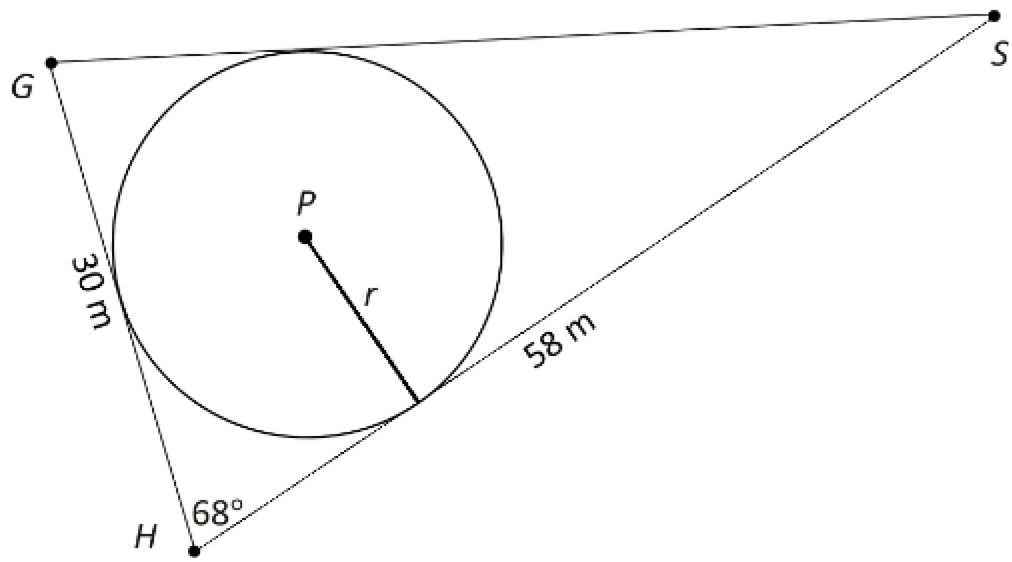


For what values of x would the triangle BED be right angled.

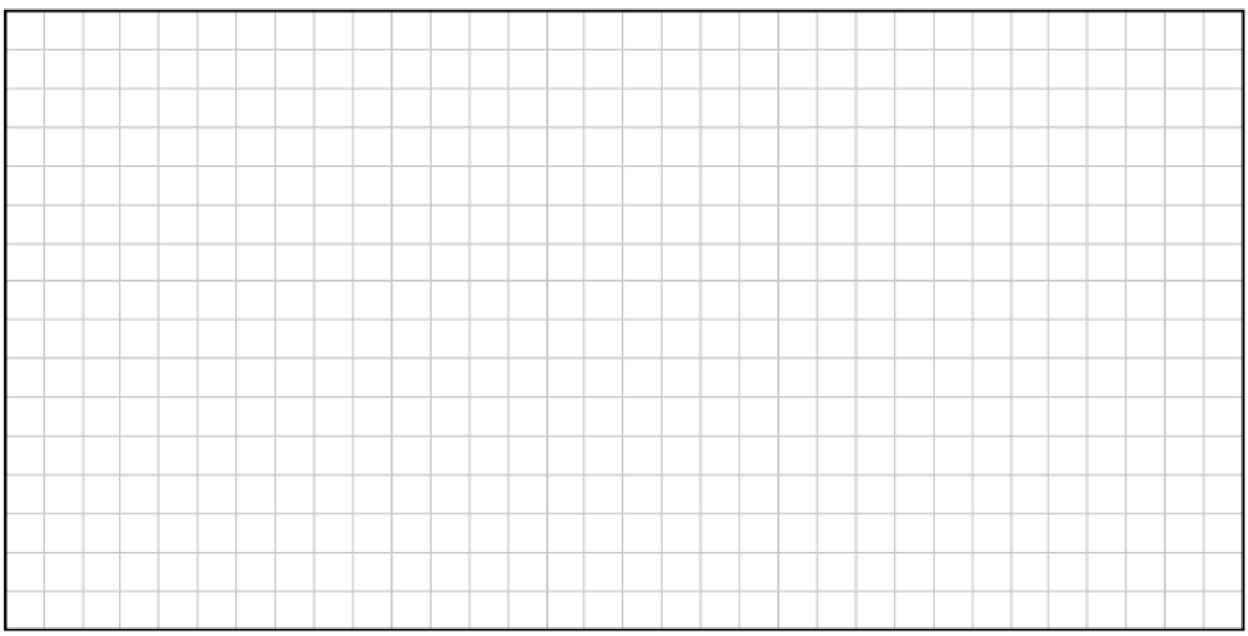


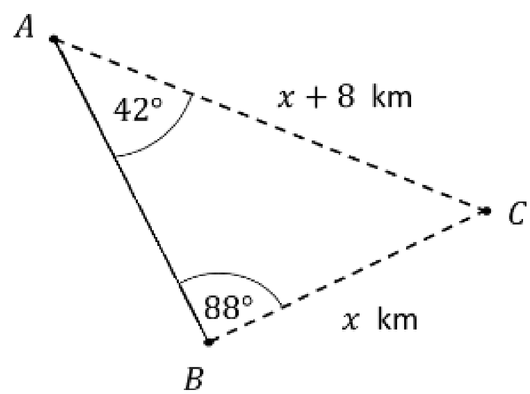


Find the perimeter of the rectangle $ABCD$ in the form $k sin\theta + j cos\theta$, where $k, j \in N, \theta \in R$.

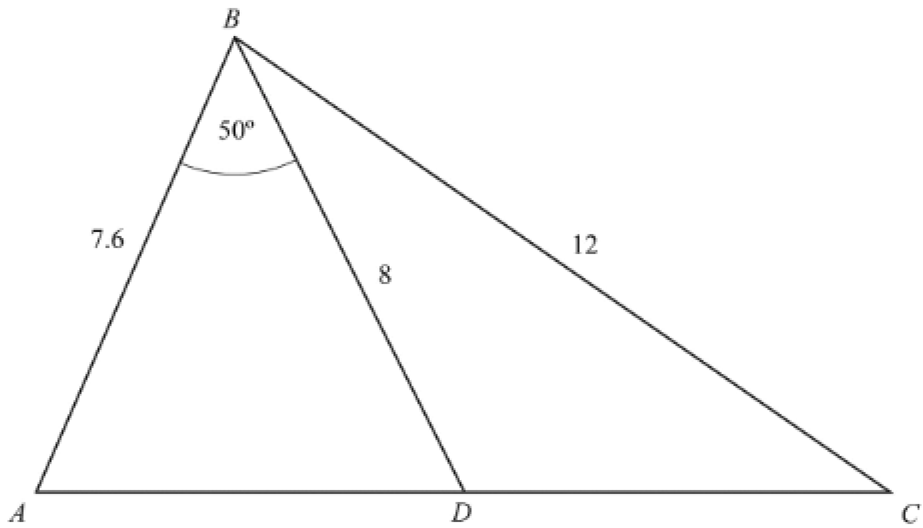


Find the value of r

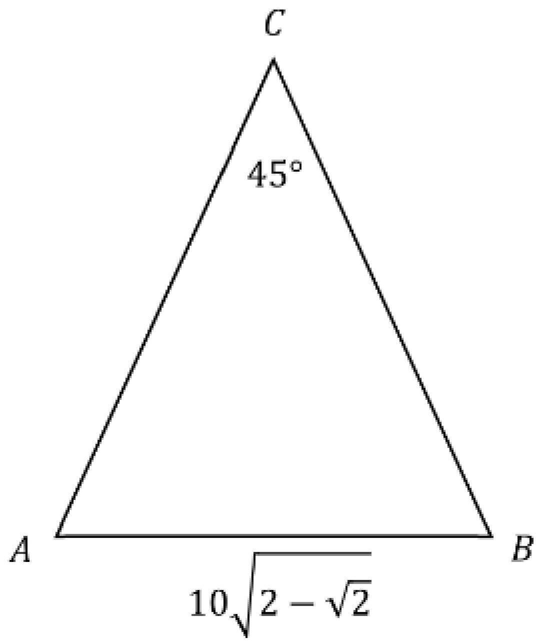




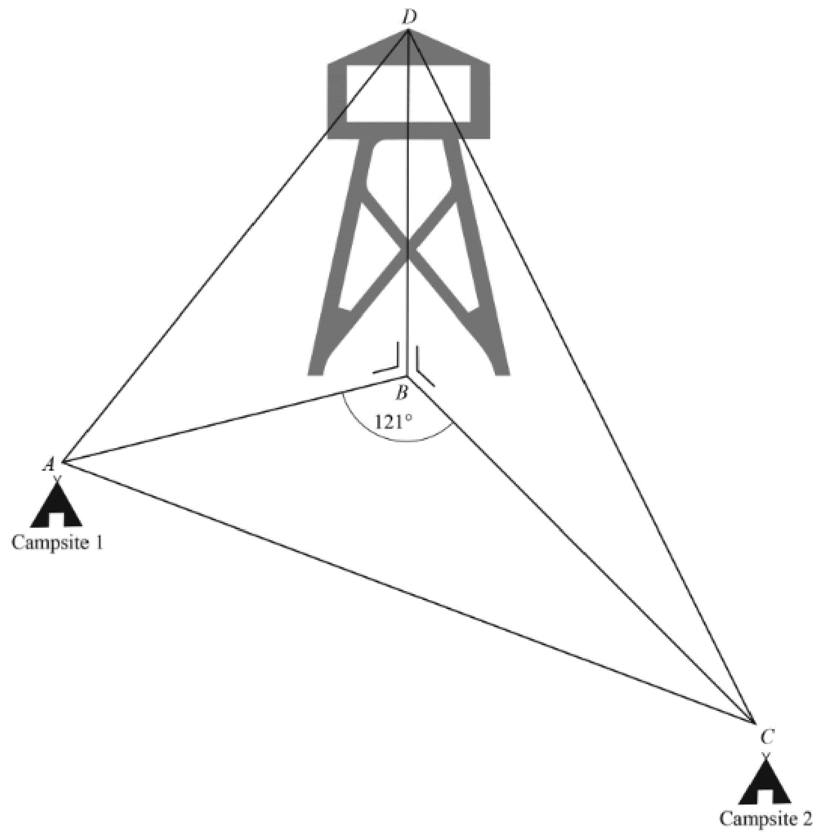
Find the value of x correct to two decimal places.



Calculate the $|\angle DCB|$ correct to the nearest degree.



ABC is an isosceles triangle. Find the length of AC.



A ranger can view two campsites from a lookout tower. The angle of depression to Camptise 1 is 37.3 degrees. The angle of depression to Campsite 2 is 18.4 degrees. Calculate the distance from Campsite 1 to Campsite 2.

Solve the equation $cos3\theta = \frac{1}{\sqrt{2}}$ for $0 \leq \theta \leq 360°$

Find the two values of θ for which $tan\frac{\theta}{2} = \frac{-1}{\sqrt{3}}$ where $0 \leq \theta \leq 4\pi$.

Solve the equation:

$$tan(B + 150°) =- \sqrt{3} \ ,$$

for $0° \leq B \leq 360°$

Derive the formula $sin2A = 2sinAcosA$ and hence solve the equation $sin2x - cosx = 0$ in the interval $0° \leq x \leq 360°$

Express $sin(3\theta) + sin(\theta)$ as a product of sine and cosine, and hence find all the solutions of

$$sin(3\theta) + sin(\theta) = 0,$$

in the interval $0° \leq x \leq 360°$.

The height of the tides on a particular beach are given by the function:

$h(t) = 7.5 - 6.5sin(\frac{\pi}{6}t)$, where $h(t)$ is in metres, and is the t time in hours from midnight. $\frac{\pi}{6}$ is expressed in radians. Find the difference between highest and lowest tides, find the period (in hours) and calculate the time at which the first high tide occurs.

A windmill turns at a constant rate, and the height at the tip of a specified blade is 20 metres above the ground at its highest point. Over a period of 15 seconds, the blade goes from its highest point (20 metres above ground) to its lowest point (2 metres above ground). Find the equation that models the height of the top of the blade above ground, taking time to start when the blade is at its highest point.

The volume of air in Daniel's lungs at any given time t, when he is resting, is given by:

$$2 - 0.4cos(\frac{\pi}{2}t).$$

When Daniel is exercising, the max capacity of his lungs is 3.6 litres. When he breathes out fully, the volume of air in his lungs is 1.3 litres. He breathes in and out twice as many times when he is exercising, as when he is resting. Find the equation for the volume of air in Daniel's lungs when he is exercising at any time t, where t is in seconds. Draw a rough sketch of this function, clearly labelling appropriate axes.

Prove that $\frac{sin\theta}{1-cos\theta} = \frac{1+cos\theta}{sin\theta}$

Prove that $cos2A = 1 - 2sin^2A$.

Prove that $sin2A = 2sinAcosA$.

Write $tan\ 15°$ in the form $\frac{\sqrt{a}-1}{\sqrt{a}+1}$, where $a \in N$.

Given that $sin\frac{\vartheta}{2} = \frac{1}{\sqrt{5}}$,

Use the formula $cos2A = cos^{2}A - sin^{2}A$ to find the value of $cos\,\theta$.

Chapter 5

FUNCTIONS

- Graphing functions
- Shifting functions
- Injective, bijective, Surjective
- Composite functions
- Inverse functions
- Complete the square

Shifting functions

Up by two units : $f(x)+2$
Down by two units : $f(x)-2$
Left by two units : $f(x+2)$
Right by two units : $f(x-2)$

Injective, bijective, surjective

Injective if : any horizontal line intersects the graph at most once.

Surjective if : every possible output value is hit by some input value

Bijective if : both injective and surjective

Inverse functions

Replace $f(x)$ with y, and rewrite the equation so that x is isolated

Example :

$$f(x) = x^2$$
$$y = x^2$$
$$x = \sqrt{y}$$
$$f^{-1}(x) = \sqrt{x} \quad \text{(inverse)}$$

Complete the Square

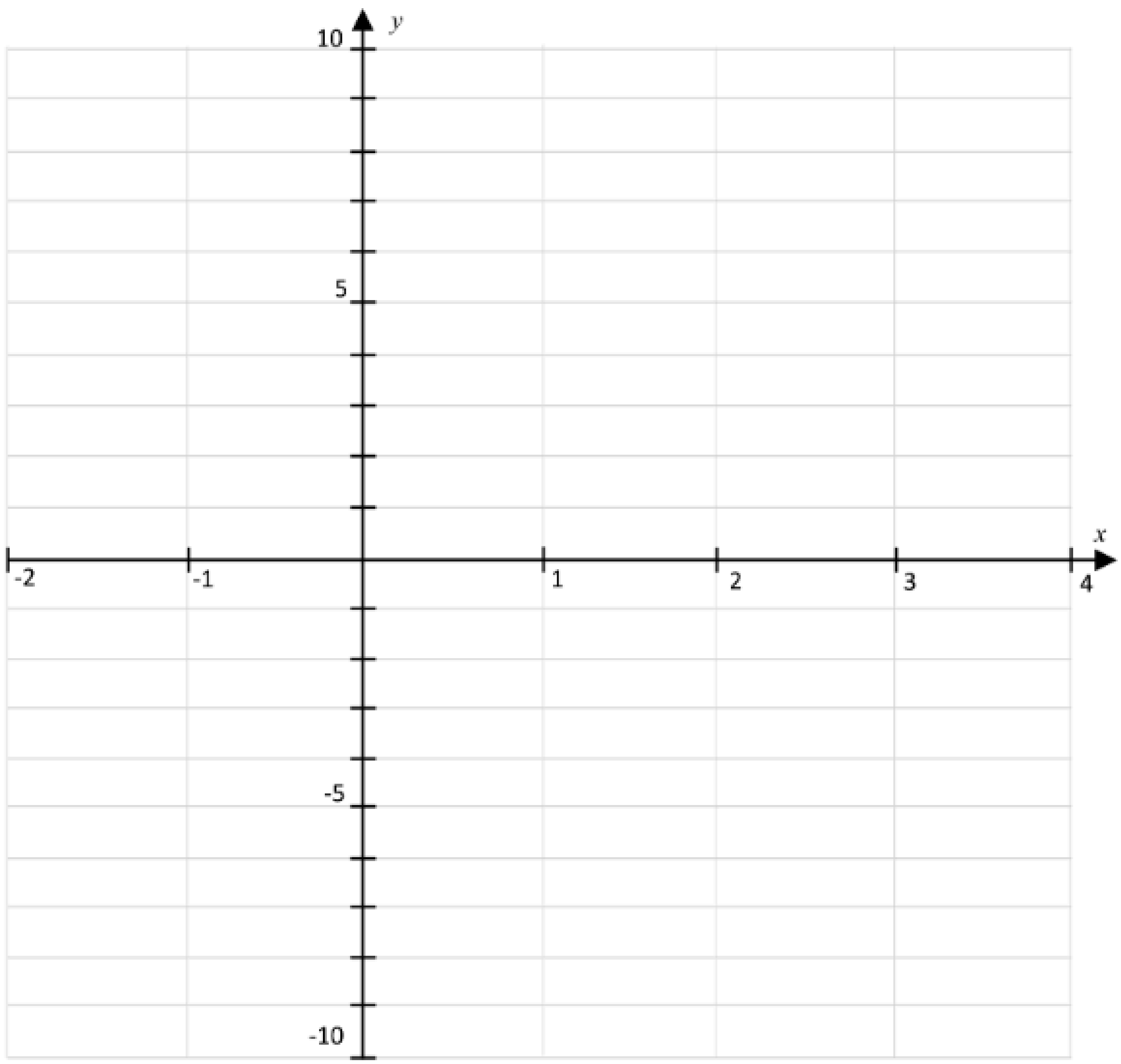
Half the coefficient of x, square it and add it and subtract it

Example : $x^2 + 4x + 7$

$$= x^2 + 4x + \left(\frac{4}{2}\right)^2 + 7 - \left(\frac{4}{2}\right)^2$$
$$= [x^2 + 4x + 4] + 7 - 4$$
$$= (x+2)^2 + 3$$
$$= (x+2) + 3$$

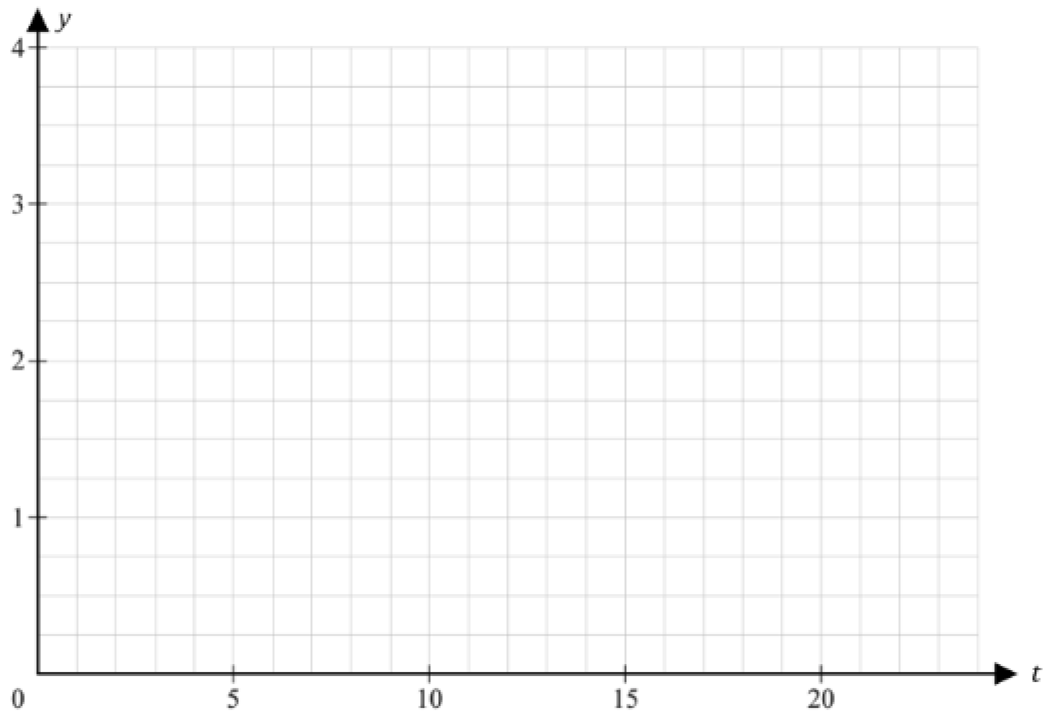
x	-2	-1	0	1	2	3	4
$h(x) = 4^x$	0·0625						
$f(x) = 2(x-1)^2 - 8$	10			-8			10

Complete the table above, and sketch the graphs of $f(x)$ and $g(x)$.



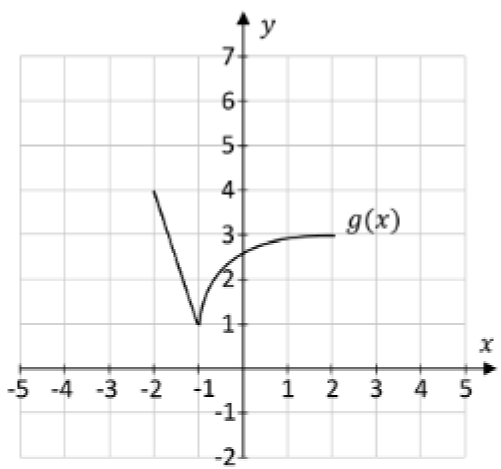
t	0	2·5	5	10	15	20
$c(t) = 15e^{-0\cdot3t} - 15e^{-0\cdot6t}$		3·74				

Complete table above and hence draw a sketch of the graph $c(t)$.

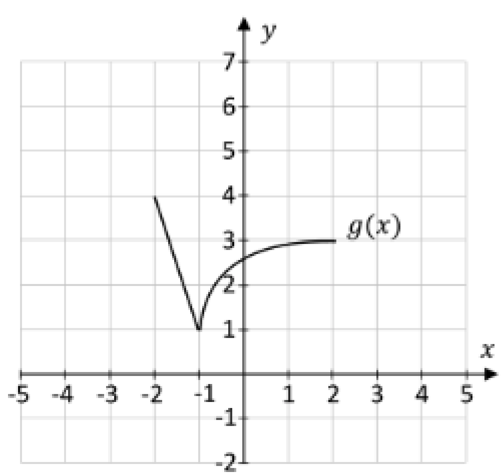


The graph of the function $2(x - 1)^2 - 8$ is shifted 2 units to the left. Write down the equation of the new graph.

Draw the graph $g(x) - 2$ below.

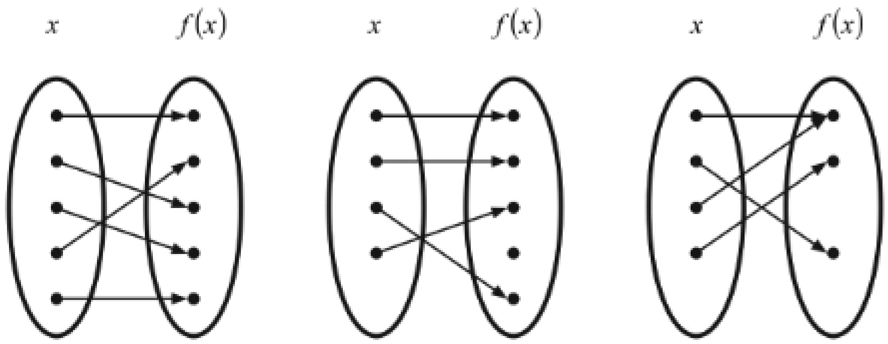


Draw the graph $g(x + 2)$ below.

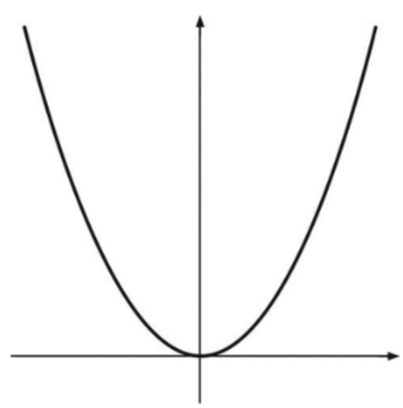


Injective, bijective, surjective

Label the following as either injective, surjective, or bijective.



Part of the graph of the function $f(x) = x^2$ is shown. State whether the function is injective, bijective, surjective, or none of these options. Explain your answer.



$h(x) = 4^x$

Show algebraically that $h(x + \frac{1}{2}) = 2h(x)$

$f(x) = 2x + 1,$ for $x \in R.$

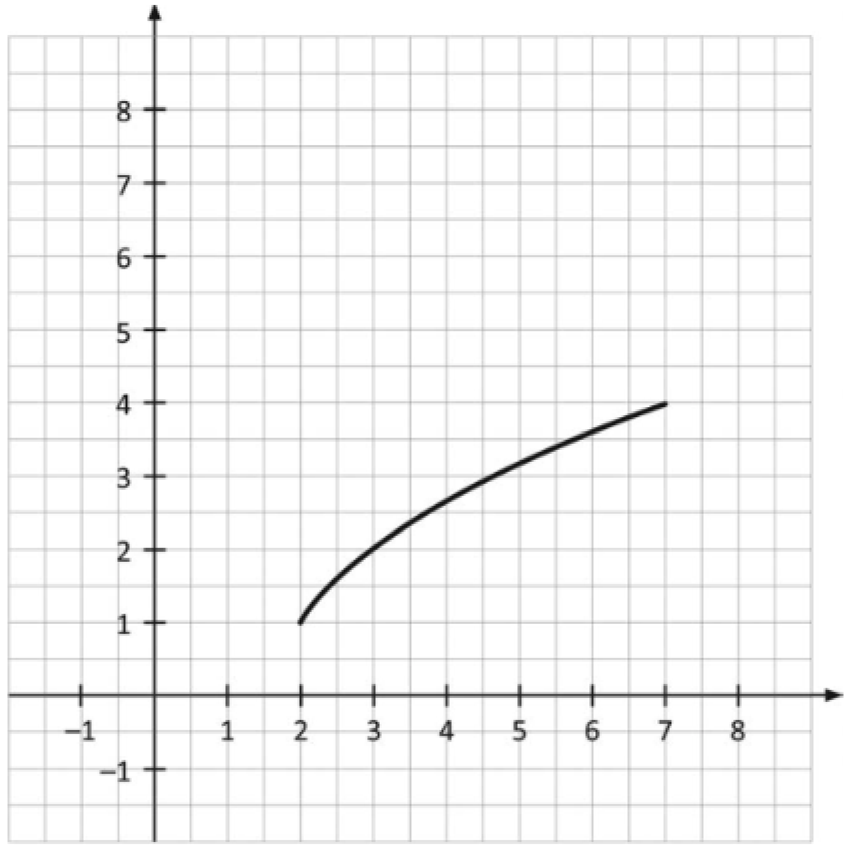
Find the value of k such that $f(x + f(x)) = kf(x)$.

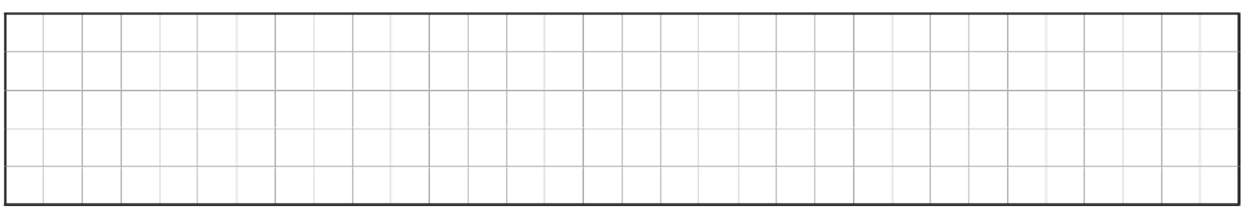
Given that $f(x) = \sqrt{3x - 5}$, find $f^{-1}(x)$.

Is $f(x)$ bijective?

Given that $f(x) = \frac{13}{5-x}$, find $f^{-1}(x)$.

Given the graph of $f(x)$ in the domain $2 \le x \le 7$ is shown below. Draw the graph of $f^{-1}(x)$ using the same scales and axes.





Complete the square

Write the function $f(x) = 2x^2 + 12x + 15$ in the form $a(x + b)^2 + c$, where $a, b, c \in Z$, and hence find the turning point of this function.

Chapter 6

PROBABILITY

- Probability Theory
- Sampling with/without replacement
- Permutation and choice
- Set Theory
- Expected Value
- Bernoulli Trials

Probabilty Theory

If one trial has m outcomes, and a second trial has n outcomes, the total number of possible outcomes is m x n

And → MULTIPLY	
Or → ADD	

Permutation and Choice

- Permutation → Arranging digits, letters in which order matters
- Choice → Selection of subjects or teams, in which order doesn't matter

Permutation

Example

In how many ways can the letters of the word COUNTER be arranged?

VS

Choice

Example

These are 7 members in the club :

Bob, Ann, Dee, Carol, Eve, Fred, Gus.

How many ways can a group of 3 be selected from the group?

Set Theory

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \setminus B) = P(A) - P(A \cap B)$$

"A less B"

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

"A given B" — Conditional Probability

A and B are independent if :

$$P(A) \times P(B) = P(A \cap B)$$

Bernoulli Trials

- 2 outcomes (success/failure)
- Fixed success rate
- Fixed number of trials

Example

$\binom{n}{r}(p)^{r}(q)^{n-r}$

$n =$ number of trials
$r =$ number of successes
$P =$ probability of success
$q =$ probabilty of failure

Jack takes 10 penalties one season.
He usually scores 80%.
What is the probability that he scores :

- (i) Exactly 6 out of 10

$n = 10 \quad r = 6 \quad p = 0.8 \quad q = 0.2$

$\binom{10}{6}(0.8)^{6}(0.2)^{4}$

- (ii) At least 8 out of 10/ 8 or more

P(8) or P(9) or P(10)

$n = 10$
$r = 8, 9, 10$
$P = 0.8$
$q = 0.2$

- (iii) At most 2 out of 10/ 2 or less

$p(0) + p(1) + P(2)$

- (iv) At most 8/ 8 or less

$1 - [P(9) + p(10)]$

- (v) His 8th penalty on his 10th attempt

$n = 9$
$r = 7$
$P = 0.8$
$q = 0.2$

$\binom{9}{7}(0.8)^{9}(0.2)^{2} \times 0.8$

A restaurant has five starters, nine main courses, and 6 desserts. How many different ways are there to order a 3-course meal?

How many different ways are there to order a meal, if you decide to get two starters, and one dessert, instead of a main course?

A circular spinner has 12 sectors as follows:

- 5 sectors labelled $6
- 3 sectors labelled $9
- The rest are labelled $0

In a game, the spinner is spun once. The spinner is equally likely to land on each sector. What is the probability that a player gets a $6, then a $9, followed by a $6, the first 3 times that they play.

If 4 students are selected at random from a class, what is the probability that all 4 were born in the same month?

There are b boys and g girls in a class, where $b, g \in N$.

$\frac{3}{5}$ of the students in the class are girls. 4 boys and 4 girls join the class. Now $\frac{4}{7}$ of the class are girls. Find the value of b and the value of g.

When John rings to Conor's house, the probability that Conor answers the door is $\frac{1}{5}$. If John rings to Conor's house for 7 consecutive days, what is the probability that Conor answers on the 2^{nd}, 4^{th} and 6^{th} days, but not on the other days.

When Michael takes penalties, he usually scores 80%. Find the probability that he scores his first 2 penalties, and misses his 3^{rd} in a season.

Sampling with/without replacement

A bag contains 5 red, 4 blue, and 6 white marbles. Marbles are drawn out of the bag and not replaced. What is the probability that the first red marble is the third marble drawn?

Two balls are taken at the same time, randomly, from a bag containing three black, three yellow, and three red balls. What is the probability that neither are red?

Separately, what is the probability that at least one is red?

Three cards are drawn at random, without replacement, from a pack of 52 playing cards. Find the probability that the 3 cards are aces. Separately, find the probability that two cards are black, and one is a diamond.

A bag contains 15 coloured discs: 6 blue discs, 4 red discs, 3 yellow discs, and 2 green discs. Four discs are chosen at random. What is the probability that the four discs are the same colour? Separately, what is the probability that the four discs are all different colours?

Twelve discs, numbers 1 to 12 are placed in a bag. Three discs are drawn at random without replacement. What is the probability that the number 9 is drawn? What is the probability that the 3 numbers drawn are even?

4 students are taken from a class containing 12 boys and 8 girls. What is the probability that the first 3 students selected will be boys, and the fourth will be a girl?

Jack needs to pick a new PIN code. It must be a 4-digit code. The code can contain numbers from 1 to 9, however, no digit can be used more than once.

Work out how many codes are possible?

How many of these codes contain the digit 2?

In how many ways can the letters of the word EDUCATION be arranged if each letter is only used once?

In how many of these arrangements will the letters A,E,I,O,U come together?

In how many of these arrangements will the letters A,E,I,O,U come together, and in this order?

In how many ways can the letters of the word HISTORY be arranged, if the H has to come first, and the vowels are together?

A security code consists of six digits chosen at random from 0 to 9. Digits may be repeated. How many codes will end with 0?.

How many codes will contain the digits 2018 and in this order?

A committee of 4 is to be chosen from 6 men and 4 women. How many different committees can be chosen? On how many committees would there be an equal number of men and women?

How many ways can a jury of 12 people be selected from a panel of 8 men and 8 women? Find the probability that one such jury contains more women than men.

300 runners take part in a road race, where each runner has a number from 1 to 300. No two runners have the same number. Two runners are picked at random from this race. What is the probability that the sum of their number is 101.

An examination is made up of 2 sections; Section A and Section B. In Section A, there are 7 questions. You must answer Q1. and 3 other questions. In Section B there are 8 questions, and you must answer 4 of them. How many different combinations of questions can be answered?

How many different teams of 15 players can be selected from a panel of 20, if the captain must be included?

How many different teams of 10 can be selected from a panel of 15 if two players refuse to play on the same team?

In a competition, Celine has a probability of $\frac{1}{20}$ of winning, a probability of $\frac{1}{10}$ of coming second, and probability of $\frac{1}{4}$ of coming third. First prize gets \$9000, second prize gets \$7000 and third prize gets \$3000. In all other cases, she gets nothing. Celine pays \$2000 to enter. Find the expected value of her loss.

A multiple choice test has 15 questions. For each question, 5 possible answers are given, of which, only 1 is correct. If a candidate selects the right answer, they get 10 points. If they select the wrong answer, they lose 2 points. Candidates must answer all questions. Find the expected score of a candidate that guesses every question.

The table below shows the prizes, in euro, a player can win in a game, as well as the probability of winning that prize. It costs \$10 to play the game once, and the game is fair, meaning the expected value of winnings, minus the cost of playing is \$0. Find the value of x.

Prize (€)	None	2	$x - 10$	x
Probability	30%	40%	28%	2%

John bought a car a number of years ago. The table below gives an estimate of the probability that each of the following three events happen to John's car in the next year.

Event	Probability
Head gasket blows	0·095
Timing belt goes	0·041
Air filters break	0·073

If the head gasket blows, it'll cost £20,000. If the head gasket is replaced now, it will cost £1450, and the probability it will blow in the next year is reduced to 0.005. Work out whether it is worth fixing the head gasket now or not.

$P(A) = 0.7,\ P(B) = 0.5,\ P(A \cap B) = 0.3.$

$(i)\ P(A \cup B)$

$(ii)\ P(A|B)$

(iii) State whether A and B are independent events.

Two events A and B are such that $P(A) = 0.2$, $P(A \cap B) = 0.5$ and $P(A' \cap B) = 0.6$.

(i) Draw a Venn Diagram to represent this information.

(ii) Find $P(A|B)$.

(iii) State whether A and B are independent events.

Two events A and B are such that $P(A|B) = 0.5$, $P(B|A) = 0.3$ and $P(A \cap B) = \frac{1}{7}$.

Find $P(A \cup B)$.

Two independent events F and S are such that:

$P(F \backslash S) = 0.25$, $P(S \backslash F) = x$, $P(F \cap S) = 0.2)$. Find the value of x.

Two events A and B are such that $P(A) = 0.75$, $P(A \cap B) = 0.5$.

(i) $P(B|A)$

(ii) $P(A \cup B) = \frac{11}{12}$. State whether A and B are independent events.

The attendance at a local rugby match is dependent on the weather. The probability of a large crowd attending if it is raining is 0.35. The probability of rain on a match day is 0.3 What is the probability of rain and a large crowd attending.

In a random sample it was found that 55% of respondents were male and 45% were female. 75% of the males surveyed and 62.5% of the females surveyed read the local newspaper. If a person is chosen at random, what is the probability that the person is female, given that the person reads the newspaper.

In a restaurant, an average of 3 out of every 5 customers order water with their meal. A random sample of 10 customers is selected.

(i) What is the probability that exactly 6 of these customers order water with their meal?

(ii) What is the probability that at least 8 of these customers order water with their meal?

(iii) What is the probability that the ninth customer is the fifth one to order water?

If 3 coins are tossed, what is the probability of getting exactly 2 tails and 1 head? If 3 coins are tossed 8 times, what is the probability of getting 2 tails and 1 head, exactly 3 times?

The probability of an archer hitting a target is $\frac{4}{7}$.

(a) If the archer fires 9 arrows, calculate the probability that their fifth hit will occur on the ninth shot.

(b) If the archer fires 9 arrows, what is the probability that he hits the target at least 7 times?

Chapter 7

STATISTICS

- Normal Distribution
- Hypothesis Test
- Confidence Interval
- Calculator Work

Normal Distribution

If the question mentions data that is normally distributed,

Use this :

$$z = \frac{x - u}{\sigma}$$

$$z = \frac{\bar{x} - u}{\frac{\sigma}{\sqrt{n}}}$$

Use if sample size is given

u = Mean

σ = Standard deviation

n = Sample size

$\bar{x}$ / x = Question number

Confidence Interval

95% confidence interval

$$\bar{x} \pm 1.96 \left(\frac{\sigma}{\sqrt{n}}\right)$$

Use if given σ

$\bar{x}$: Mean of new sample

or

$$\hat{p} \pm 1.96 \left(\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$$

$\hat{p}$: Proportion of new sample

Use otherwise

Hypothesis Test

[5% level of significance]

H_0 : Nothing happens (there is no effect or difference)

H_1 : Something happens (there is effect or difference)

Construct confidence interval

IF	OR
If original value lies Inside the conf. Int, Fail to reject H_0	If original value lies Outside the conf. Int, reject H_0

P-Value

1) Form Z score from previous part of question

2) Find 1 - [p (z < Z Score)]

3) Multiply your answer by 2

4) If >0.05 Fail to reject H_0
<0.05 Reject H_0

Calculator Work

Mean → Average

Mode → Most frequent

Median → Middle term of an ordered list

Range → Biggest - smallest

Interquartile Range → $Q_3 - Q_1$

First Quartile (Q1): The value that separates the lowest 25% of the data from the rest

Third Quartile (Q3): The value that separates the lowest 75% of the data from highest 25%.

Q3-Q1 = Interquartile range

Be able to find :

- σ
- u
- r
- $\bar{X}$, $\bar{Y}$
- Line of best fit

On your calculator

On a farm, the weights of chickens are normally distributed, with a mean weight of 78.6kg and a standard deviation of 5.03kg.

(a) Find the probability that a randomly selected chicken will weigh less than 82.2kg.

(b) Find the probability that a randomly selected chicken will weigh less than 75.2g.

(c) Find the probability that a randomly selected chicken will weigh between 81.6kg and 76.8kg

(d) 20% of chickens weigh less than t kg. Find the value of t correct to two decimal places.

Josh and Karen's school are running standardised tests, where the results are normally distributed, with a mean score 62 and a standard deviation of 13.

(a) Josh scored 77 on the test. Investigate if this places him in the top 10% of the country?

(b) The students who scored in the top 4% nationwide will receive a book token. What is the minimum whole number score required in order to be awarded a book token?

Sorcha ran two different marathons. The table below gives her finishing time for the two marathons. For each marathon, the finishing times were normally distributed.

Sorcha's position in each marathon was based on her finishing time. Sorcha came 5265th in the Windy marathon.

Sorcha's finishing time for both marathons was the same.

Estimate Sorcha's position in the Sunny marathon.

	Mean finishing time (minutes)	**Standard deviation of finishing times** (minutes)	**Number of runners**	**Sorcha's position**
Windy Marathon	254	38	6000	5265th
Sunny Marathon	247	29	2000	

In the Rugby World Cup, it is widely believed that the average number of tries scored per match is 5.9, with a standard deviation of 1.8 tries. To investigate if this claim still holds, a sample of 36 matches from the 2020 Rugby World Cup is selected, and the mean number of tries scored per match is found to be 6.4.

(a) Form a 95% confidence interval using the above information

(b) Conduct a hypothesis test at 5% level of significance to determine if there is enough evidence to suggest that the mean number of tries scored per match has changed.

(c) Calculate the p-value for this hypothesis test and interpret this value in the context of the question.

Across the country, test results are normally distributed with a mean score of 62 and a standard deviation of 13. Josh's school scored a mean of 62. The principal wanted to improve the school's performance and so during the following year the school ran some extra preparation classes prior to the standardised test. The principal then took the scores of the 200 students who had sat the test in his school and he found that they had a mean of 64. Investigate to a 5% level of significance if there is evidence to suggest there has been a change in the school's performance.

For a particular game, players in Ireland's scores are approximately normally distributed with a mean of 3.87 and a standard deviation of 0.36. A random sample of 64 Galway players have a mean score of 3.74. Based on this, a local newspaper claims that Galway players have a different mean score to players in Ireland. Use this information about the sample to construct a 95% confidence interval for the mean score of all Galway players. Use a standard deviation of 0.36 in your calculations.

Carry out a hypothesis test at the 5% level of significance to test the newspaper's claim that Galway players have a different mean score to players in Ireland. State your null hypothesis, state your alternative hypothesis, and state your conclusion. Give a reason for your conclusion.

The speeds of 150 randomly selected cars were recorded as they passed a checkpoint on a motorway. The mean speed of the cars was 115 kilometres per hour and the standard deviation was 24 kilometres per hour. The speed limit on the motorway is 112 kilometres per hour.

(a) Test the hypothesis, at the 5% level of significance, that the mean speed of cars passing the checkpoint is greater than this speed limit. State the null hypothesis and alternative hypothesis. Give your conclusion.

(b) Find the p-value for the mean speed of cars passing the checkpoint. Comment on what can be concluded from its value, in a two-tailed hypothesis test at the 5% level of significance in relation to the speed of the cars.

(c) Find the smallest sample size for which the result could be regarded as significant at the 5% level.

The National Lottery claims that 42% of adults in Ireland play the Lottery weekly. A competitor lottery company wants to test this claim. They surveyed 1000 people at random and found that 408 of them played the National Lottery weekly. Use this information to test the National Lottery's claim at a 5% level of significance, clearly stating the null and alternative hypothesis.

Calculator work

Patient	A	B	C	D	E	F	G	H	I	J	K	L
Cotinine level, x	160	390	169	175	125	420	171	250	210	258	186	243

(a) Calculate the mean and standard deviation of the above information.

(b) Find the median, upper quartile and lower quartile of this data.

A magician is testing the likelihood of pulling certain colour cubes from a bag, given that there are 5 red cubes, 5 green cubes, and 5 blue cubes.

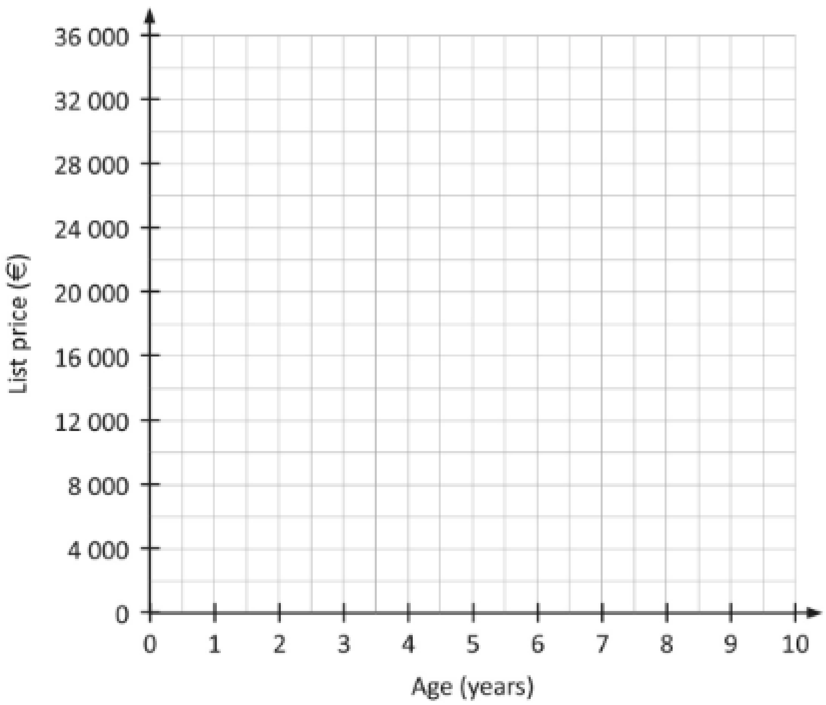
Trial	A	B	C	D	E	F	G
Number of red cubes	0	3	2	2	4	5	1
Number of green cubes	4	2	0	3	0	0	2
Number of blue cubes	1	0	3	0	1	0	2

(a) Calculate the mean and standard deviation of the number of red cubes per trial.

(b) Work out the correlation coefficient (r) between the number of red cubes, and the number of green cubes per trial.

Age (yrs)	1	2	3	3	3	4	4	5	6	7	8
List price (€) (thousands)	32·5	26	24	21·5	20	20	17·5	19·5	13	11·5	11·5

(a) Display this data in a scatter plot.



(b) Work out the correlation coefficient (r)

(c) Find the coordinates of the point where the mean of each variable lies. Plot this point, and hence or otherwise, find and draw the line of best fit.

Chapter 8

SEQUENCES AND SERIES

$T_n = ar^{n-1}$

3, 5, 7

Arithmetic

a = 1st term
d = Common difference

2, 5, 8, 11

a = 2
d = 3

Common difference

$$T_2 - T_1 = T_3 - T_2$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Use when question says total

Geometric

a = 1st term
r = Common ratio

2, 6, 18, 54

a = 2
r = 3

Common ratio

$$\frac{T_3}{T_2} = \frac{T_2}{T_1}$$

$$T_n = ar^{(n-1)}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

Use when question says total

$$S_\infty = \frac{a}{1-r}$$

"Infinity/Indefinitely"
Goes on forever
When $|r| < 1$

The steps of a ladder decrease uniformly in length. The bottom step is 88cm and each successive step is 2.75cm shorter than the previous one. If there are 13 steps on each ladder, what is the length of the top step?

Sean bought some roller blades and he is keen to practise as much as he can. On the first evening after work, he rollerblades 6 km. Each evening he increases this by 1.25 km more than the previous evening. If his first day of rollerblading was on October 1st, what was the total distance rollerbladed by the end of October (31 days)?

The first term of an arithmetic series is a and the common difference is d. The 18th term of the series is 25 and the 21st term of the series is 32.5. Use this information to find the value of a and to find the value of d.

An athlete prepares for a race by completing a practice run on each of 11 consecutive days. On each day after the first day, he runs further than he ran on the previous day. The lengths of his 11 practice runs form an arithmetic sequence with first term a and common difference d. He runs 9 km on the 11th day and runs a total of 77 km over the 11-day period. Find the value of a and the value of d.

The first three terms of an arithmetic sequence are as follows:

$$5e^{-k},\ 13,\ 5e^{k}$$

By letting $y = e^{k}$, show that:

$$5y^{2} - 25y + 5 = 0$$

Paul and his teammates are doing sprint training in an Athletics Club. He sets a cone 8 metres from the start point and places a further 23 counts in a line 4 metres apart. He sprints from the start to the first cone, touches it and sprints back to the start. He then sprints to the second cone, touches it and sprints back to the start; he continues this until he has touched all cones. Calculate total distance Paul will cover in one round of his training.

$p,\ p + 7, p + 14,\ p + 21...$ is an arithmetic sequence, where $p \in N$.

By finding T_n, find the smallest value of p for which 2021 is a term in this sequence.

Justin noticed a plant on the sun deck of a house. The height of the plant was 95 cm, and each week he noticed that it grew upwards by another 4%. Calculate the height of the plant at the end of week 10 correct to the nearest cm .

A geometric series is as follows: $lnx, \; lnx^2, \; lnx^4, \; lnx^8, \; ...$

Find r, the common ratio, and hence find x, if $T_8 - T_6 = 45$.

Show by using an infinite geometric series that $0.\overline{18}$ can be written as $\frac{2}{11}$

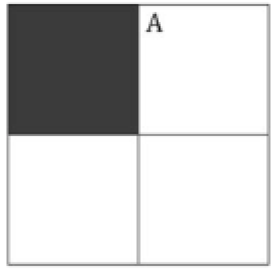


Diagram 1

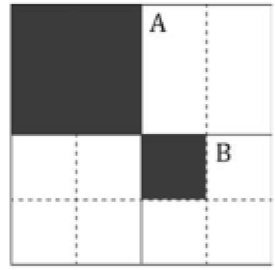


Diagram 2

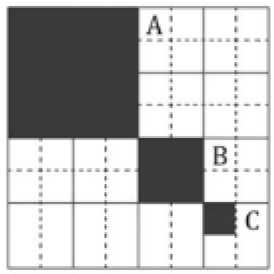


Diagram 3

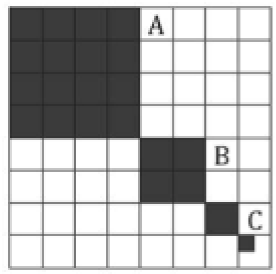


Diagram 4

The diagram above shows the first four squares in a sequence of squares which are subdivided in half. The area of the shaded square A is $\frac{1}{4}$ square units.

(i) Show that the areas of the squares A, B, C and D are in geometric progression.

(ii) Find the total area shaded in the 8th diagram of this sequence

(iii) The dividing and shading process illustrated is continued indefinitely. Find the total area shaded.

A depth charge testing facility collects data on how far the shock wave of an explosion travels in each second after the explosion. They collect the following observations:

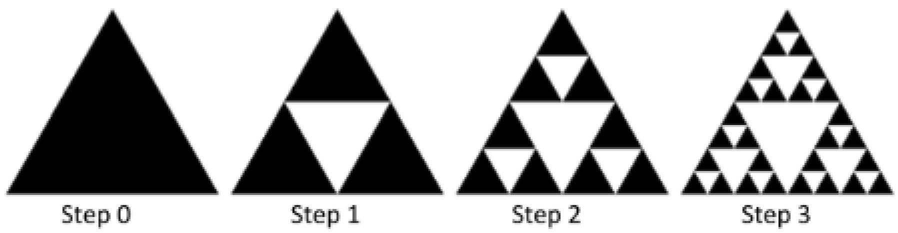
Time (seconds)	1	2	3	4
Distance travelled in the previous second (metres)	777·6	518·4	345·6	230·4

(i) Find the total distance travelled by the wave in the first 5 seconds.

(ii) In which second does the wave travel less than 100m for the first time?

The first 3 terms of a geometric sequence are x^2, $5x - 8$ and $x + 8$ where $x \in R$.

Use the common ratio to show that $x^3 - 17x^2 + 80x - 64 = 0$.



This sequence begins with a black equilateral triangle. Each step is formed by removing an equilateral triangle from the centre of each black triangle in the previous step, as shown. Each equilateral triangle that is removed is formed by joining the midpoints of the sides of a black triangle from the previous step.

(i) Write an expression in terms of n for the number of black triangles in step n of the pattern.

(ii) Step h is the first step of the pattern in which the number of black triangles exceeds 1 billion for the first time. Find h.

(iii) Step k is the first step of the pattern in which the the fraction of the original triangle remaining is less than $\frac{1}{100}$ of the original triangle. Find the value of k.

(iv) What fraction of the original triangle is remaining after an infinite number of steps.

Chapter 9

€ FINANCIAL MATHS $ £

From AER to monthly

Example :

24% AER → monthly

$(1+i)^{12} = 1.024$

$i = \left[\sqrt[12]{1.024} - 1\right] \times 100$

$= 0.1978\ \%$

$= 0.001978$

From AER to weekly

3.6% AER → weekly

$(1+i)^{52} = 1.036$

$i = \left[\sqrt[52]{1.036} - 1\right] \times 100$

Example :

$= 0.068\ \%$

$= 0.00068$

From monthly to AER

0.2 % Monthly → AER

Example :

$(1+0.002)^{12} = 1.024$

$= 2.4\ \%$

Compound Interest

$F = P(1+i)^t$

Investing money
Putting money into a bank
Paying into a pension
Is it a once off deposit, or are we continually depositing?

If continuous : Geometric Series → $\frac{a(1-r^n)}{1-r}$

Present Value

$P = \frac{F}{(1-i)^t}$

The amount of money that needs to be invested now to reach a specific amount in the future, considering a given interest rate over a certain number of years

Geometric Series if :

- Paying off a loan early
- Paying off a credit card/mortgage as a lumpsum

$\frac{a(1-r^n)}{1-r}$

Depreciation

When something loses value over time

Amortisation

pg 31 log tables

Calculating repayments at equal intervals

AER →monthly/weekly and vice versa

Find the monthly interest rate that is equivalent to an annual percentage rate (APR) of 5.53%. Leave your answer correct to two decimal places.

Find the annual percentage rate (APR) that is equivalent to a monthly interest rate of 0.526%. Leave your answer correct to two decimal places.

Calculate the weekly rate that is equivalent to an AER of 7.5%, correct to two decimal places.

Calculate the quarterly rate that is equivalent to an APR of 6.8%, correct to two decimal places.

Sean puts €3000 in a savings account. Interest is added annually at a rate of 2.4% per year. Work out the amount in Sean's account after five years correct to the nearest cent

Jack puts money in a savings account with an interest rate of 2.4% per year. After six years, Jack has €4,000 in the account. Work out how much Jack put into the account initially, correct to the nearest cent.

Write down the present value of a future payment of €30,000 in one year's time, at an interest rate of 4%, correct to the nearest cent.

If you borrow €1000 for one year at an interest rate of 12% per year compounded quarterly, how much do you owe at the end of the year?

Dan buys a car for €54,000 which drops in value each year. The dealership advised Dan that the car will have a value of €39,015 at the end of two years. Assuming the annual percentage loss remains constant, find the annual depreciation rate and hence deduce the value of the car at the end of the first, third and fourth years.

Connor decides to invest 20,000 in the stock market for five years, withdrawing it on his 30th birthday. He hopes that his investment will increase by 5% per annum. If the investment achieves the growth rate of 5% per annum, what will the investment be worth at the end of the five years, correct the nearest euro?

Mark buys a house for €280,000. It is estimated that in 10 years time the house will have a value of €350,000. Find i, the expected rate of increase over this time.

Paul breeds and sells rabbits for a living. He is planning for his retirement by contributing to a retirement fund. He will invest €500 on each birthday from age 25 to 64 inclusive. That is, he will make 40 contributions to the fund. The retirement fund pays interest on the Investments at a rate of 8% per annum, compounded annually. How much money will be in Paul's fund on his 65th birthday?

Dan has won a prize in a lottery game. When he goes to collect his prize, he is offered the following option:

Receive a payment of €2,200 at the beginning of each month for 25 years, starting immediately.

The bank is offering a rate of interest which corresponds to an annual equivalent rate of 2.8%. Dan allows the monthly repayments to build up in the bank account over a six month period. Find the amount in the bank account at the end of the six months

Ron wants to put the same amount of money in a savings account at the start of each month for 36 months so that, at the end of three years, he will have a total of €12,000 in the account. Interest is calculated at a rate of 0.11% per month. Find how much he has to invest each month so that there will be a total of €12,000 in the account after three years.

Connor decides that when he turns 25 years old he will start saving €500 per month, lodging the savings on the first day of each month. He will continue his regular savings until his 30th birthday. He will not make a lodgement on the day of his 30th birthday. His bank will offer an annual rate of interest on a regular savings of 2.5%. Find, to the nearest euro, the value of his savings after five years.

James is 30 years old and plans to retire when he is 65 years old. He contributes €700 at the start of each month to a pension fund earning 4% AER. The pension will pay a lump sum on his retirement date. Find out how much James will receive, correct to the nearest Euro as a lump sum when he retires.

Dan won the Lotto. He decided to receive his winnings over a 25 year period, with €2200 being deposited into his account every month, for 6 months. At the end of these 6 months, he requests to have the remaining monthly payments paid immediately as a lump sum. Based on an AER of 2.8%, calculate how much Dan would expect to receive as the lump sum.

Jack wants to buy a car. He agrees on a four-year loan, with monthly repayments of €336.90 that include interest of 0.6%. After three years, Jack gets a bonus in work and decides to use the money to pay the remaining balance of the loan in order to save himself paying interest on the last year. Calculate how much Jack needs to pay to clear the loan after the three years.

A couple agree to take out a mortgage of €350,000, at a rate of 0.3% per month, in order to purchase a new home. This loan is to be paid back monthly over 25 years with the repayments due at the end of each month. The amount of each repayment is €1771. After exactly 11 years of repayments, the couple receive a financial windfall. They decide to repay the remaining balance on the mortgage. Find out how much the couple will need to repay in order to clear their mortgage entirely.

A couple agrees to take out a €250,000 mortgage in order to purchase a new home. The loan is to be paid back monthly over 25 years with the repayments due at the end of each month. The bank charges an annual percentage rate which is equivalent to a monthly rate of 0.287%. Using the amortisation formula, or otherwise, find the couple's monthly payment on the mortgage. Give your answer correct to the nearest cent.

Mark and Ashley buy a house for €280,000. If they pay 15% of the house off from their savings, and they take out a mortgage for the rest, work out their monthly repayments, given that the mortgage is 25 years at an interest rate of 0.36% per month.

After 35 years of work, John will receive €632,045 as a lump sum from his pension. John decides to invest his lump sum at 3% APR. He receives a regular payment at the end of each month for the next 25 years. Calculate the value of his monthly payment correct to the nearest euro.

Alex has a credit card debt of €5,000. The APR charged on the debt by the credit card company is 21.75% fixed for the term of the debt. Assume Alex pays the fixed monthly repayment each month and does not have any further transactions on that card. Complete the table below to show how the balance of the debt of €5,000 is reducing each month for the first three months, assuming an APR of 21.75% charged and compounded monthly. Using the amortisation formula, find out how long it would take to pay off a credit card debt of €5,000 using this repayment method. [The amount of each repayment is 2.5% of the original debt].

Payment number	Fixed monthly payment, €A	€A		New balance of debt (€)
		Interest	Previous balance reduced by (€)	
0				5000
1			42·50	4957·50
2				
3				

Chapter 10

INDUCTION

$n = k + 1$ $\quad 1^2 + 2^2 + \ldots n^2$

- Divisibility
- Series
- Inequality

1) Prove true for base case

2) Assume true for $n = k$

3) Prove true for $n = k + 1$

4) Conclusion

Prove by induction that $8^n - 3^n$ is divisible by 5 for all $n \in N$.

Prove by induction that $4^n + 6n - 1$ is divisible by 3 for all $n \in N$.

Prove using induction that 8 is a factor of $7^{2n+1} + 1$, for all $n \in N$.

Prove by induction that $2^{3n-1} + 3$ is divisible by 7 for all $n \in N$.

Prove by induction that $n(n + 1)(2n + 1)$ is divisible by 6 for all $n \in N$.

Prove using induction that:

$$1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Prove using induction that:

$$1 + 2 + 3 + \ldots\ldots + n = \frac{n(n+1)}{2}$$

Hence or otherwise prove that the sum of the first even natural number is given by $n^2 + n$.

Prove by induction that $n^2 \geq 2n + 3$, for $n \geq 3$.

Chapter 11

COMPLEX NUMBERS

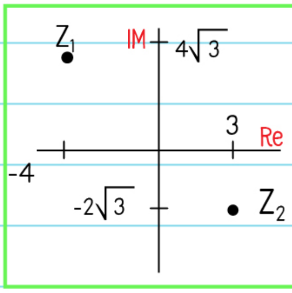
$Z = a + bi$ $\quad i = \sqrt{-1}$ $\quad i^2 = -1$

$i = \sqrt{-1}$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$

Example:

i^{57}
$= (i^4)^{14} \times i^1$
$= 1 \times i$
$= i$

i^{67}
$= (i^4)^{16} \times i^3$
$= 1 \times -i$
$= -i$



$Z_1 = -4 + 4\sqrt{3}i$

$Z_2 = 3 - 2\sqrt{3}i$

i) $Z_1 - Z_2$

$$-4 + 4\sqrt{3}i - (3 - 2\sqrt{3}i)$$
$$= -7 + 6\sqrt{3}i$$

ii) $Z_1 Z_2$

$$(-4 + 4\sqrt{3}i)(3 - 2\sqrt{3}i) \quad (-1)$$
$$= -12 + 8\sqrt{3}i + 12\sqrt{3}i - 24i^2$$
$$= 12 + 20\sqrt{3}i$$

iii) $\overline{Z_1}$ and $\overline{Z_2}$

$Z_1 = -4 + 4\sqrt{3}i \qquad Z_2 = 3 - 2\sqrt{3}i$

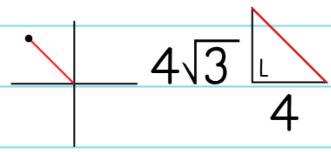
$\overline{Z_1} = -4 - 4\sqrt{3}i \qquad \overline{Z_2} = 3 + 2\sqrt{3}i$

iv) $\dfrac{Z_1}{Z_2}$

$$\frac{-4 + 4\sqrt{3}i}{3 - 2\sqrt{3}i} \times \frac{3 + 2\sqrt{3}i}{3 + 2\sqrt{3}i}$$

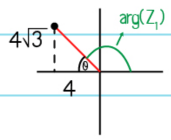
Fraction? Multiply top and bottom by conjugate of bottom

v) $|Z_1|$



$$(4)^2 + (4\sqrt{3})^2 = |Z_1|^2$$
$$|Z_1| = 8$$

vi) $\arg(Z_1)$



$$\tan^{-1}\left(\frac{4\sqrt{3}}{4}\right) = 60^\circ$$
$$180^\circ - 60^\circ = 120^\circ$$

vii) Put Z_1 into polar form

$$8\left(\cos(120^\circ) + i\sin(120^\circ)\right)$$

viii) Complex Quadratics

$z^2 - (\text{SUM of roots})z + (\text{products of roots})$

If Z is a root, $\overline{Z}$ is also a root

Tip

ix) De Moivre

ANYTIME we see indices in Complex numbers → De Moivre's theorem

Normal

Example: $Z_1 = -4 + 4\sqrt{3}i$

Find $(Z_1)^3$

$Z_1 = 8(\cos(120^\circ) + i\sin(120^\circ))$

$(Z_1)^3 = [8(\cos(120^\circ) + i\sin(120^\circ))]^3$

$= 8^3(\cos(120^\circ \times 3) + i\sin(120^\circ \times 3))$

$= 512(\cos(360^\circ) + i\sin(360^\circ))$

+360n

Example: $Z_1 = -4 + 4\sqrt{3}i$

$W^3 = Z_1$

Find the solutions of W

$W^3 = 8(\cos(120^\circ) + i\sin(120^\circ))$

$W = 8(\cos(120^\circ) + i\sin(120^\circ))^{1/3}$

$= 8^{1/3}\left(\cos\left(\frac{120^\circ+360n}{3}\right) + i\sin\left(\frac{120^\circ+360n}{3}\right)\right)$

$= 2(\cos(40^\circ+120n) + i\sin(40^\circ+120n))$

$n = 0$
$n = 1$
$n = 2$
sub these in

continue

+360n → "Solve" "Find the Solutions" "Find the Roots"

Normal → "Rewrite" "Find"

$w = d + 5i$ and $z = 3 - 4i$. Find the value of d if $wz = 38 - 9i$.

Find the real numbers p and q such that:

$$2(p + iq) + i(p - iq) = 5 + i$$

If $z = 1 + 1$ and $w = \frac{1}{z} + i$, find the exact value of $arg(w)$.

$w_1 = a + ib$ and $w_2 = c + id$.

Prove that $(\overline{w_1 w_2}) = (\overline{w_1})(\overline{w_2})$

Given that $z\bar{z} = 18$, where z is a complex number in the form $a + bi$, write an equation in terms of a and b

If $z = 2 + 3i$, write $\frac{26}{z}$ in the form $a + bi$.

$(\sqrt{3} - i)^9$ can be written in the form $a + bi$. Find the value of a and the value of b.

$z = \sqrt{3} + 1$. Find z^2 in the form $a + bi$.

Given that $z = 2 + 2\sqrt{3}i$, find z^4 in the form $a + bi$.

$z = 1 - \sqrt{3}i$. Write $(1 - \sqrt{3}i)^6$ in rectangular form.

De Moivre's theorem: $+\ 360n$

Solve the equation: $z^4 = -\ 2\ -\ 2\sqrt{3}i$. Leave your answers in polar form.

Find the roots of $z^3 = -\ 8$. Give each answer in the form $a\ +\ bi$.

$v = 2 - 2\sqrt{3}i$

Find the two possible values of w, where $w^2 = v$

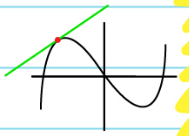
$3 + 2i$ is a root of $z^2 + pz + q = 0$, where $p, q \in R$.

Find the value of p and the value of q.

$(1 + i)$ is a root of the equation $z^2 + (-2 + i)z + 3 - i = 0$. Find its other root.

Solve the equation $z^3 - 2z^2 + 5z + 26 = 0$

Chapter 12



DIFFERENTIATION

- Rules
- Rates of change
- Applications
- 1st Principles

Rules + Examples

Chain

U′ is the derivative of U

- $\sin(U) \rightarrow \cos(U) \times U'$
- $\cos(U) \rightarrow -\sin(U) \times U'$
- $\cos^{-1}\left(\frac{U}{a}\right) \rightarrow -\frac{1}{\sqrt{a^2 - U^2}} \times U'$
- $\sin^{-1}\left(\frac{U}{a}\right) \rightarrow \frac{1}{\sqrt{a^2 - U^2}} \times U'$

Example:

$3\sin(2x^3) \rightarrow 3\cos(2x^3).6x^2$

$= 18x^2\cos(2x^3)$

$-4\cos(2x) \rightarrow 4\sin(2x).2$

$= 8\sin(2x)$

- $\ln(U) \rightarrow \frac{U'}{U}$

Example: $\ln(3x^2) \rightarrow \frac{6x}{3x^2}$

- $e^U \rightarrow e^U.U'$

Example:

$20e^{-3t^2} \rightarrow 20e^{-3t^2}.-6t \quad = -120te^{-3t^2}$

Example: $(5x+6)^4 \rightarrow 4(5x+6)^3 \times 5$

$= 20(5x+6)^3$

Product

Two terms being multiplied

Quotient

Two terms being divided

Applications

$f'(x) > 0$ function is increasing

$f'(x) < 0$ function is decreasing

$f'(x) = 0$ Stationary point (Max/Min)

$f''(x) > 0$ Minimum point

$f''(x) < 0$ Maximum point

$f''(x) = 0$ Point of inflection

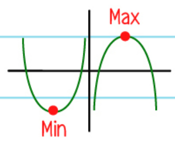
If you sub X into the original function, you shold be outputted Y

If you sub X into derivative function, you shold be outputted the slope of a tangent @ X

Max/Min Problems

If it is a *trig function*, use [a-b, a+b] from chapter 4.

If it is a *quadratic function*, it will either have a max or a min. It Cannot have both. Thus there is no need for the 2nd derivative test.



Otherwise

1) Find 1st derivative
2) Let it = 0 and solve
3) Find the 2nd derivative
4) Sub solutions from Step 2) into the 2nd derivative

$f''(x) > 0$ Min
$f''(x) < 0$ Max ✱

5) Sub that value back into the original function if necessary

Rates of Change

Rate means $\frac{?}{dt}$

Volume is changing at a rate of...

→ $\frac{dv}{dt}$

Surface area changes at a rate of...

→ $\frac{dA}{dt}$

Radius is increasing at a rate of...

→ $\frac{dr}{dt}$

$V = \frac{4}{3}\pi r^3$	$o = g^4$
$\frac{dv}{dt} = 4\pi r^2$	$\frac{do}{dg} = 4g^3$

$\frac{dr}{dt} = \frac{dr}{*} \times \frac{*}{dt}$

✱ =something

$\frac{dy}{dx} = \frac{dy}{*} \times \frac{*}{dx}$

1st Principles

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Given that $y = x cos 2x,\ x \in R,\ find\ \frac{dy}{dx}$.

$c(t) = 15e^{-0.3t} - 15e^{-0.6t}$.

Find $c'(t)$.

$P(t) = t^2 ln(3t) + 6$. Find $P'(t)$.

$f(x) = \frac{1}{5x^2+7}$

Find $f'(x)$.

Find $g'(\frac{\pi}{2})$, where $a, b \in R$ if $g(x) = (tan(\frac{x}{2}))(ln\, x)$.

$y = (2x^2 - 5x + 2)(e^{-x})$

Find $\frac{dy}{dx}$.

$v = 1.2sin(\frac{4\pi}{3}t)$

Find $\frac{dv}{dt}$.

$y = (5x + 1)^7$

Find $\frac{dy}{dx}$.

$y = ln\sqrt{\frac{5x}{x-2}}$, find $\frac{dy}{dx}$.

$g(x) = x^2 - \frac{1}{x}$.

Find $g'(x)$.

Differentiate $cos^{-1}(\frac{x}{4})$ with respect to x.

Differentiate $cos^{-1}(\frac{4}{x})$ with respect to x.

There are two points on $f(x) = 2x^3 - x^2 + 2x - 13$ where the slope of a tangent to the curve is 10. Find the coordinates of these two points.

A heated metal ball is dropped into a liquid. As the ball cools, its temperature, T, t minutes after it enters the liquid ,is given by: $T(t) = 400e^{-0.05t} + 25$, $t \geq 0$. Find the rate at which the temperature of the ball is decreasing at the instant when $t = 50$.

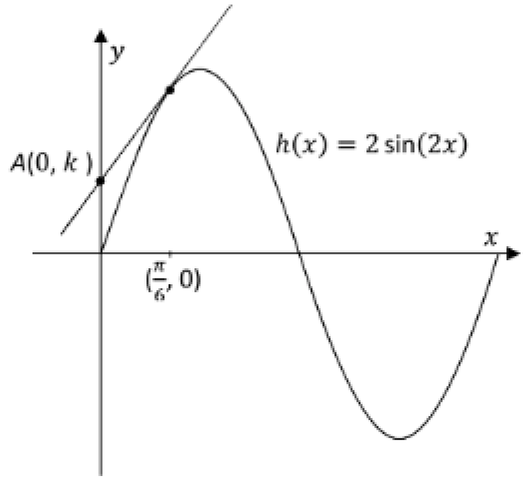
$N(t) = \frac{300}{1+6.5e^{-0.814t}}$, gives the number of trout in a lake at any time t. Find, in terms of t, the rate at which the number of trout in the lake is increasing.

Fiona is driving on a motorway. She passes point A on the motorway. Her speed is given by: $v(t) = \frac{2}{3}t^3 - 6t^2 + 13t + 109$, where v is her speed in km/hr t mins after passing the point A, for $0 \le t \le 5$. Work out Fiona's acceleration (that is, the rate at which her speed is increasing), 5 minutes after she passes A.

$V = 2 - 0.4cos(\frac{\pi}{2}t)$ gives the volume of air in Olga's lungs at any time t, where t is in seconds.

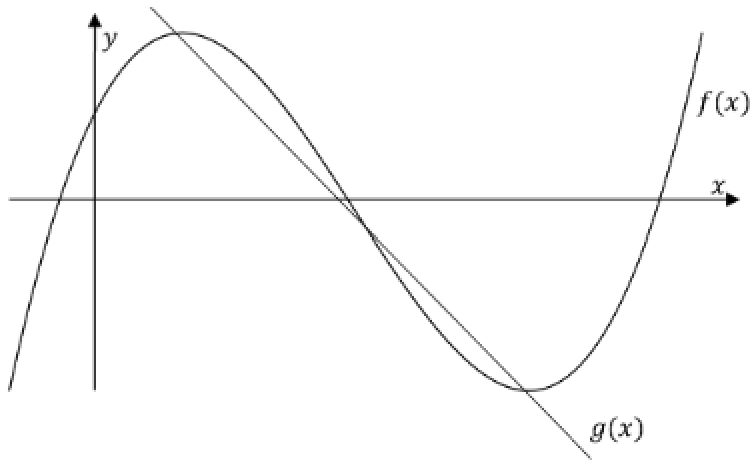
Find out whether the volume of air in her lungs is increasing or decreasing half a second after $t = 0$.

$T(t) = 75e^{-0.081t} + 20$, gives us the temperature of coffee t minutes after being brewed. Find, correct to one decimal place, the temperature the coffee has reached when $T'(t) = -4.05$, where $T'(t)$ is the rate at which the coffee is cooling per minute.



The diagram above show a tangent line to $h(x) = 2sin(2x)$, where $0 \le x \le \pi$, at the point where $x = \frac{\pi}{6}$. Find the value of k, the point where the tangent crosses the y-axis.

Applications: stationary points, max/min and points of inflection



$f(x) = x^3 - 9x^2 + 15x + 8$, where $x \in R$.

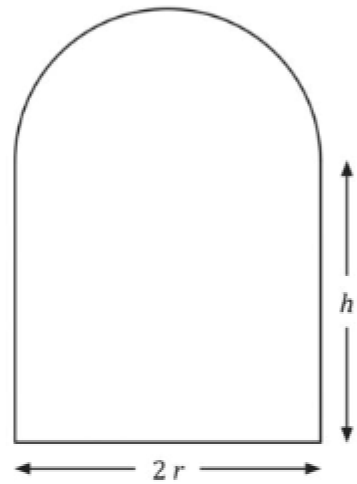
The line $g(x)$ passes through the two turning points of $f(x)$. Find the equation of $g(x)$ and hence verify whether $g(x)$ contains the point of inflection of $f(x)$.

$y = (2x^2 - 5x + 2)(e^{-x})$.

Find the turning points of this function.

$g{:}\rightarrow 4x^3 - 40x^2 + 77x$.

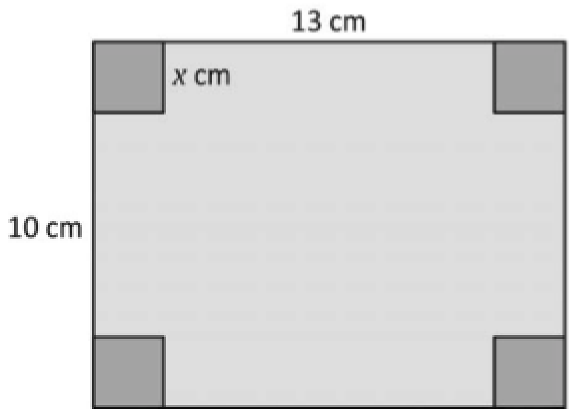
Use calculus to find the coordinates of the local maximum and the local minimum of g.

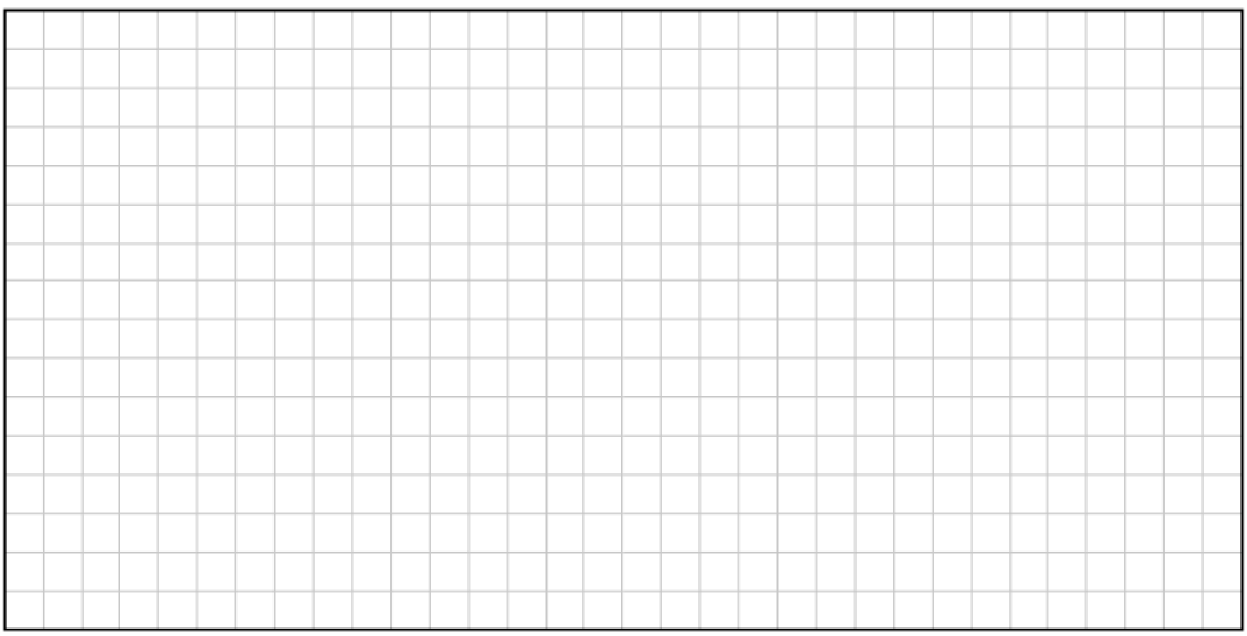


The perimeter of the above window is 20 metres. Express h in terms of r , and hence show that the area of the window is given by $20r - 2r^2 - 0.5\pi r^2$

Find the dimensions of the window that will maximise the amount of light that gets in.

Find the value of x that will maximise the internal volume of the box below.





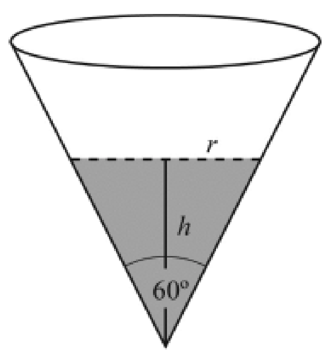
A spherical balloon is being inflated at a rate of 3 cm^3 per second.

(i) Find the rate at which the radius of the balloon is increasing at the instance when the radius is 3 cm.

(ii) Find the rate of change of the surface area of the balloon when the radius is 3 cm.

A particle moves along the curve $y = ln\sqrt{\frac{5x}{x-2}}$, in such a way that the x-coordinate is increasing at a constant rate of 0.4 units per second. Find the rate at which the y-coordinate of the particle is increasing at the instant when $x = 2.5$.

Water is poured into an empty conical container as shown, at a rate of 0.02 litres per second.



(i) Using this information, write an equation to represent the volume of water in the cone at any time t.

(ii) Write the radius of the cone r in terms of h.

(iii) Show that the height of the water h at any time t can be expressed as $h = \sqrt[3]{\frac{0.18t}{\pi}}$

(iv) Find the rate of change in the height of the water after 3 seconds.

Differentiate $5x^2 - 3x + 8 = 0$ with respect to x from first principles.

Differentiate $f(x) = 2x^2 + 4x$ with respect to x from first principles.

Differentiate $(2x + 3)^2 = g(x)$ with respect to x from first principles.

Chapter 13

INTEGRATION

- Rules
- Area under curve
- Average Value

Rules

Indefinite

$\int$ add + C

definite

$\int_a^b$ No C

$\text{Cos}(x) \rightarrow \text{Sin}(x)$

$\text{Sin}(x) \rightarrow -\text{Cos}(x)$

$\text{Cos}(ax) \rightarrow \dfrac{\text{Sin}(ax)}{a}$

$\text{Sin}(ax) \rightarrow \dfrac{-\text{Cos}(ax)}{a}$

Example :

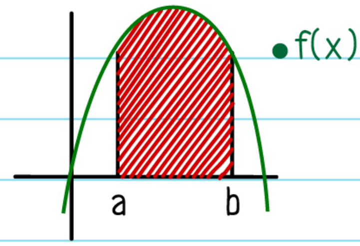
$5\text{Cos}(3x) \rightarrow \dfrac{5\text{Sin}(3x)}{3}$

Example :

$e^{ax} \rightarrow \dfrac{e^{ax}}{a}$

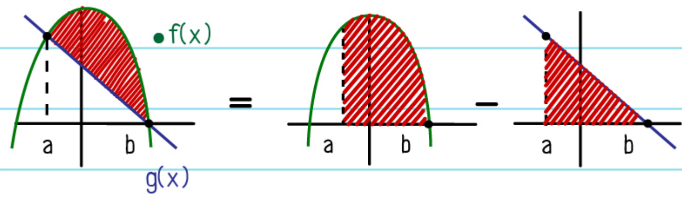
$e^{3x} \rightarrow \dfrac{e^{3x}}{3}$

Area Under Curve



$$\int_a^b f(x)$$

Area Bound



Area Bound $= \int_a^b f(x) - \int_a^b g(x)$

Average Value

$$\frac{1}{b-a}\int_a^b f(x)$$

$g(x) = 2x^2 + 5x + 6$

Find $\int g(x)\,dx$

Integrate $\int (2x + 3)^2 dx$

Integrate $\int sin(5\theta)\,d\theta$

Integrate $\int - 3cos(6\theta)\,d\theta$

$T(t) = 400e^{-0.05t}$

Find $\int T(t)\,dt$

Integrate $\int (x^2 + e^{3x} + 2)dx$

Integrate $\int_0^{\frac{\pi}{3}} (3sin(2x) - 4cos(2x))dx$

Evaluate $\int_0^{\frac{\pi}{6}} (sin4xcos2x)\,dx$

$b \in R$ and b is a positive constant.

$$\int_0^b be^{bx} dx = e$$

Work out the value of b.

Given that $f'(x) = 6x^2 - 54x + 109$, $show\ that\ f(x) = 2x^3 - 27x^2 + 109x - 126$, given that $f(x)$ crosses the x-axis at $(2, 0)$.

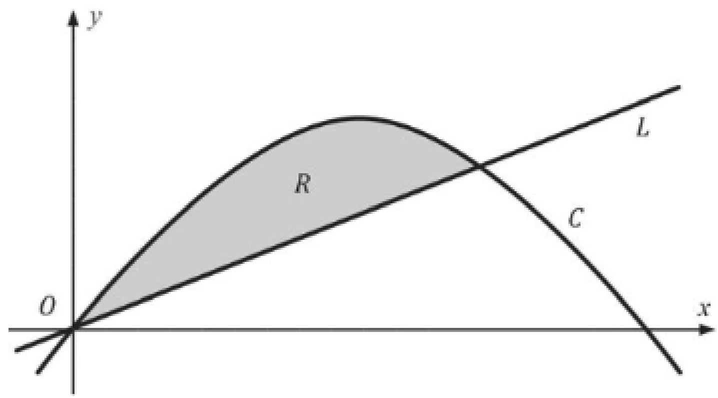
A package dropped from an aircraft moves with velocity:

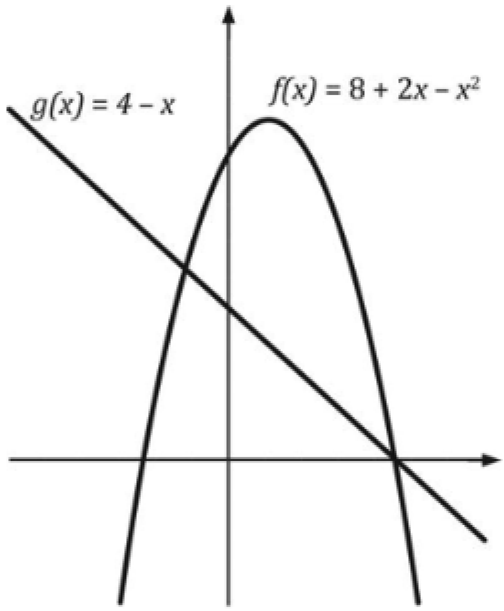
$$v(t) = 75(1 - e^{\frac{-t}{10}}),$$

Where t is the time in seconds from when the package was released.

Use integration to find the distance travelled by the package between $t = 0$ and $t = 14$. Give your answer correct to one decimal place.

In the diagram below, $C = 6x - x^2$ and $L = 2x$. Find the area of R.

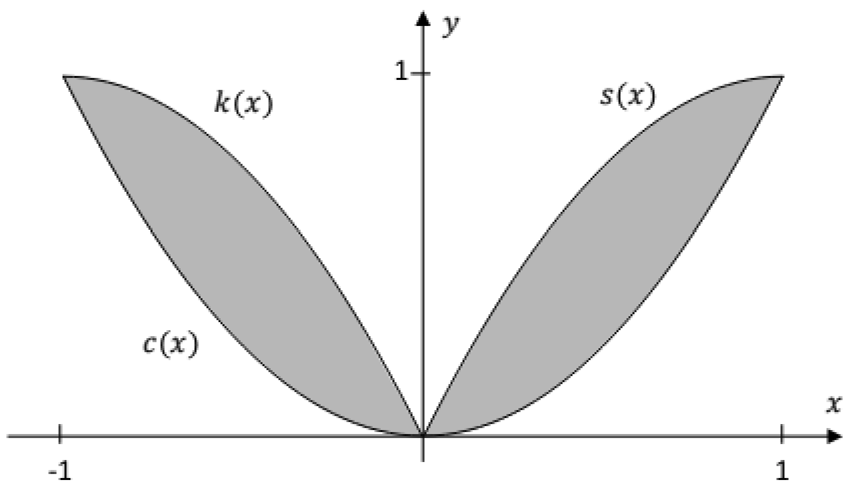




Find the area enclosed by $g(x)$ and $f(x)$.

$c(x) = x^2 \ (-1 \le x \le 1)$, $s(x) = 2x - x^2 (0 \le x \le 1)$, $k(x)$ is the image of $s(x)$ under axial symmetry in the y-axis.

Work out the area of the shaded region.



Find the area enclosed by the x-axis , and the curve $-x^2 + 2x + 3$.

Find the average value of the function $f(x) = 3x^3 - 2x^2 + x + 2$ on the interval $[1, 4]$.

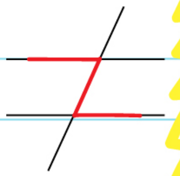
The approximate length of the day in Galway, measured in hours from sunrise to sunset, may be calculated using the function:

$$f(t) = 12.25 + 4.75sin(\frac{2\pi}{365}t)$$

Where t is the number of days after March 21st.

Use integration to find the average length of the day in Galway over the 6 months from March 21st September 21st (184 days). Give your answer in hours and minutes correct to the nearest minute.

Chapter 14



GEOMETRY AND AREA + VOLUME

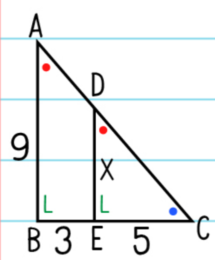
Note: *Geometry* is a paper 2 topic and *area + volume* is a paper 1 topic, however, the underlying concepts tie in quite well, and therefore it is beneficial to attack them together

Circumcentre:
The point in a triangle where the lines that cut each side in half at right angles meet. It's the center of the circle that goes through all three corners of the triangle.

Orthocentre:
The point in a triangle where the lines drawn straight down from each corner to the opposite side meet.

Similar Triangles

Triangles with all 3 angles the same



$|\angle ABC| = |\angle DEC|$
$|\angle ACB| = |\angle DCE|$
$|\angle BAC| = |\angle EDC|$

$$\frac{9}{X} = \frac{8}{5}$$

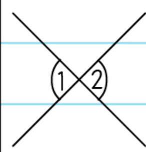
continue

Congruent Triangles

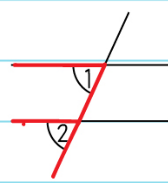
→ Identical

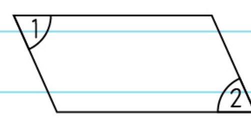
Proofs : SSS
SAS
ASA
RHS

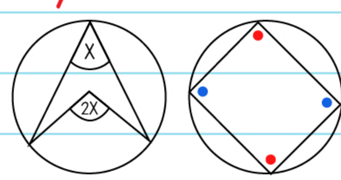
Things to know



$|\angle 1| = |\angle 2|$ for all examples



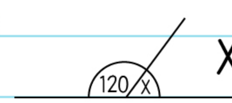




Cyclic Quadrilateral
= opposite angles add to 180^0

Straight lines add to 180^0

Example

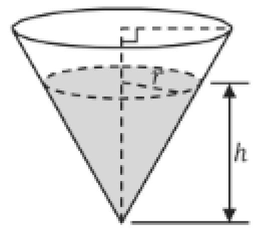


$X = 60^0$

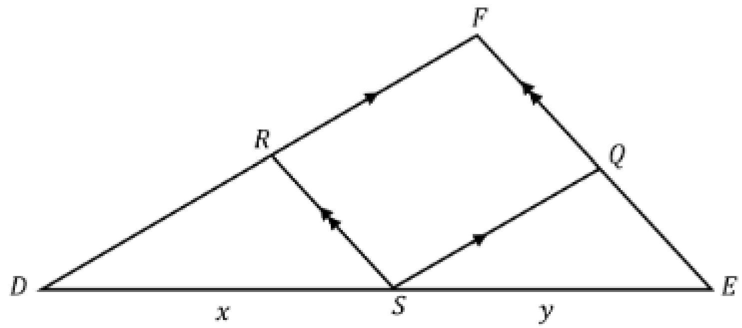
Area and Volume

All formulas appear in the log tables !!

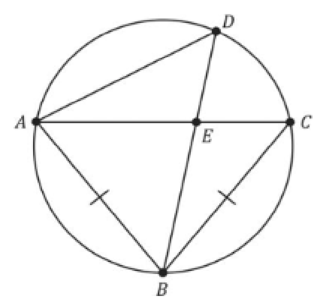
The radius of the cone below is $5\sqrt{3}$ metres and the height is 10 metres.. Given that r is the radius of the surface of the water in a cone when the depth is h, show that $r = \frac{\sqrt{3}}{2}h$.

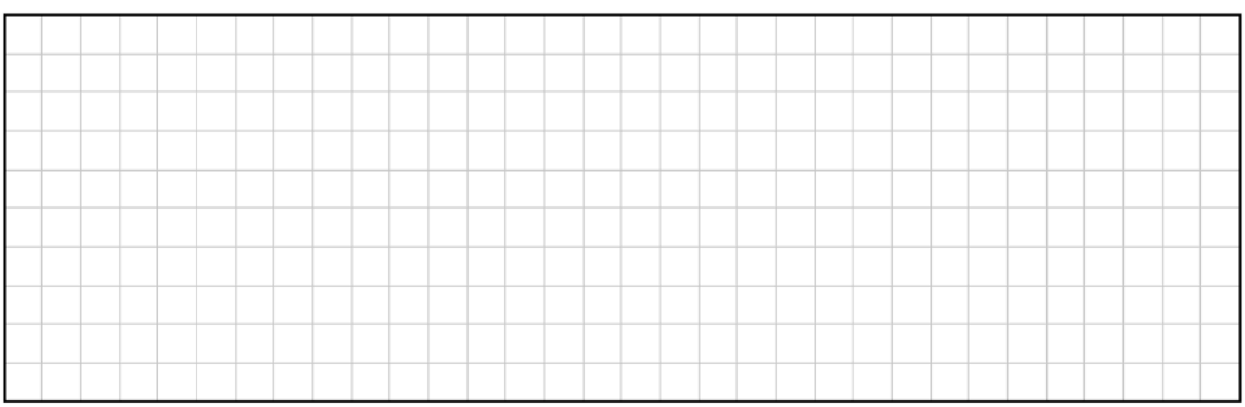


Show that ΔDEF is similar to ΔSEQ

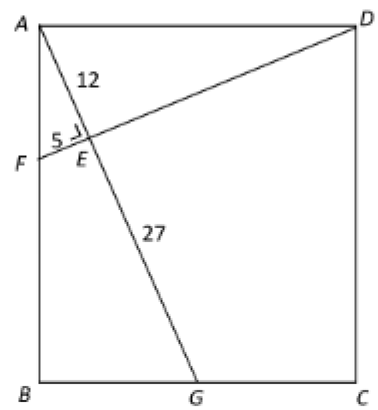


Show that ΔABE is similar to ΔABD





Show that ΔAFE is similar to ΔDAE and hence find $|AD|$



In Diagram B, $|BD|$ and $|GE|$ are extended, and they meet at the point H. Prove that $|FE| = |EH|$. Use congruent triangles.

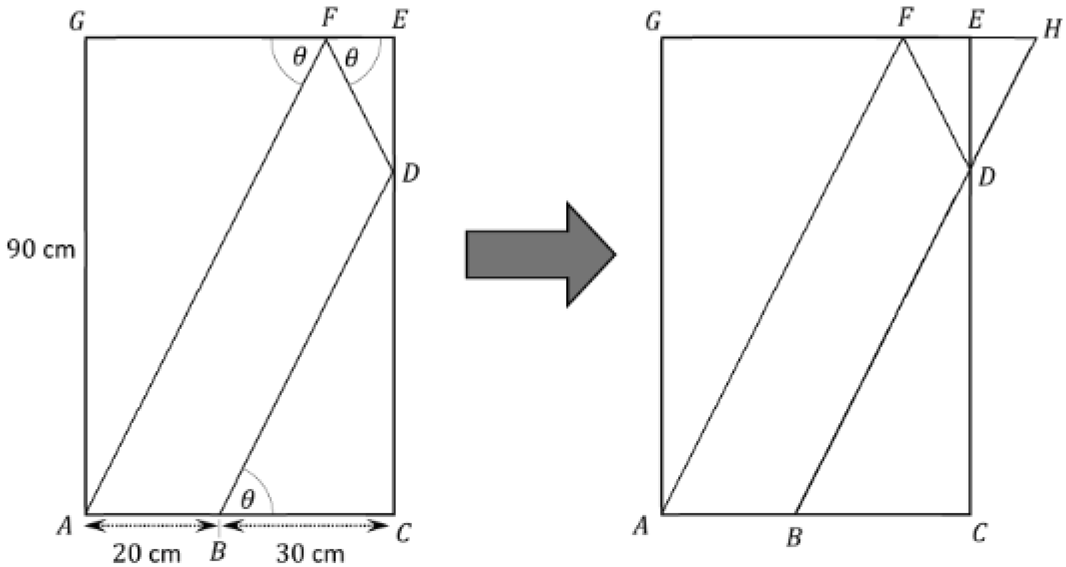
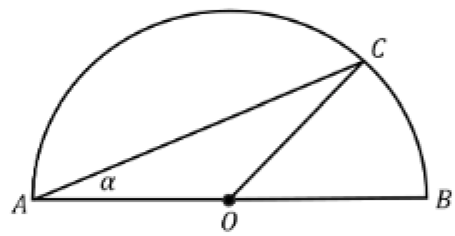
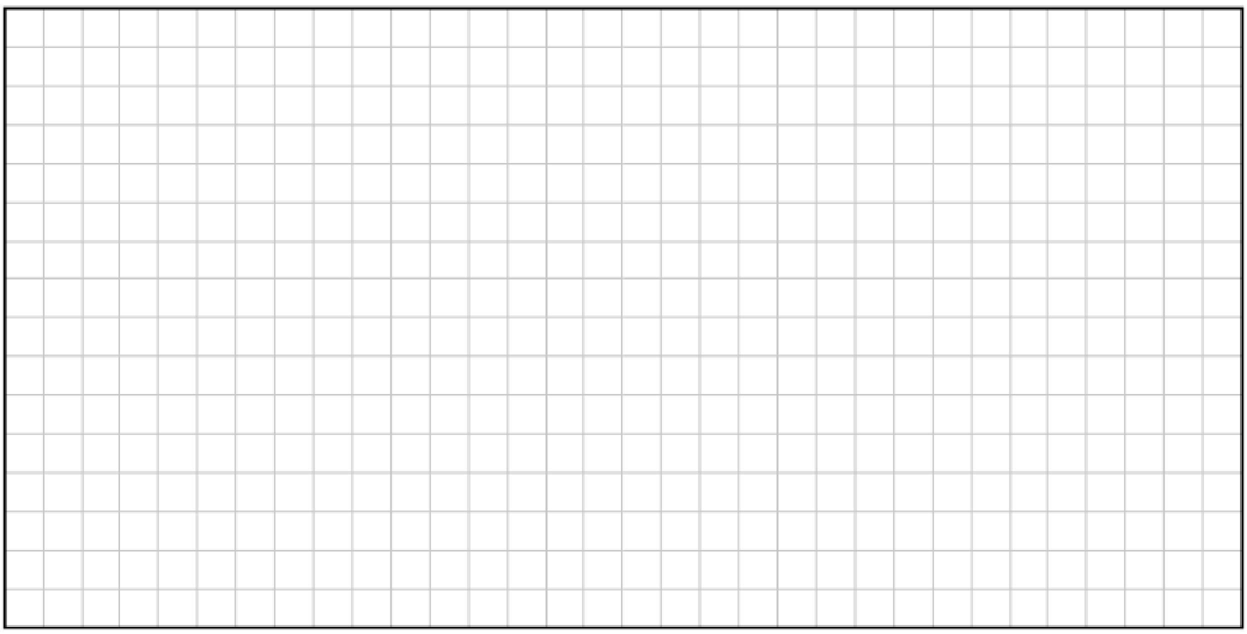


Diagram A

Diagram B

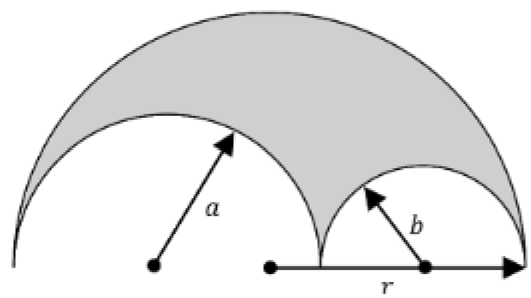
Show that the area of sector OBC is $r^2\alpha$.



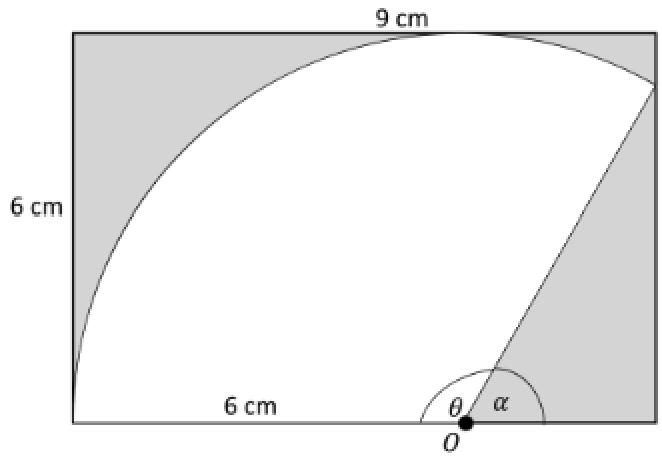


$r = a + b$

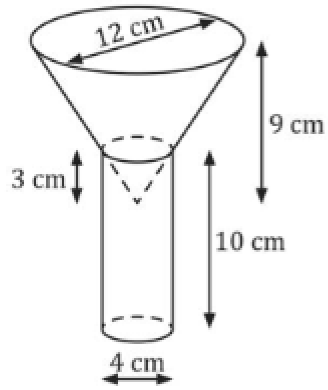
Show that the perimeter of the arbelos below is independent of the values of a and b.



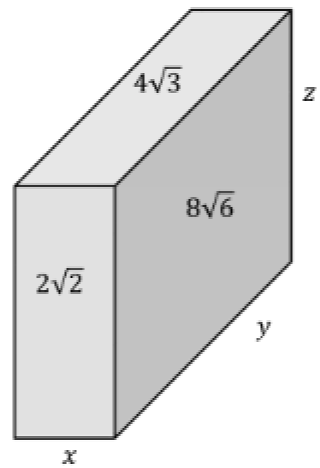
A sector of a circle of centre O and radius 6 cm lies inside a rectangle. Find the measure of angle a, AND find the area of the shaded region, leaving your answer correct to one decimal place.



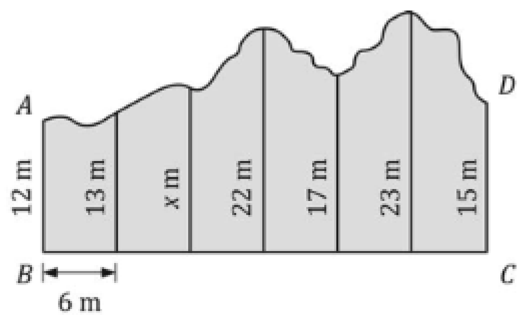
To make a funnel, a cone is inverted and the tip of the cone is removed. The remaining shape is then placed on top of a cylinder. Calculate the volume of the funnel.



Find the volume of this cuboid in the form $a\sqrt{b}\ cm^3$ where $a, b \in R$.

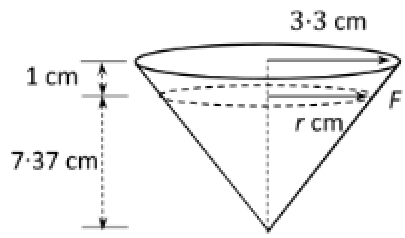


Given that the area of the below shape is $627m^2$, find the value of x.

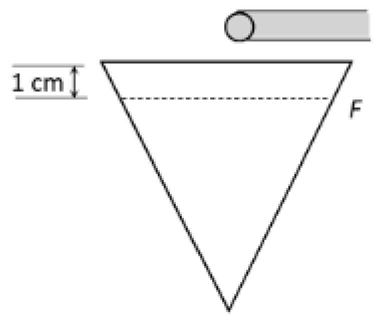


In order to avoid spillages, conical cups are marked at the dotted line marked F, which is 1 cm vertically below the top of the cup, as shown.

(i) Find the volume of water in the cup when it is filled as far as the dotted line.



(ii) Water flows into these cups through a cylindrical pipe with radius 0.8 cm at a rate of 2.5 cm/sec. Find how long, to the nearest second, it will take to fill one of these cups to the line at F.



Chapter 15

BANKERS

- Proofs
- Constructions

Paper 1 Proofs / Constructions

- Amortisation
- Prove $\sqrt{2}$ is irrational
- De Moivre's theorem
- Prove S_n and S_∞ theorem

Paper 2 Proofs / Constructions

- Construct $\sqrt{2}$ / $\sqrt{3}$
- Prove $\cos^2 A + \sin^2 A = 1$
- Prove the sine rule
- Prove the cosine rule
- Prove $\cos(A-B) = \cos A \cos B + \sin A \sin B$
- Prove $\cos(A+B) = \cos A \cos B - \sin A \sin B$
- Prove $\cos(2A) = \cos^2 A - \sin^2 A$
- Prove $\sin(A+B) = \sin A \cos B + \cos A \sin B$
- Prove $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$
- Theorem 11
- Theorem 12
- Theorem 13

- Cicumcentre and circumcircle of triangle.
- Incentre and Incircle of a triangle.
- Angle of 60^0
- Centroid of a triangle.
- Orthocentre of a triangle.

Paper 1 bankers

Show how the formula $A = P\,\frac{i(1+i)^t}{(1+i)^t-1}$ is derived

Prove that $\sqrt{2}$ is irrational.

Hence, explain what is meant by proof by contradiction

De Moivre's theorem states that $(cos\theta + isin\theta)^n = cos(n\theta) + isin(n\theta)$. Prove De Moivre's theorem by induction.

De Moivre's theorem states that $r(cos\theta + isin\theta)^n = r^n[cos(n\theta) + isin(n\theta)]$. Prove De Moivre's theorem by induction.

Using a binomial expansion, expand $(cos\theta + isin\theta)^3$ fully.

Use De Moivre's theorem to prove that $sin3\theta = 3sin\theta - 4sin^3\theta$

Use De Moivre's theorem to prove that $cos3\theta = 4cos^3\theta - 3cos\theta$

Prove $S_n = \frac{a(1-r^n)}{1-r}$

Hence, or otherwise, prove that the sum to infinity of a geometric series can be written as:

$$\frac{a}{1-r}$$

Paper 2 bankers

Prove $cos^2A + sin^2A = 1$

Prove the sine rule

Prove the cosine rule

Prove $cos(A - B) = cosAcosB + sinAsinB$

Prove $cos(A + B) = cosAcosB - sinAsinB$

Prove $cos2A = cos^2 A - sin^2 A$

Prove $sin(A + B) = sinAcosB + cosAsinB$

Prove $tan(A + B) = \frac{tanA + tanB}{1 - tanAtanB}$

Prove that, if three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal line.

Diagram:
Given:
To prove:
Construction:
Proof:

Prove that, if two triangles ΔABC and $\Delta A'B'C'$ are similar, then their sides are proportional, in order:

$$\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CA}{C'A'}$$

Diagram:

Given:

To prove:

Construction:

Proof:

Prove that the angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.

Diagram:

Given:

To prove:

Construction:

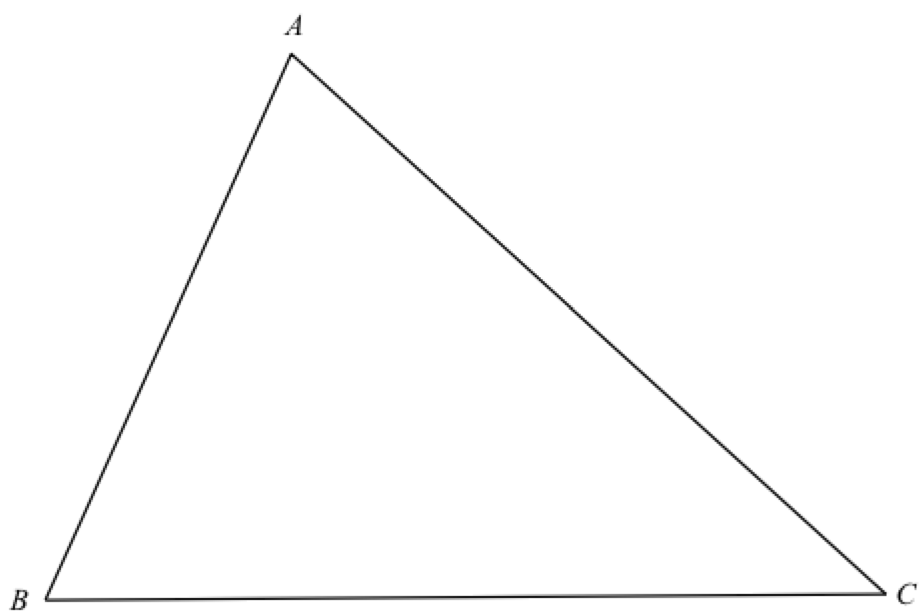
Proof:

Let ABC be a triangle. Prove that if a line l is parallel to BC and cuts $[AB]$ in the ratio $s:t$ where $s, t \in N$, then it also cuts $[AC]$ in the same ratio.

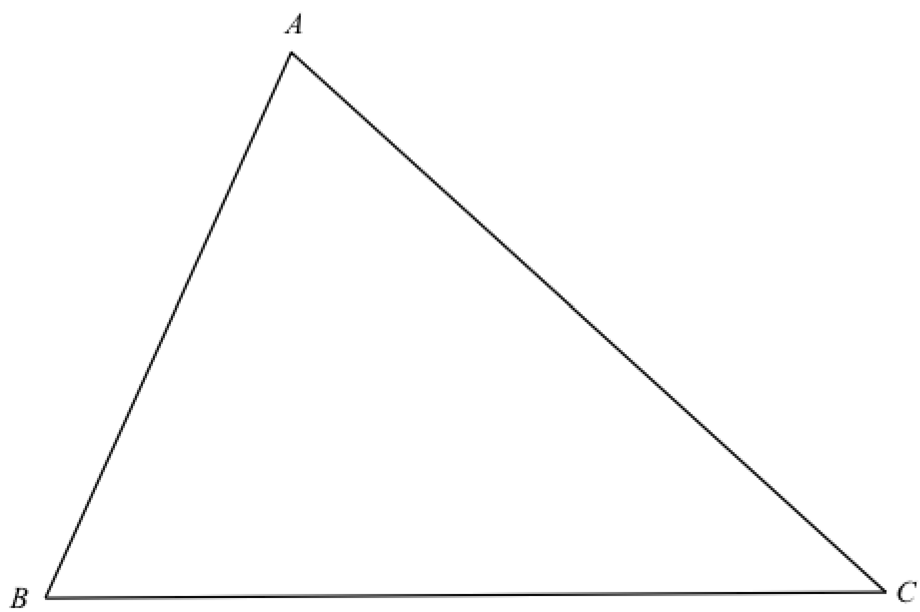
Diagram:
Given:
To prove:
Construction:
Proof:

Given the line segment $[BC]$, construct, without using a protractor or set square, a point A such that $|\angle ABC|$ 60°. Show your construction lines.

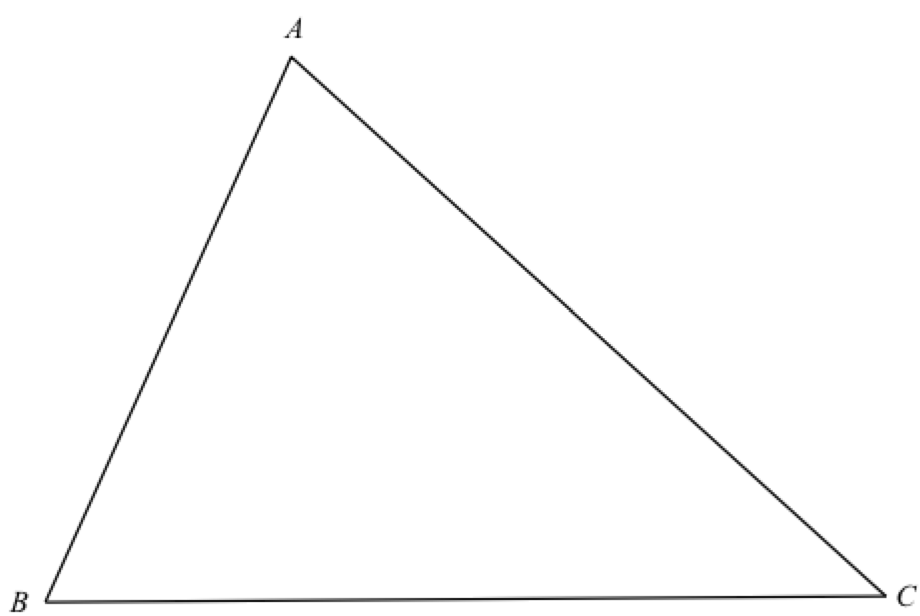
Construct the centroid of triangle ABC below.



Construct the circumcircle of triangle ABC below.



Construct the orthocentre of triangle ABC below.



Construct the incircle of triangle ABC below.