


ORDINARY LEVEL

LEAVING CERT MATHS

BY DUBLIN MATHS

"The Essential LC OL Study Guide"



INTRODUCTION

The aim of this revision book is to help you enhance your grade in your Leaving Certificate. It does this by breaking exam questions down by subtopic, in a way that is easy to understand, helping the student to recognise what the question is *really* asking. This book is most effective when the questions are answered **in order**.

At the start of each section, there is a link to a collection of similar questions and solutions, which can be used for **extra study and practice**. Each chapter of this book is covered in more detail during our weekly free group grind that takes place on www.dublinmaths.ie.

We strongly encourage you to attend these sessions.

Recordings are also available for playback.

The Leaving Cert curriculum is broad, and daunting. Don't be discouraged by a challenging question. As in the actual exam, difficult questions can sometimes begin with one or two simple parts. You should **answer as much as you can**. We hope that this book offers even a small beacon of hope as you prepare for the big day.

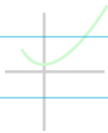
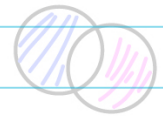
Thank you for trusting us. We hope it pays off in spades!



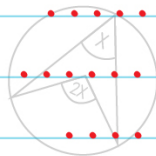
TABLE OF CONTENTS

$$T_n = a + (n-1)d$$

$$x^2 + 7x + 10$$



• Algebra	1
• Functions	35
• Coordinate geometry : the line	48
• Coordinate geometry : the circle	62
• Trigonometry	70
• Geometry	88
• Area and volume	101
• Financial maths	122
• Complex numbers	134
• Sequences and series	140
• Probability	151
• Statistics	168
• Differentiation	189
• Constructions	190
• Miscellaneous	200

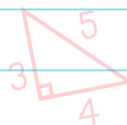
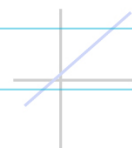


$$f(x) = x^2$$



$$y = x^2$$

$$-b \pm$$



Chapter 1

ALGEBRA

- Solving
- Inequalities
- Indices
- Factorising + Expressional
- Simultaneous Equations

Variable → The unknown
[x, y, z, θ]

Constant → The fixed value term
[Normal numbers]

Coefficient → The number directly
before a variable

Index → The power

$3x^2 + 7x + 10$ — Constant
Coefficient Variable Index

• Solving

i) Linear

when you have tidied the equation up as much as possible, and the highest power of a variable is 1 e.g. (no x^2 , no y^3 etc)

Rule :

Bring variables to the left hand side
Bring constants to the right hand side

When a number moves across an = it changes sign e.g. $x+3=15$
 $x=15-3$

ii) Quadratic

$$ax^2 + bx + c$$

a = in front of x^2
b = in front of x
c = constant

Always let quadratics = 0

Use

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

log tables
pg 20

Factorising + Expressional

i) HCF

Example: $4x^2 - 18x$
 $= 2x(2x - 9)$

ii) Grouping

4 terms

$$6cx - 3bc + 4dx - 2bd$$

Example: $= 3c(2x - b) + 2d(2x - b)$
 $= (3c + 2d)(2x - b)$

iii) Quadratic

- an x^2
- an x
- a constant

Example: $x^2 + 7x + 10$ $\begin{matrix} \textcircled{10} \\ \swarrow \searrow \\ 10 + 1 = 11 \quad \times \\ 2 + 5 = 7 \quad \checkmark \end{matrix}$

$$= x^2 + 2x + 5x + 10$$
$$= x(x+2) + 5(x+2)$$
$$= (x+5)(x+2)$$

Example: $x^2 + 6x - 27$ $\begin{matrix} \textcircled{-27} \\ \swarrow \searrow \\ 9 \quad -3 \end{matrix}$

$$= x^2 + 9x - 3x - 27$$
$$= (x+9)(x-3)$$

Example:

$$3x^2 - 5x - 2 \quad \begin{matrix} (3)(-2) \quad \textcircled{-6} \\ \swarrow \searrow \\ -6 \quad +1 \end{matrix}$$
$$= 3x^2 - 6x + 1x - 2$$
$$= 3x(x-2) + 1(x-2)$$
$$= (3x+1)(x-2)$$

iv) Difference of two squares

- 2 squares numbers
- Separated by a minus (-) sign

Examples:

- $4x^2 - 25$
 $= (2x+5)(2x-5)$
- $9x^2 - 49y^2$
 $= (3x+7y)(3x-7y)$

Why is $2x^2 + 50$ not a difference of 2 squares?

v) Combining fractions

Example:

$$\frac{3}{2x} - \frac{4}{3x+1}$$
$$= \frac{3(3x+1) - 4(2x)}{(2x)(3x+1)}$$
$$= \frac{9x+3-8x}{(2x)(3x+1)} = \frac{x+3}{(2x)(3x+1)}$$

● Inequalities

N → Natural numbers
(positive whole numbers)

Z → Integers
(positive and negative whole numbers)

R → Real numbers
(ANY number on a number line)

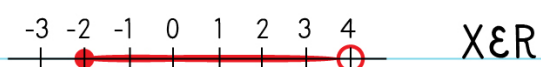
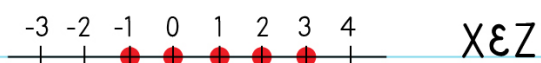
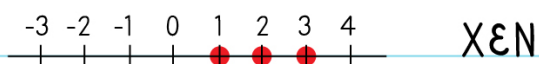
If you divide/multiply by a negative number, flip the inequality sign

Example:

$$\begin{aligned}4(3-x) &< 20 \\12-4x &< 20 \\-4x &< 20-12 \\-4x &< 8 \\ \frac{-4x}{-4} &< \frac{8}{-4} \\ x &\geq -2\end{aligned}$$

Little number < variable < big number

$$\boxed{-3 \leq x < 4}$$



● Simultaneous Equations

Equations that have more than 1 variable

(i) Both linear

(ii) One linear, One non linear

● Indices

$$(2^3)^2 = 2^6$$

$$2^0 = 1$$

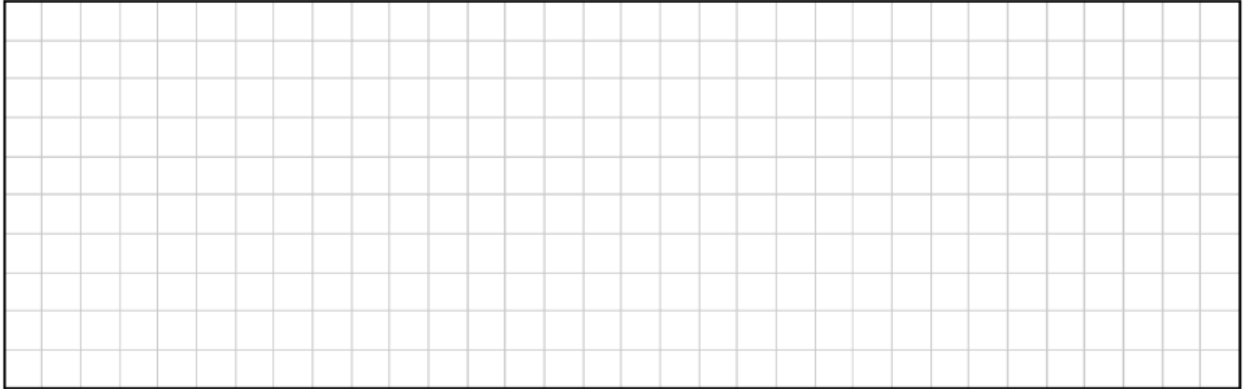
$$2^3 \times 2^2 = 2^5$$

$$\sqrt{2} = 2^{1/2}$$

$$\frac{2^6}{2^4} = 2^2$$

Solve the following equation in x :

$$3(2x + 4) - 5 = 3$$



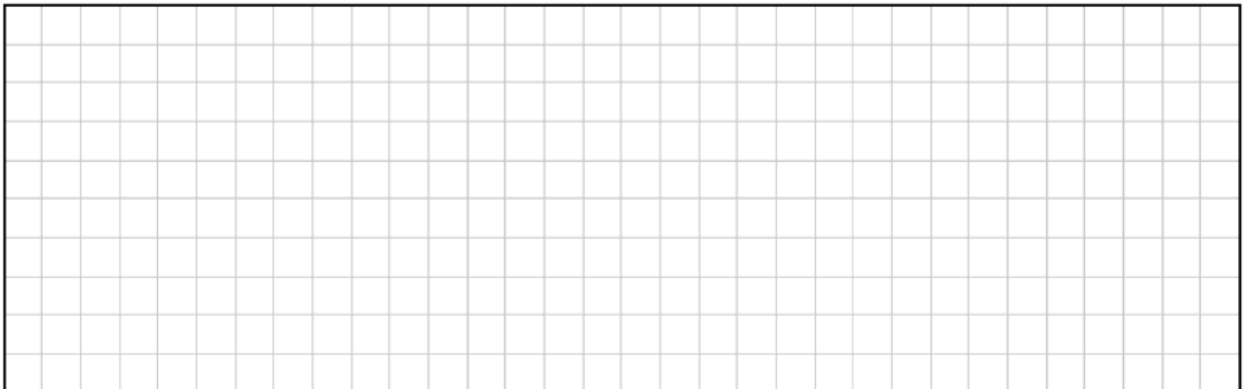
Solve the following equation in x :

$$2(3x - 5) + 8 = 4x - 5$$



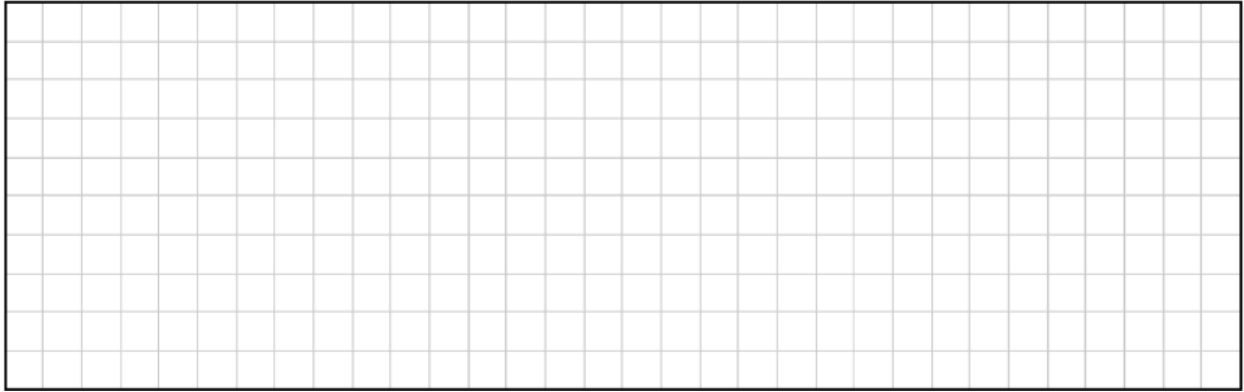
Solve the following equation in x :

$$3(x - 7) + 5(x - 4) = 14, \text{ where } x \in R$$



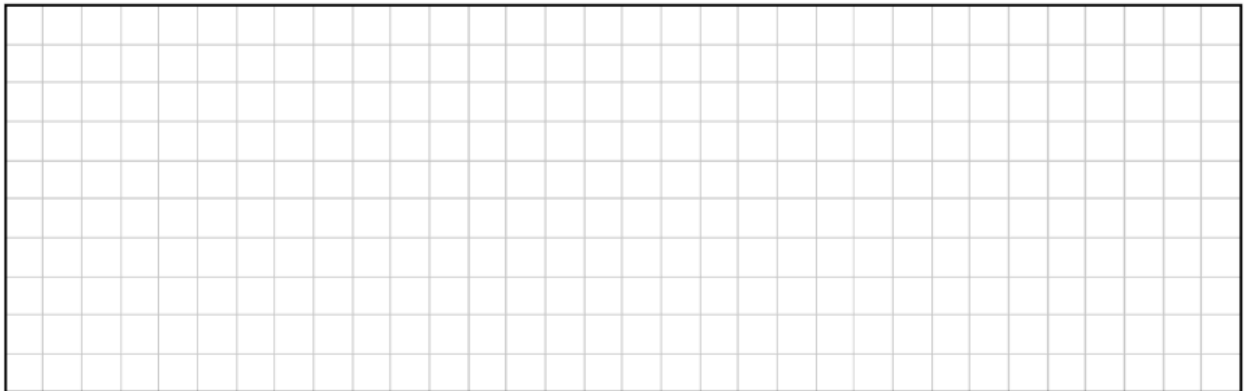
Solve the following equation in x :

$$(x + 5)(3x - 4) - 3(x^2 + 2) + 4 = 0$$



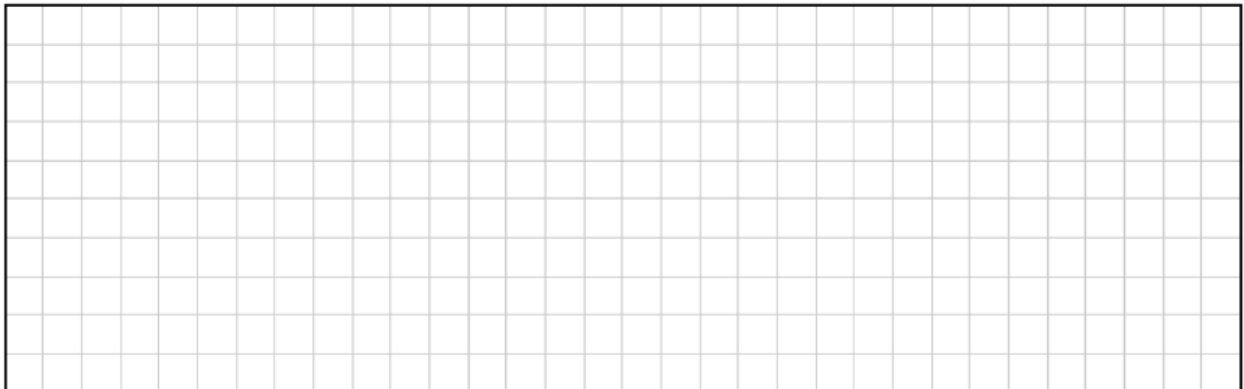
Solve the following equation in x :

$$5(x + 4) - 6(2x + 7) = 9(x + 3) - 17.$$



Solve the following equation in x :

$$2(4 - 3x) + 12 = 7x - 5(2x - 7).$$



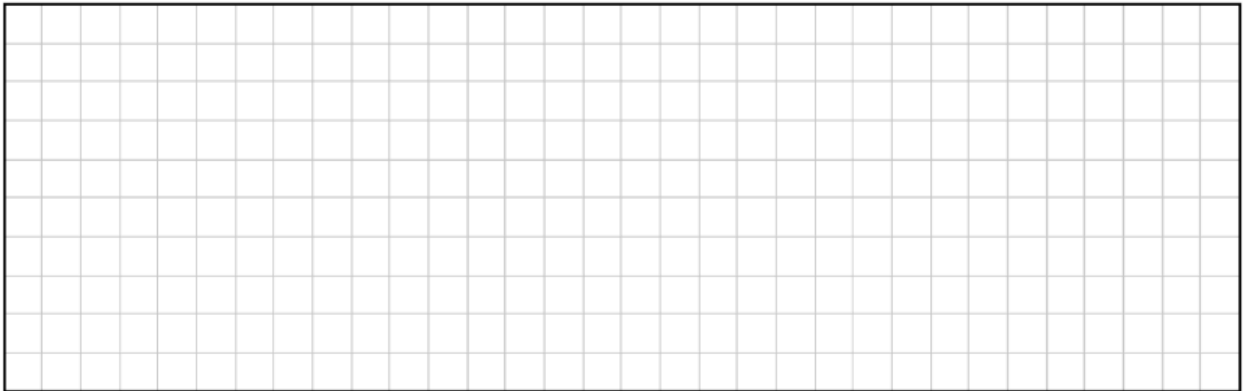
Solve the following equation in x :

$$x^2 + 7x + 10 = 0$$



Solve the following equation in x :

$$2x^2 + x - 10 = 0$$



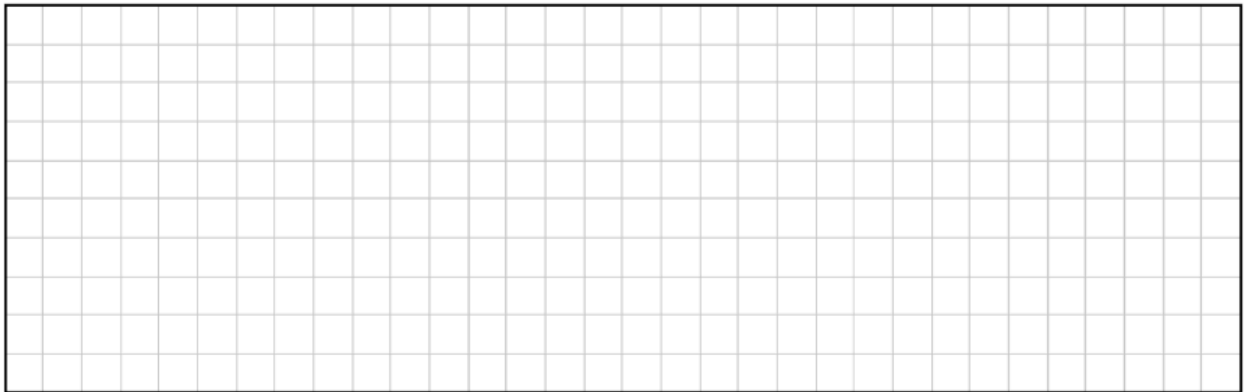
Solve the following equation in x , leaving your answer correct to two decimal places:

$$x^2 - 6x = 23$$



Solve the following equation in x :

$$-21 + 6x^2 = 39x$$



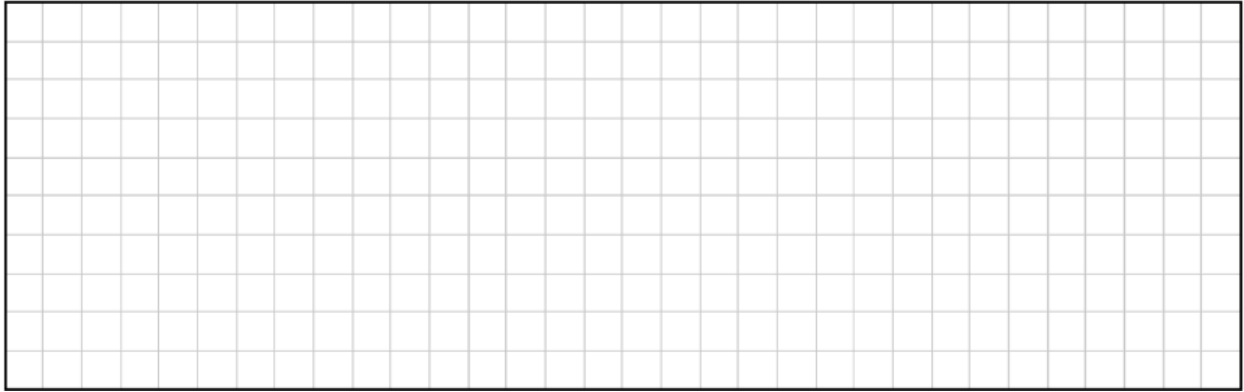
Solve the following equation in x : $3x^2 - 2x - 3 = 0$.

Give each answer in its simplest *surd* form.

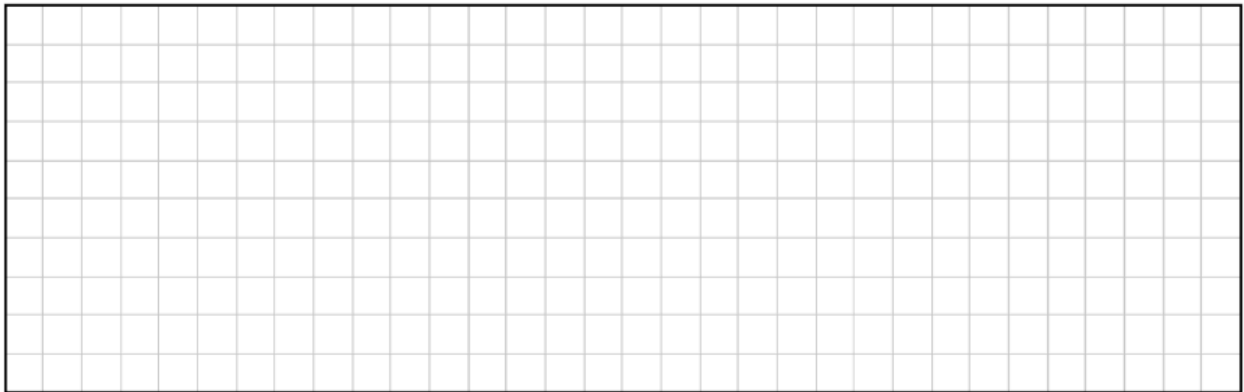


Solve the following equation in x :

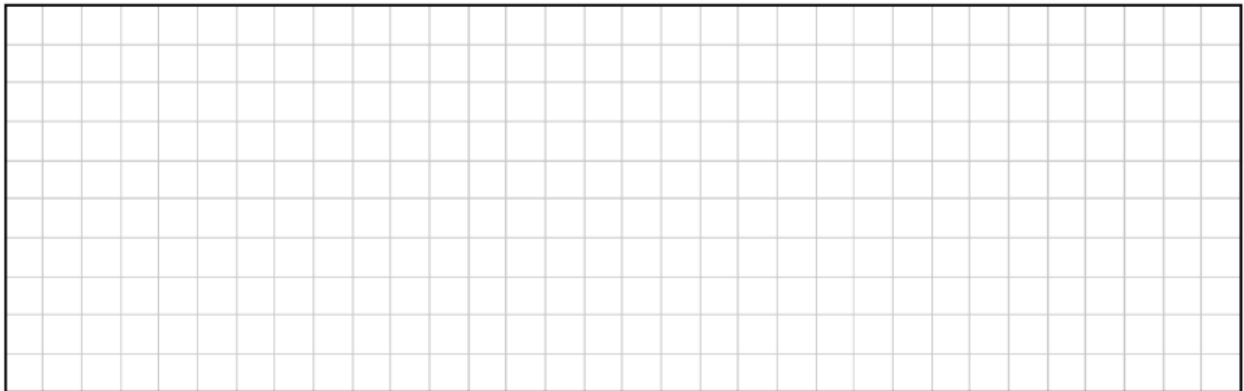
$$x^2 + (10 - 2x)^2 = 25$$



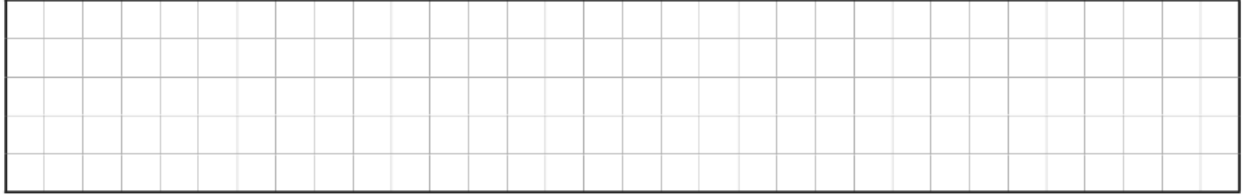
Show that $x = 4$ is a solution of the equation $x^2 - 2x - 8 = 0$.



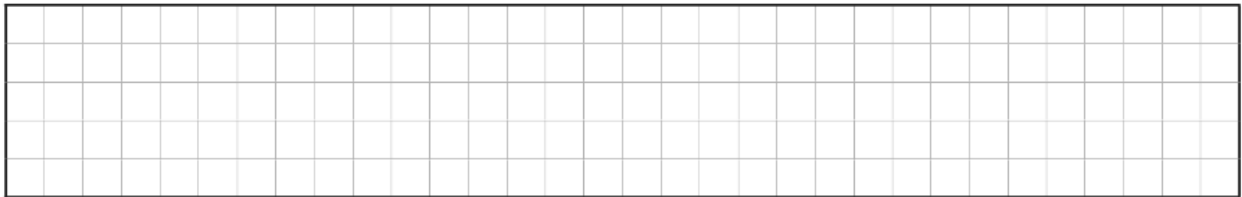
The equation $x^2 + ax + b = 0$, where $a, b \in \mathbb{Z}$, has solutions $x = 5$ and $x = -2$. Find the value of a and the value of b .



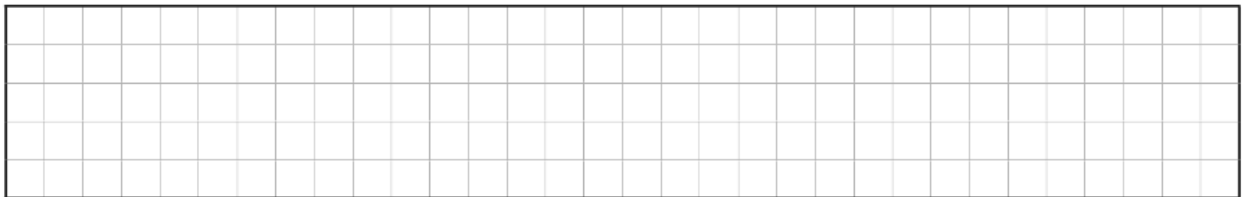
Factorise fully $4a^2 - 12ab$



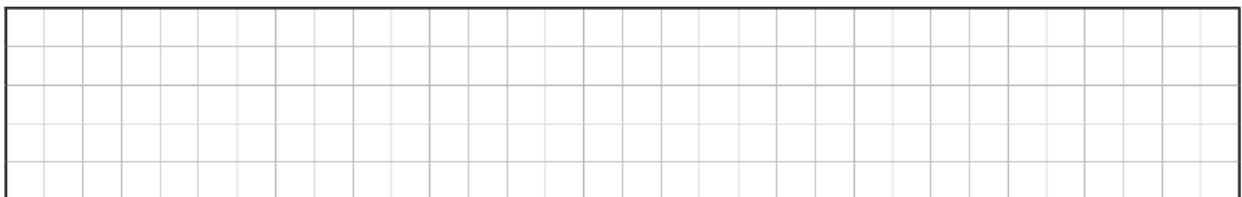
Factorise fully $9xy - 27y^2$



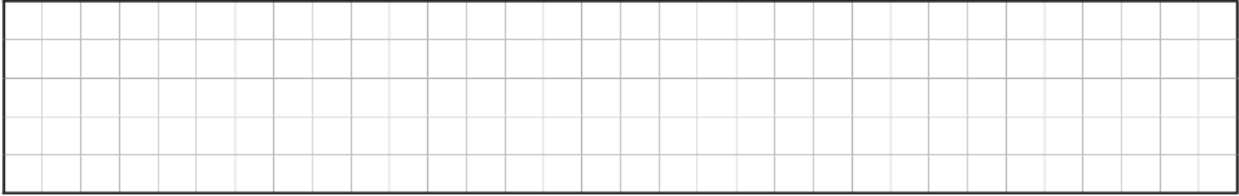
Factorise fully $4xy^2 - 7x^2y$



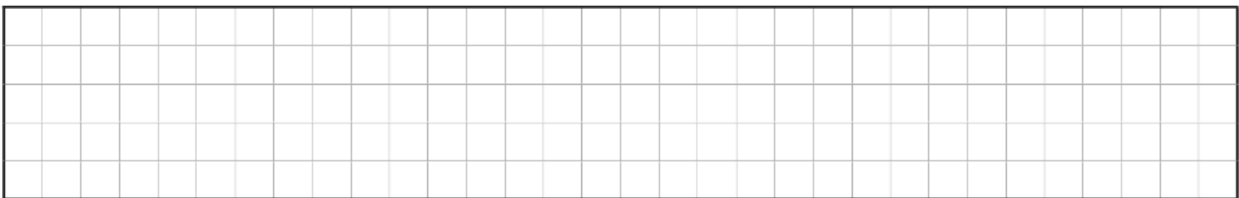
Factorise fully $36z^3y - 24z$



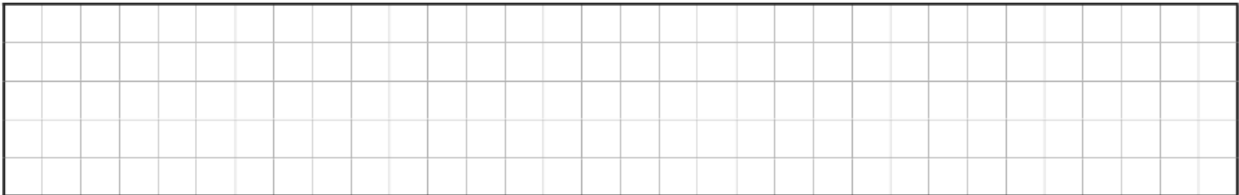
Factorise fully $6ay - 3by - 9bx + 2ay$



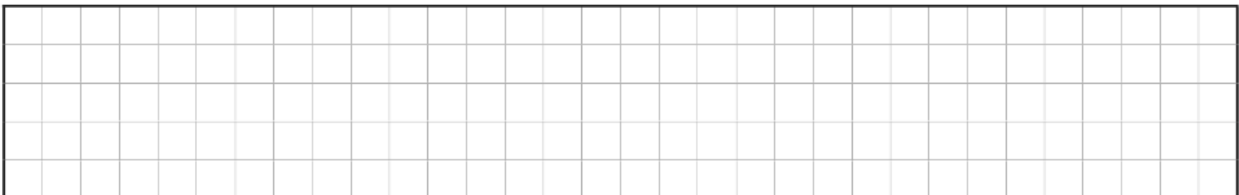
Factorise fully $2ab + 3cd - cb - 6ad$



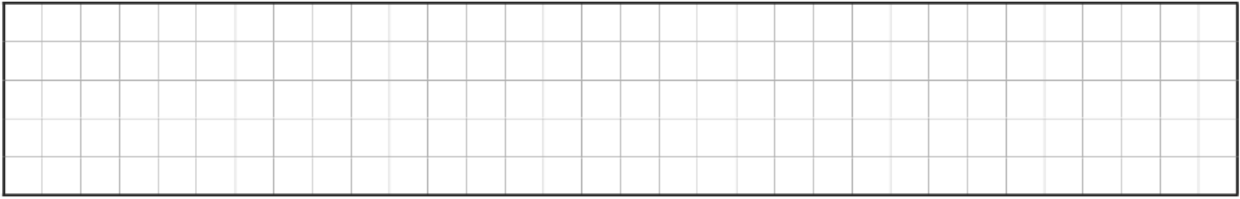
Factorise fully $wy - y - 1 + w$



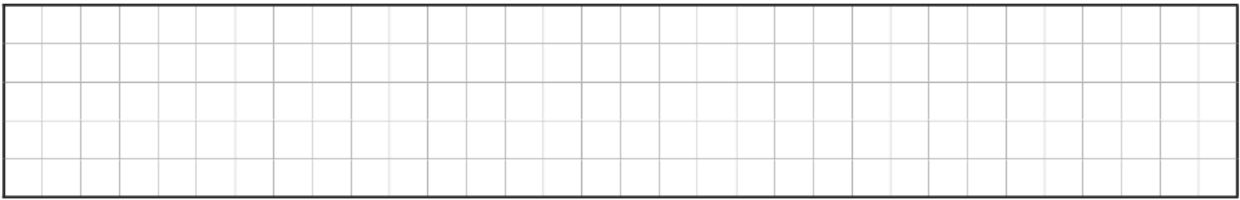
Factorise fully $15p^2 - 10pq - 6ps + 4qs$



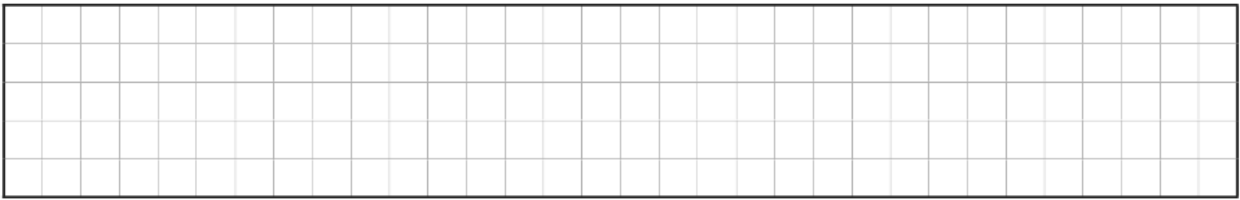
Factorise fully $8ax - 14bx + 4ay - 7by$



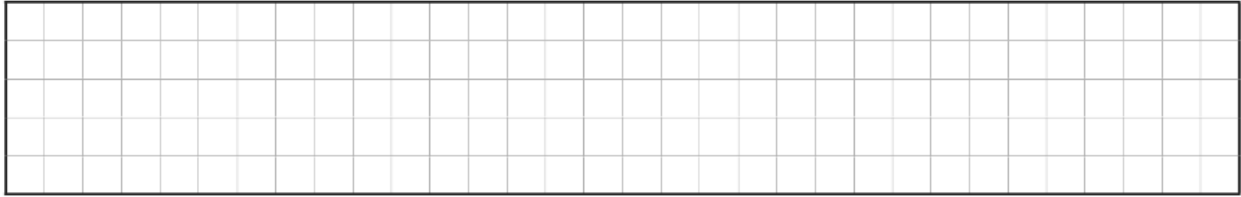
Factorise fully $6xy - 2x + 3z - 9zy$



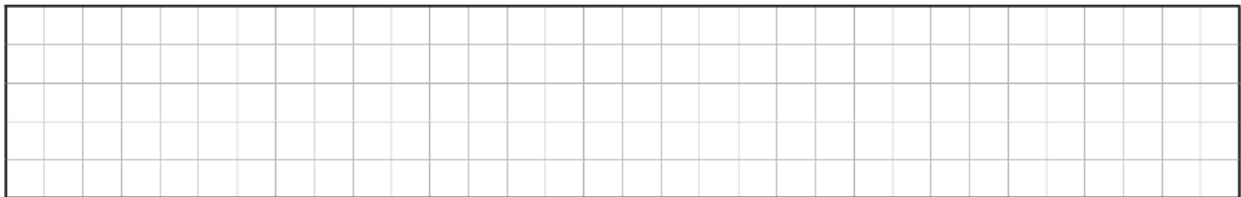
Factorise fully $3f^2 + 4qp - 2fp - 6qf$



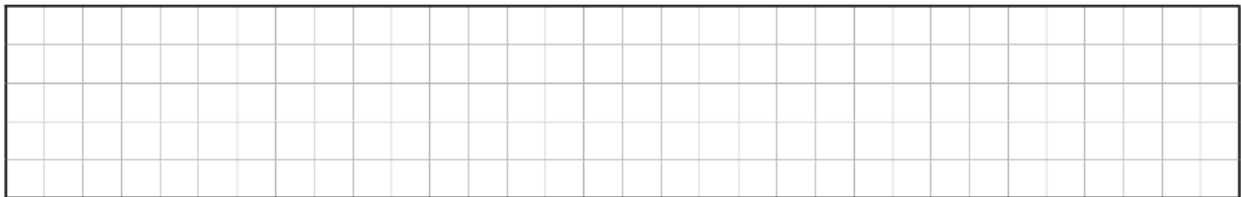
Factorise fully $x^2 + 13x + 30$



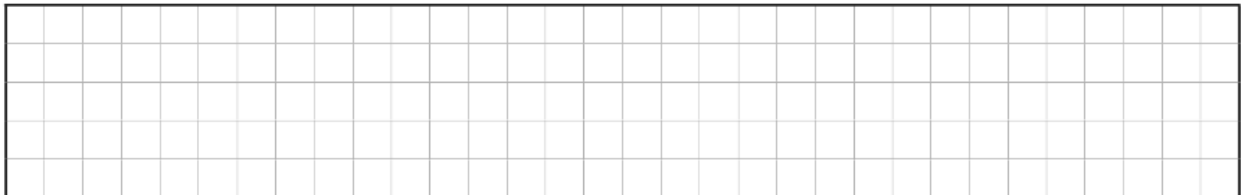
Factorise fully $x^2 + 3x - 18$



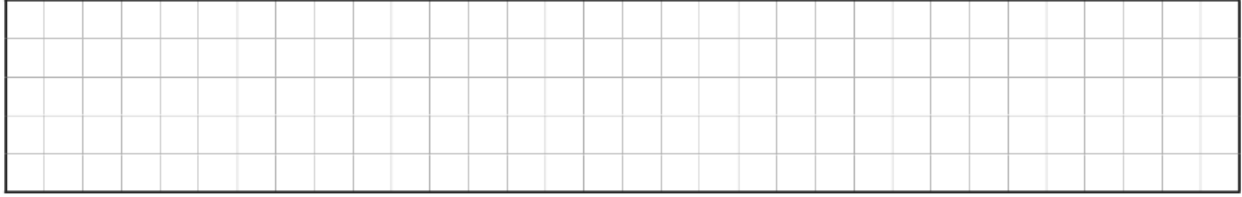
Factorise fully $x^2 - x - 20$



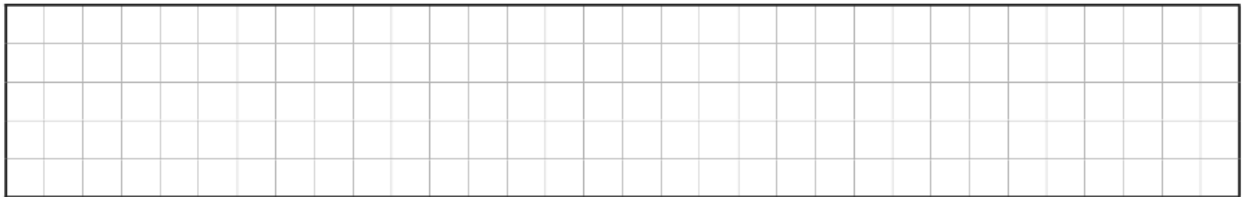
Factorise fully $x^2 - 11x + 18$



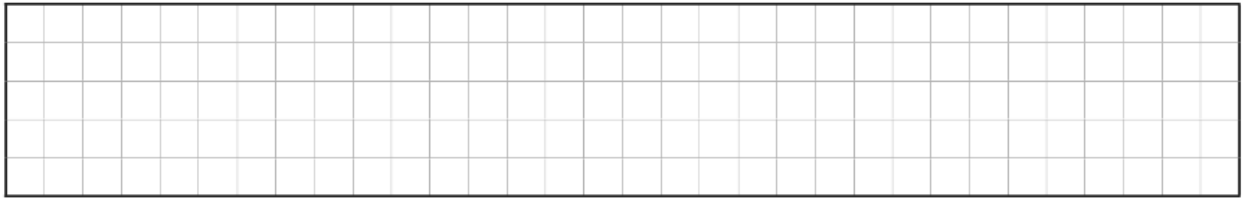
Factorise fully $3x^2 + 5x - 12$



Factorise fully $2n^2 + 7n - 15$

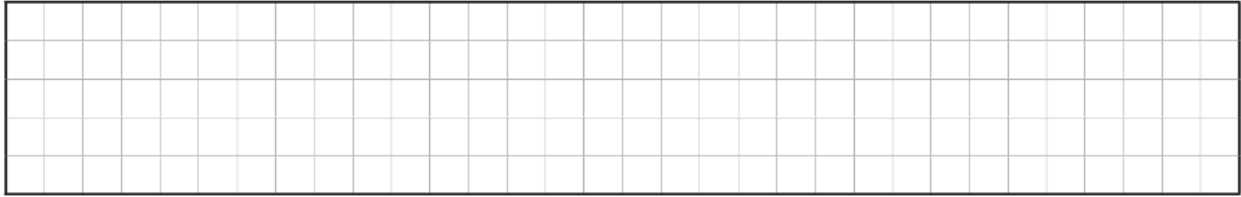


Factorise fully $3y^2 + 2y - 5$

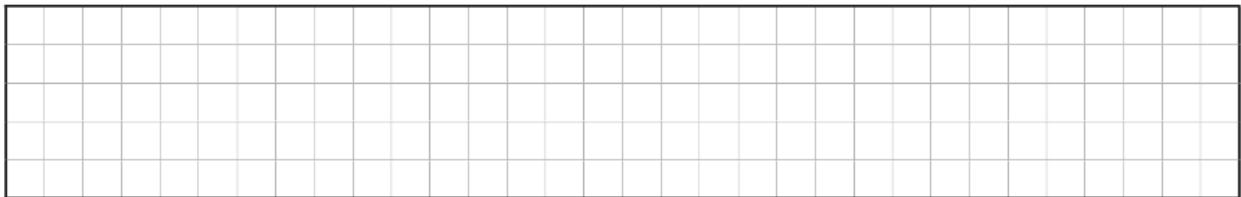


Difference of two squares

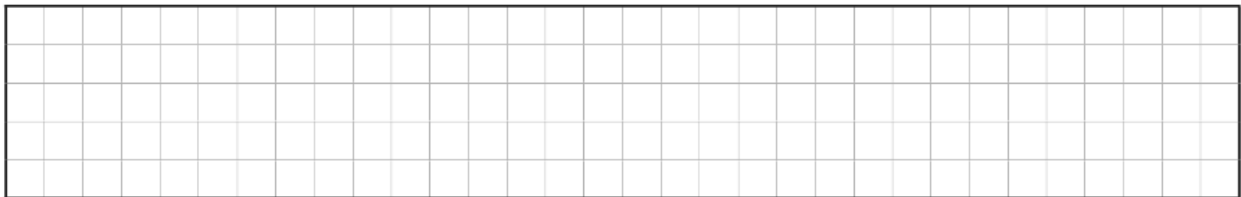
Factorise fully $x^2 - y^2$



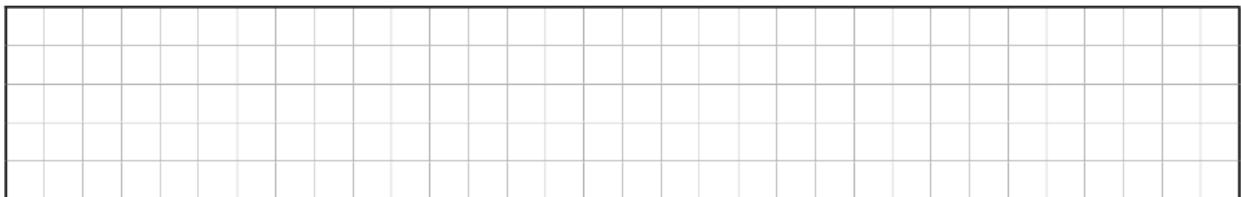
Factorise fully $4a^2 - 16$



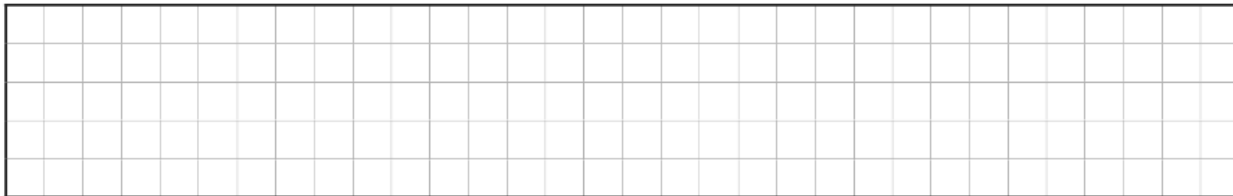
Factorise fully $81 - 9y^2$



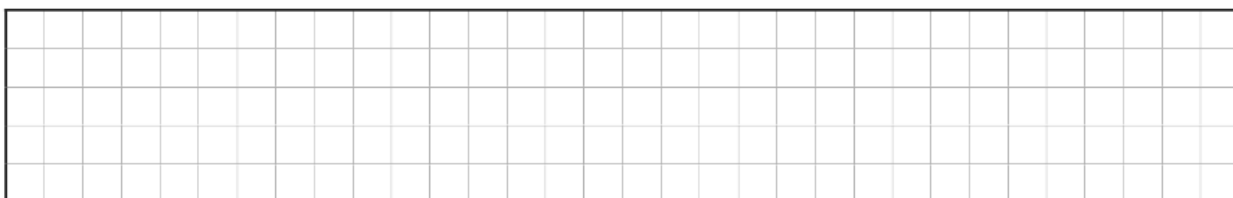
Factorise fully $4x^2 - 49y^2$



Factorise fully $2x^2 - 50$

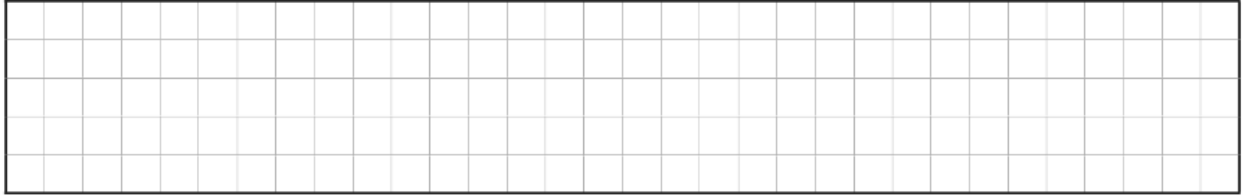


Factorise fully $2x^2 - 2$

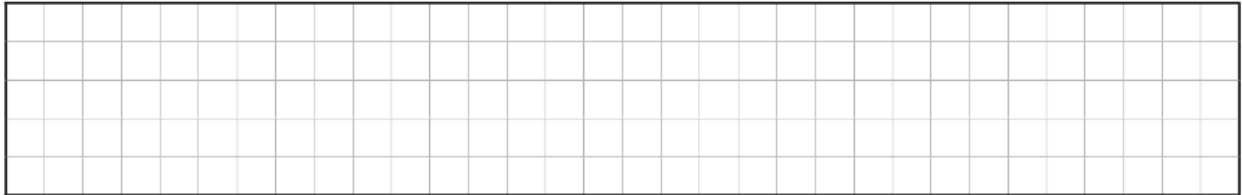


Combining and simplifying fractions

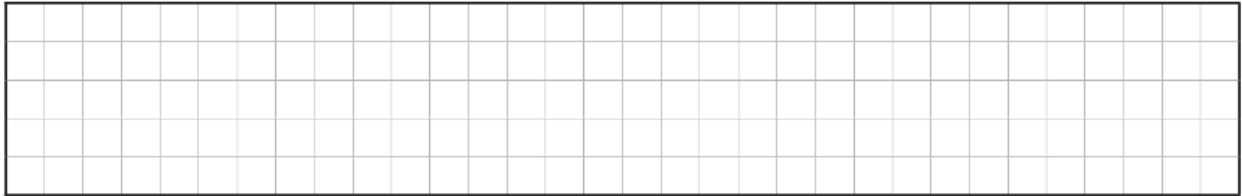
Express in its simplest form: $\frac{5-x}{5} + \frac{x-4}{4}$



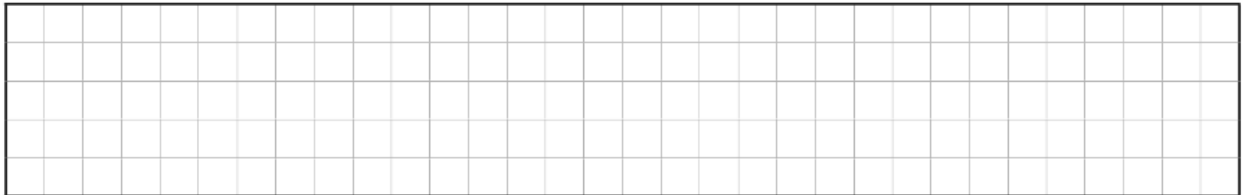
Express in its simplest form: $\frac{x+2}{3} - \frac{x-3}{4}$



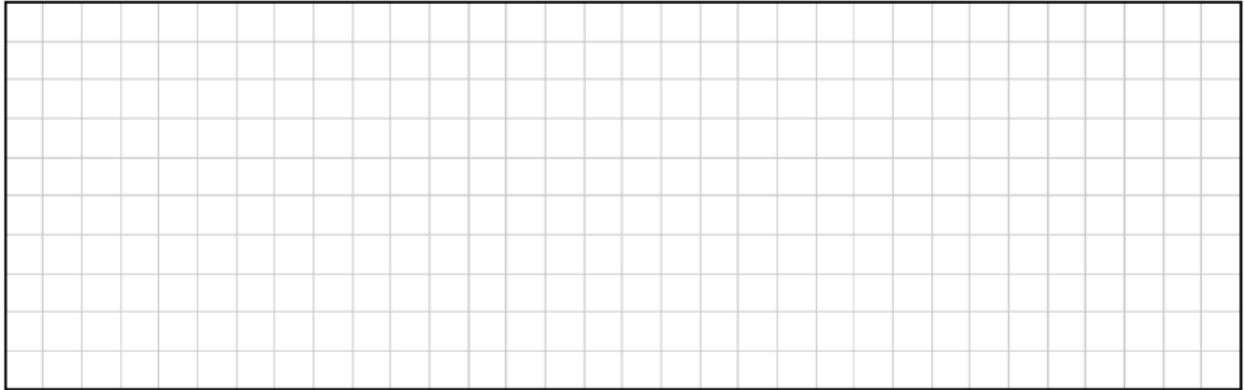
Express in its simplest form: $\frac{2}{n-3} - \frac{5}{2n+5}$



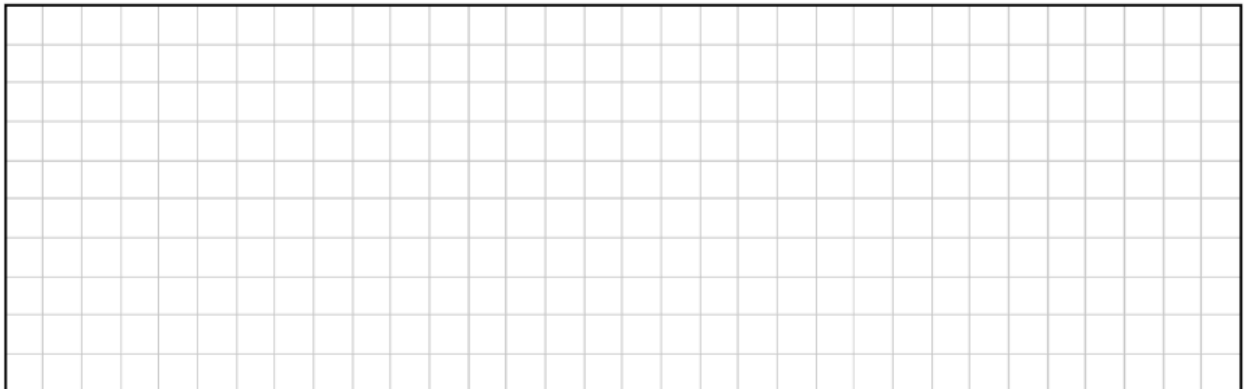
Express in its simplest form: $\frac{3}{a} - \frac{a-b}{2a+b}$



Use factors to simplify: $\frac{2n^2+n-15}{n^2-9}$

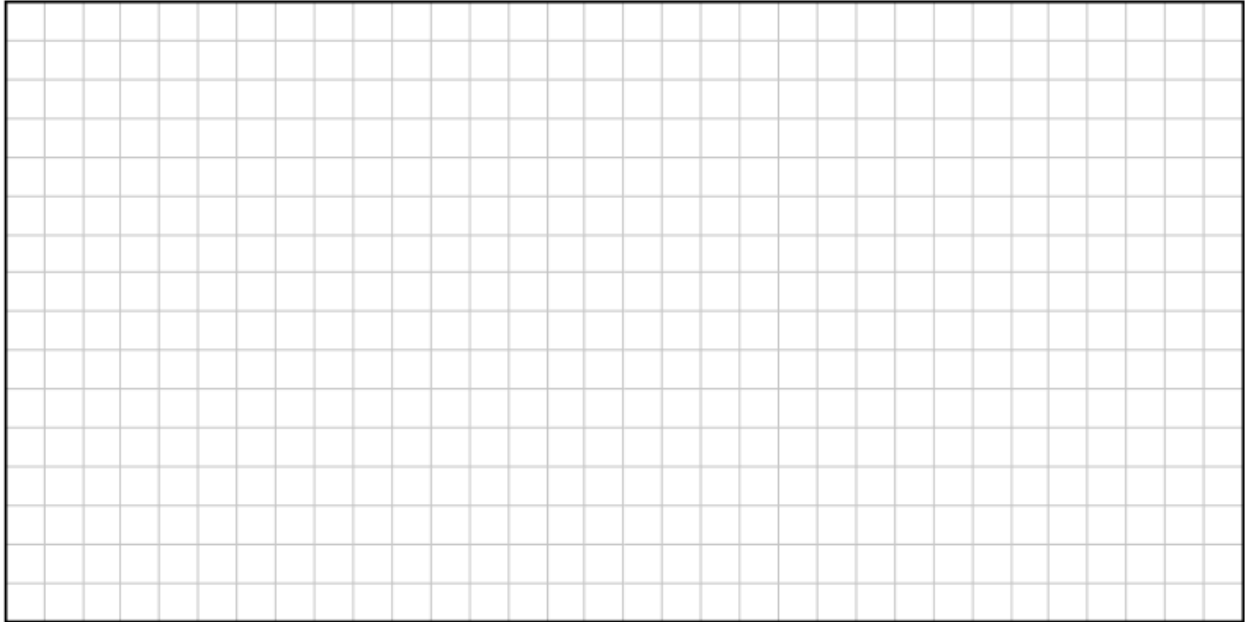


Use factors to simplify: $\frac{4e^2-9}{2e^2+3e-9}$



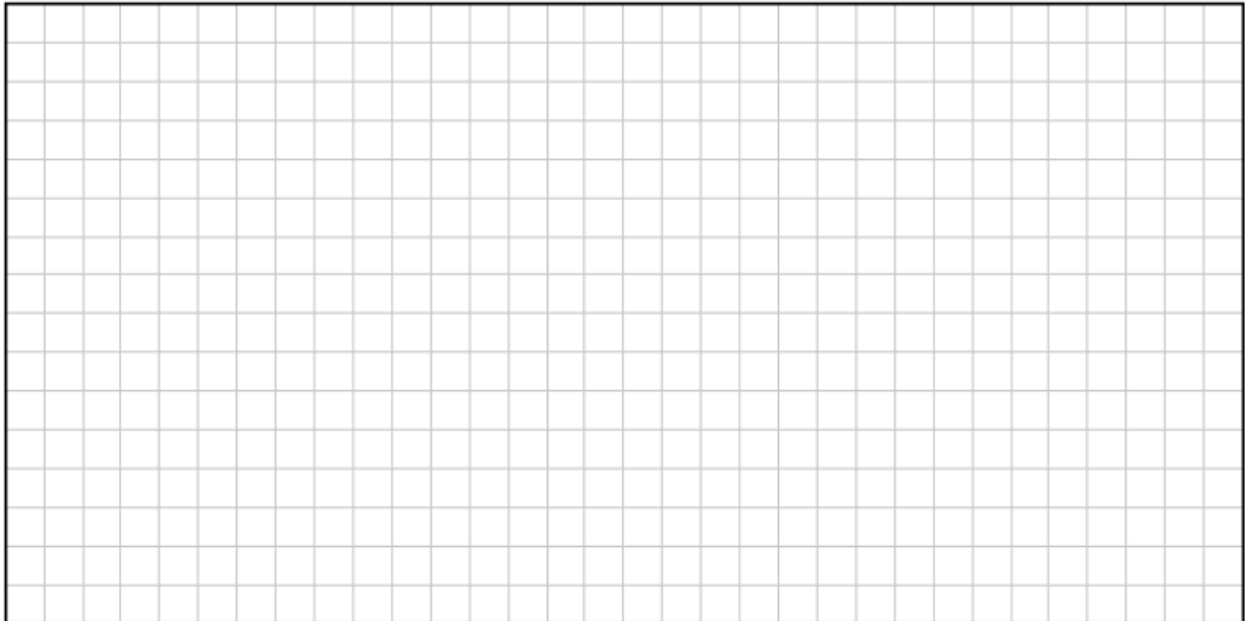
Solve the equation:

$$\frac{5x+2}{3} - \frac{6x+4}{4} = 7$$



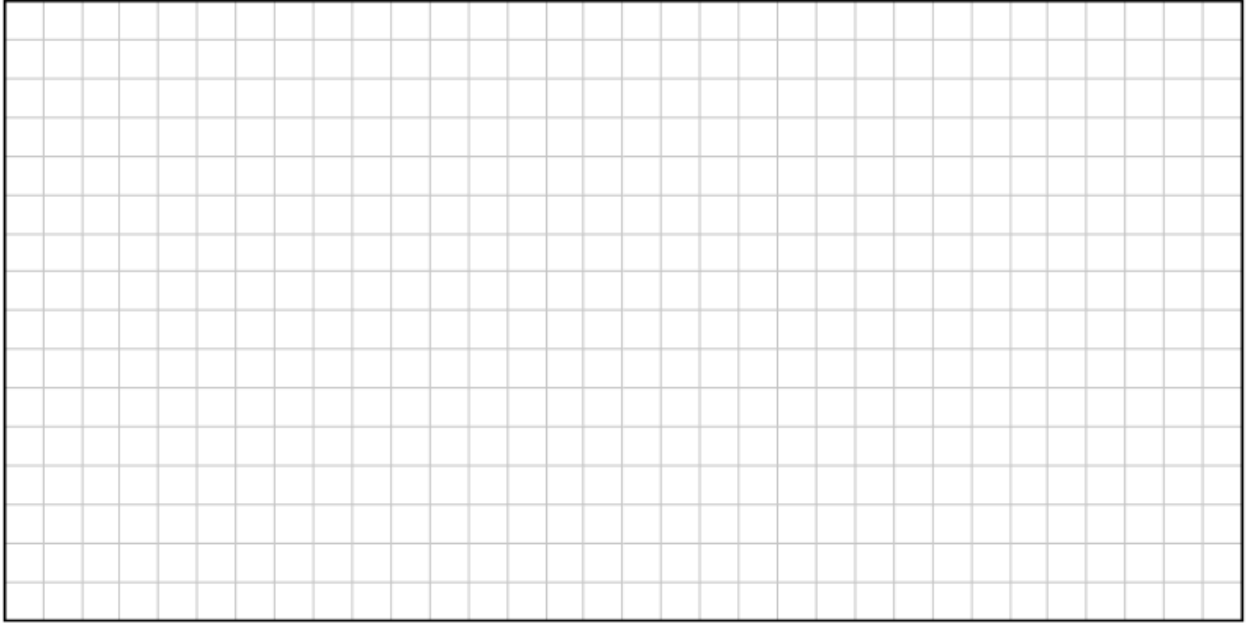
Solve the equation:

$$\frac{9x-6}{2} = \frac{3x-14}{3} + \frac{9x}{4}$$



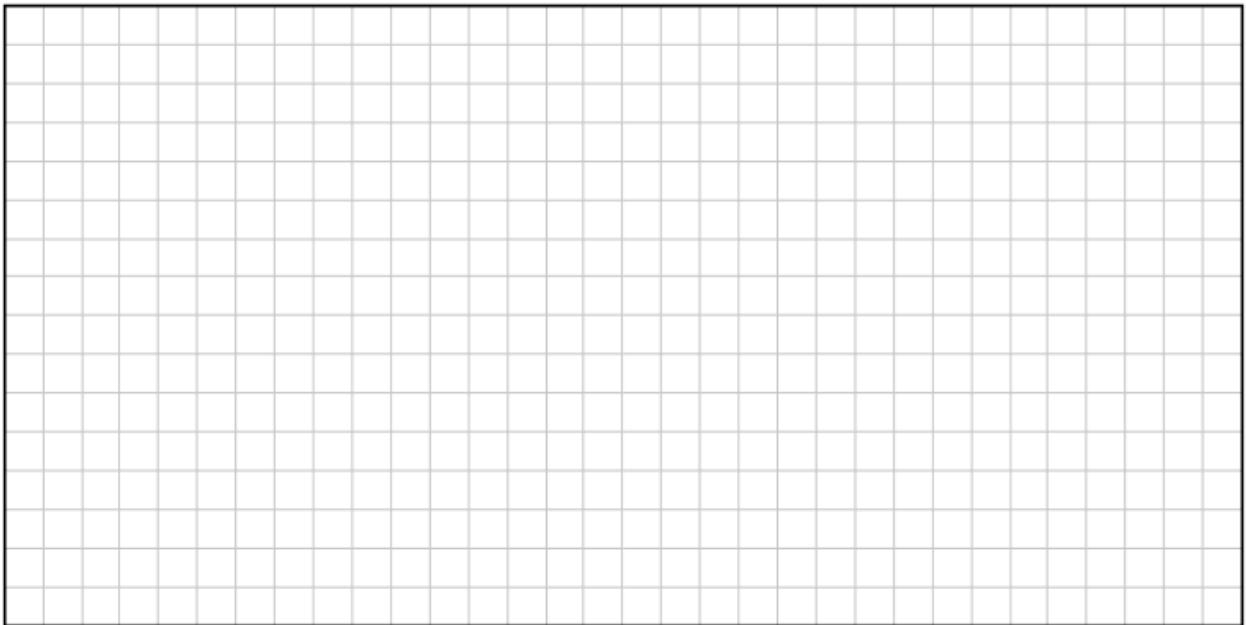
Solve the equation:

$$\frac{2x-3}{3} = \frac{x-2}{5}$$



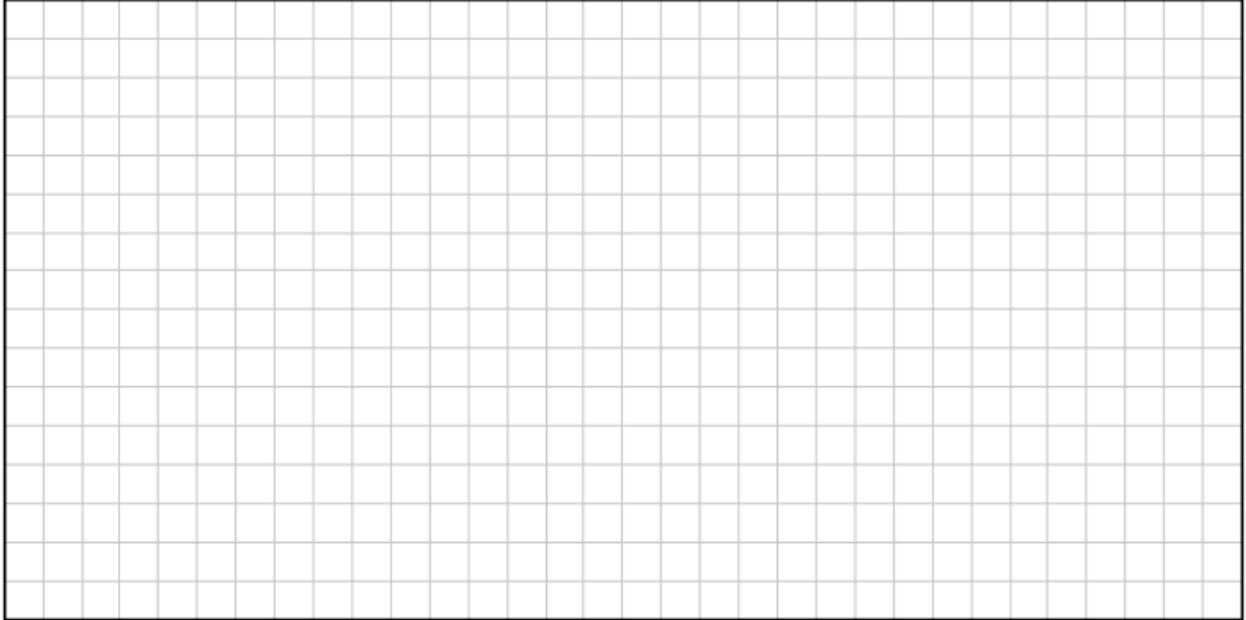
Solve the equation:

$$\frac{2x}{3} - 13 = x - 2$$



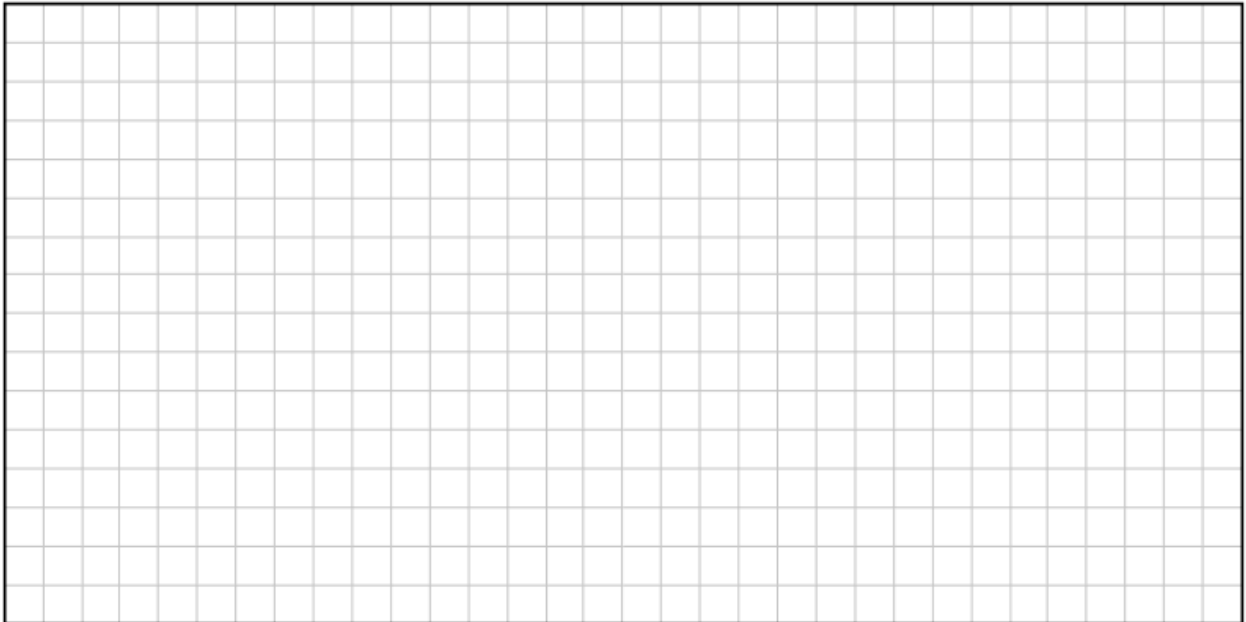
Solve the equation:

$$\frac{3}{x+1} - \frac{2}{x+4} = \frac{1}{3}$$



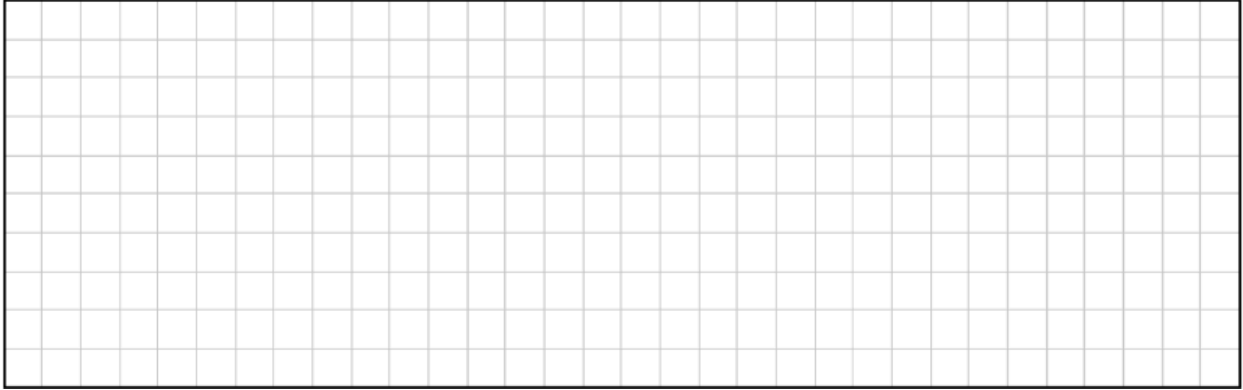
Solve the equation:

$$\frac{2}{3x-4} - \frac{1}{2x+1} = \frac{1}{2}$$



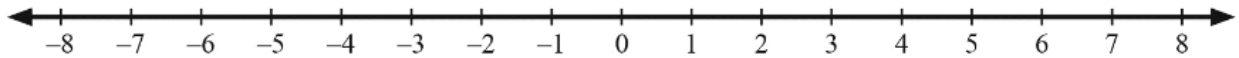
Solve the following inequality and list the possible values for x :

$$3x - 5 \leq 4, \text{ where } x \in \mathbb{N}.$$



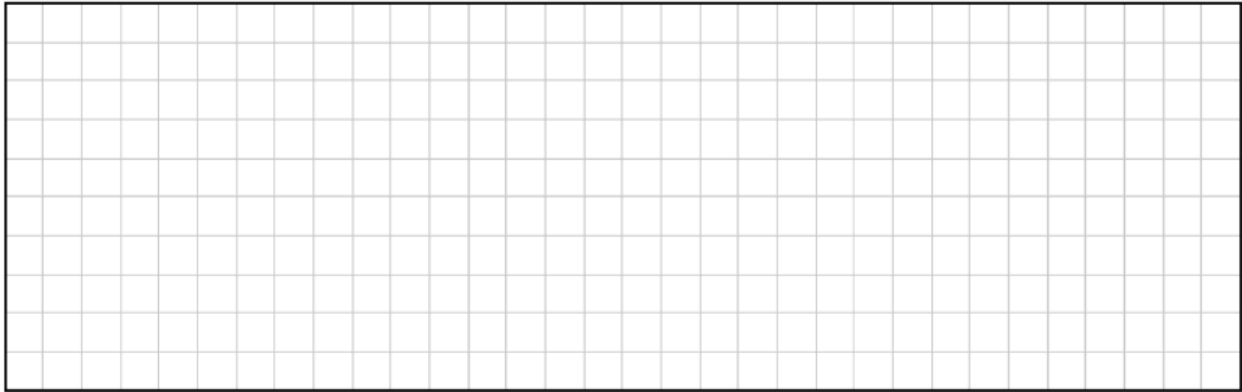
Solve the following inequality and graph the values of x :

$$4 - x \leq 2x - 5, \text{ where } x \in \mathbb{N}.$$



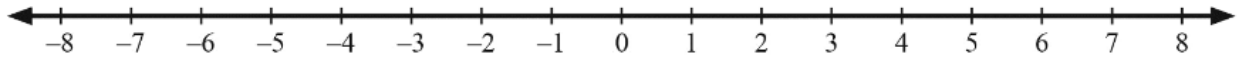
Solve the following inequality and list the possible values for x :

$$2(2x - 3) + 6x < 25, \text{ where } x \in N.$$

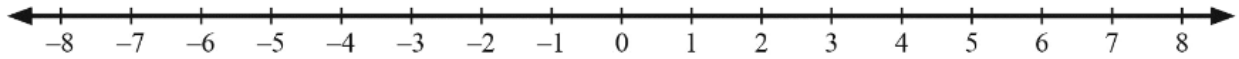


Graph each of the following inequalities on the given number line.

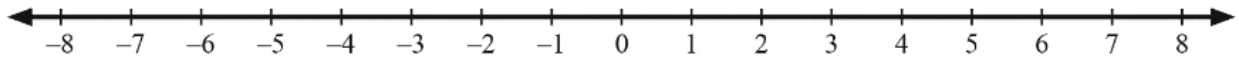
(i) $x \leq 2.8$, where $x \in R$



(ii) $x \leq 2.8$, where $x \in Z$

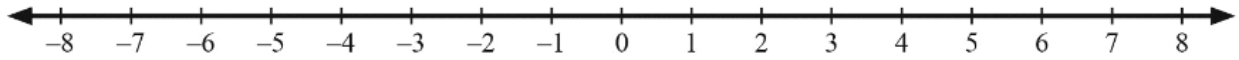
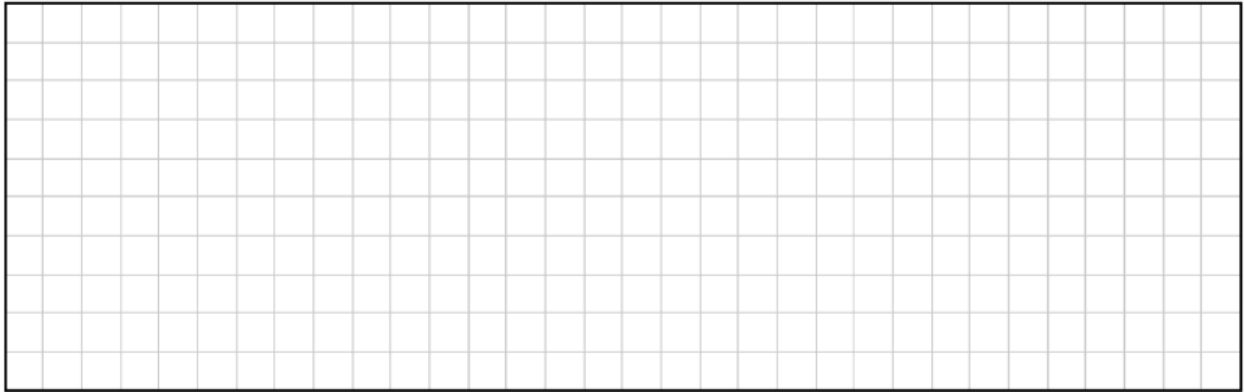


(iii) $x \leq 2.8$, where $x \in N$



Solve the following inequality for $x \in \mathbb{Z}$ and show your solutions on the following number line.

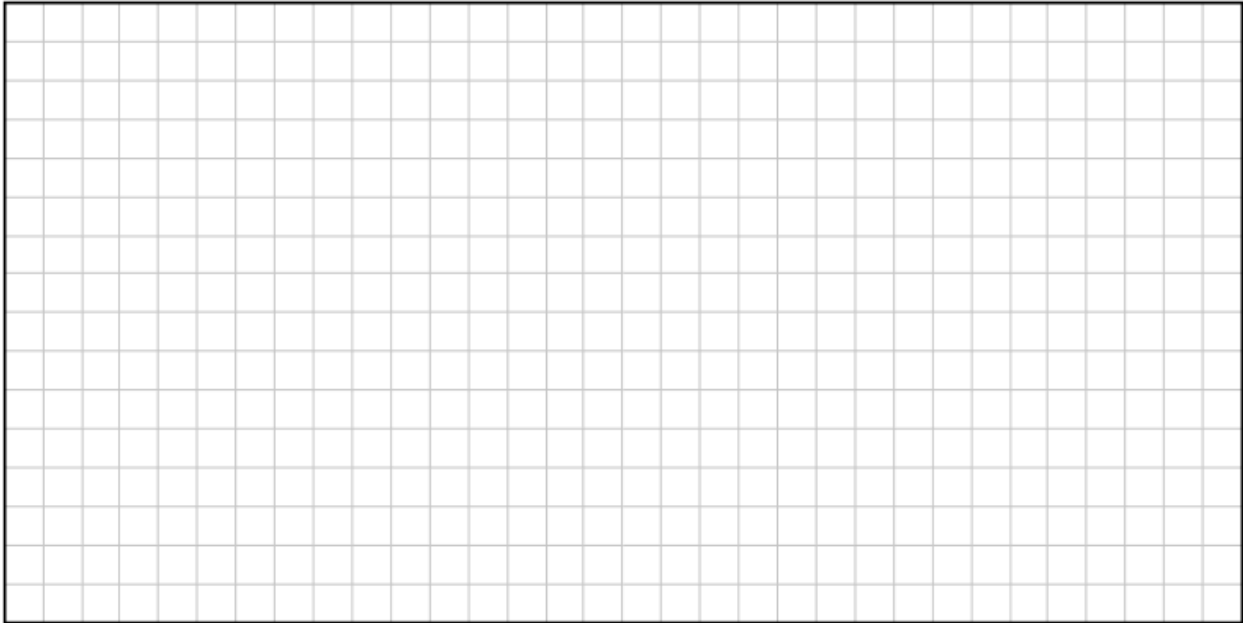
$$4(3 - x) > 8$$



Simultaneous equations: both linear

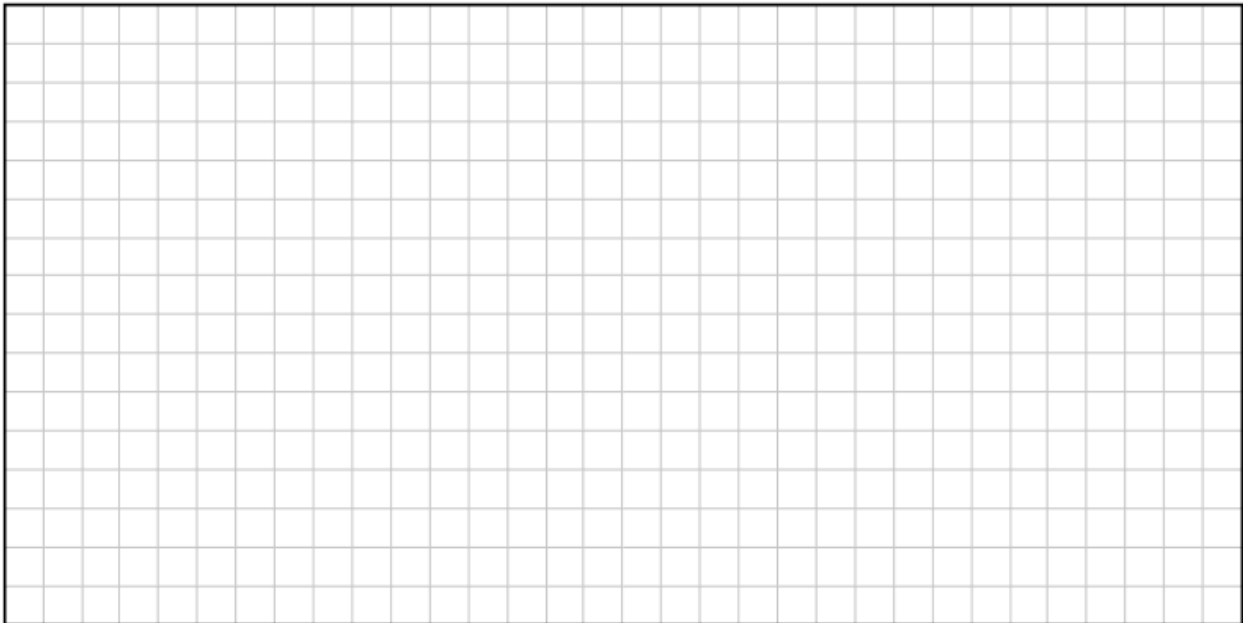
$$5x + 6y = 4$$

$$7x + 8y = 8$$



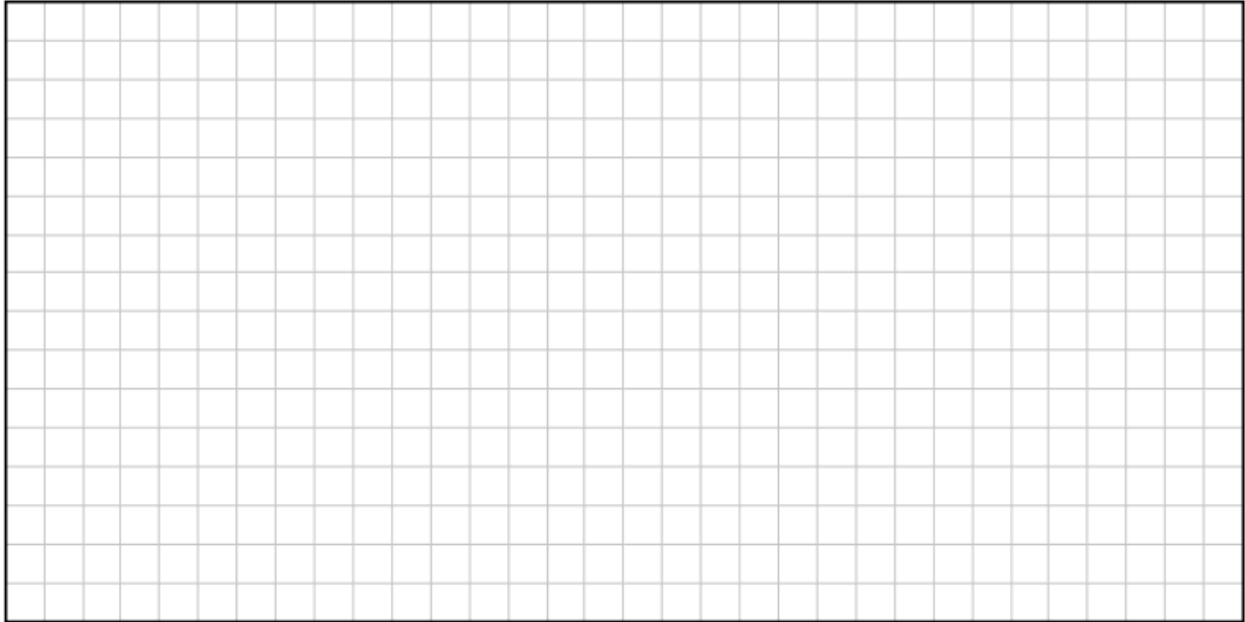
$$3x + 2y = 1$$

$$7x + 5y = -2$$



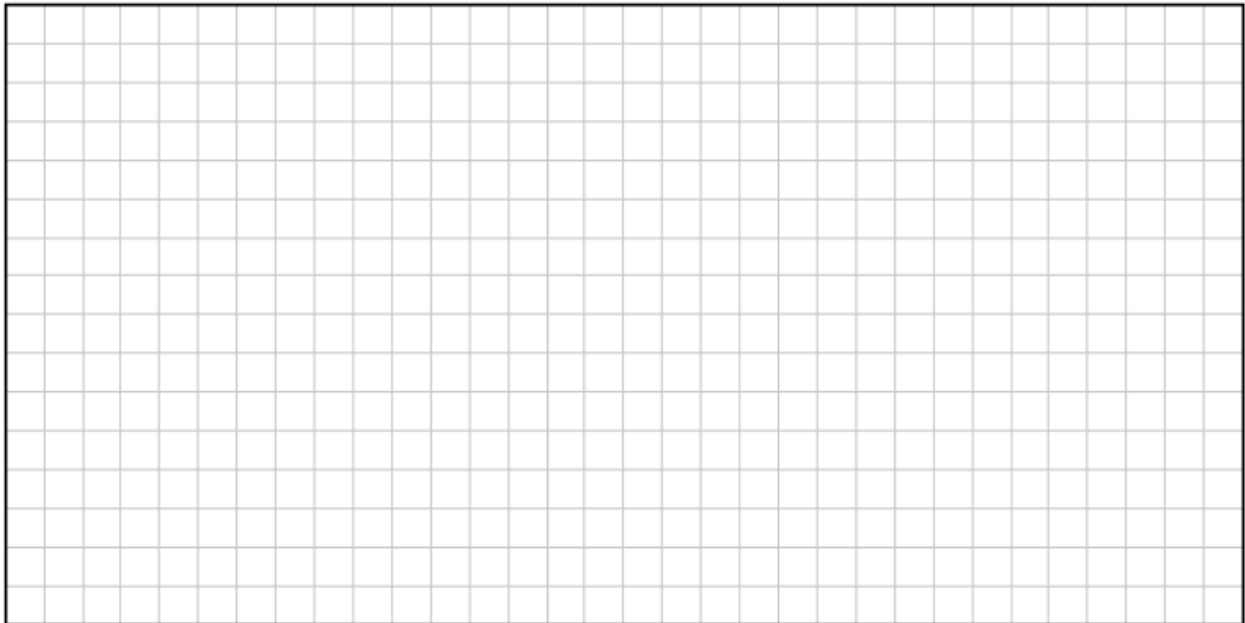
$$p + 5q = 17$$

$$5p + q = 13$$



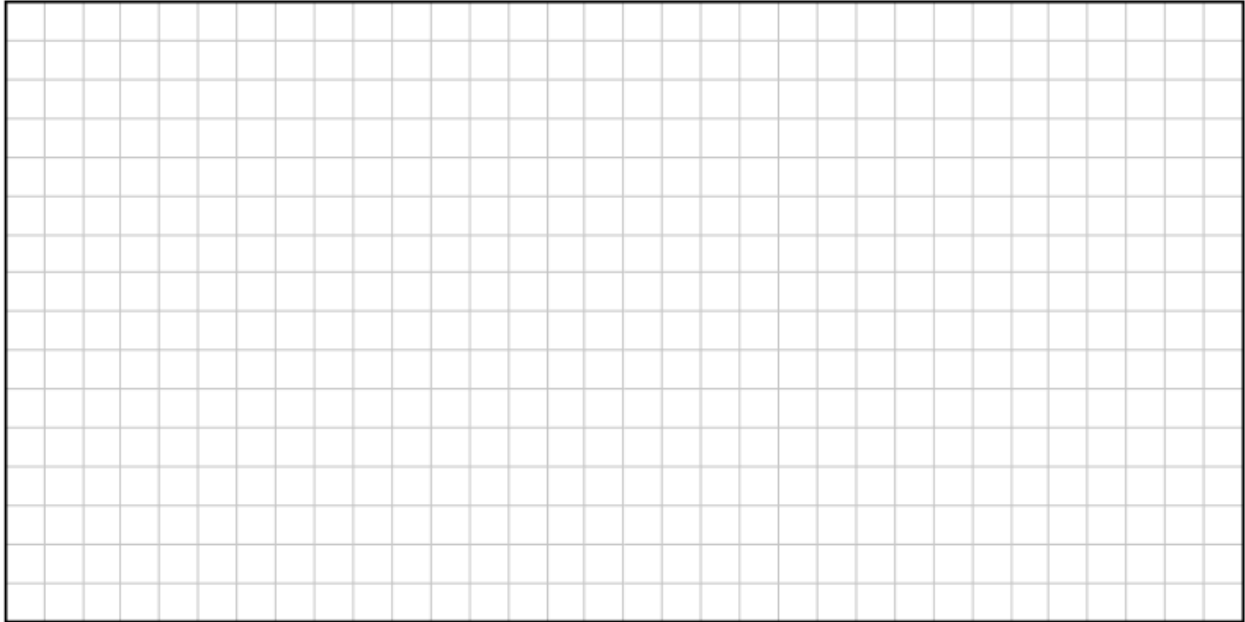
$$x = -1 + y$$

$$2x + 4y = 19$$



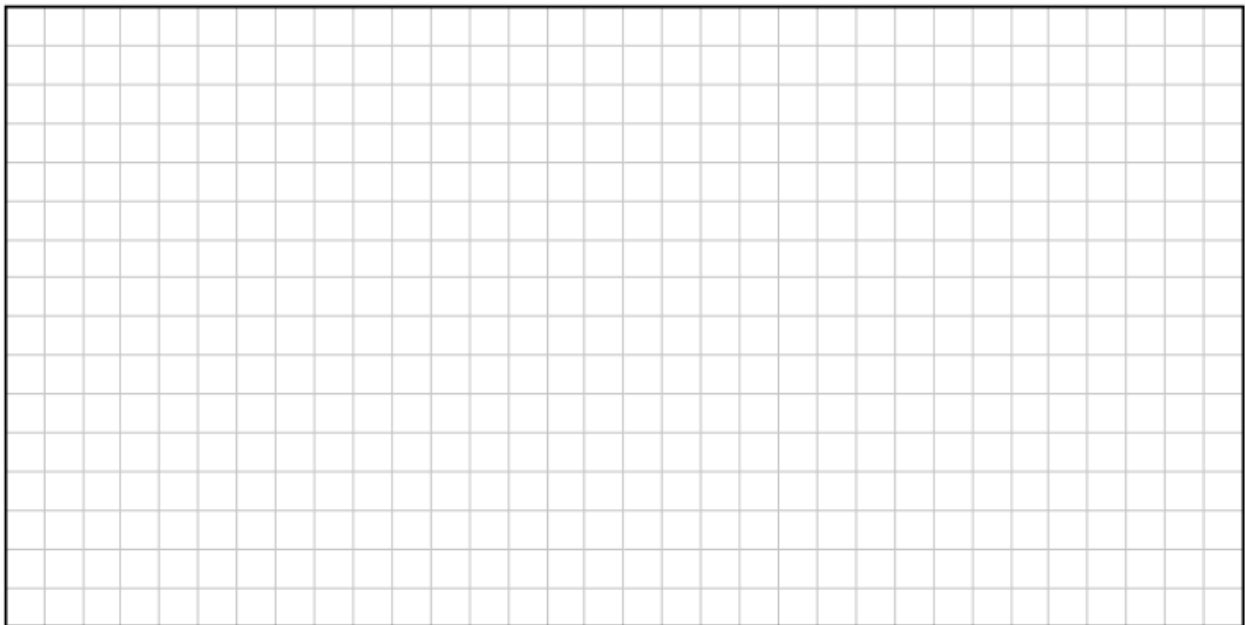
$$3x - 10 = 4y$$

$$7 + 3y = 2x$$

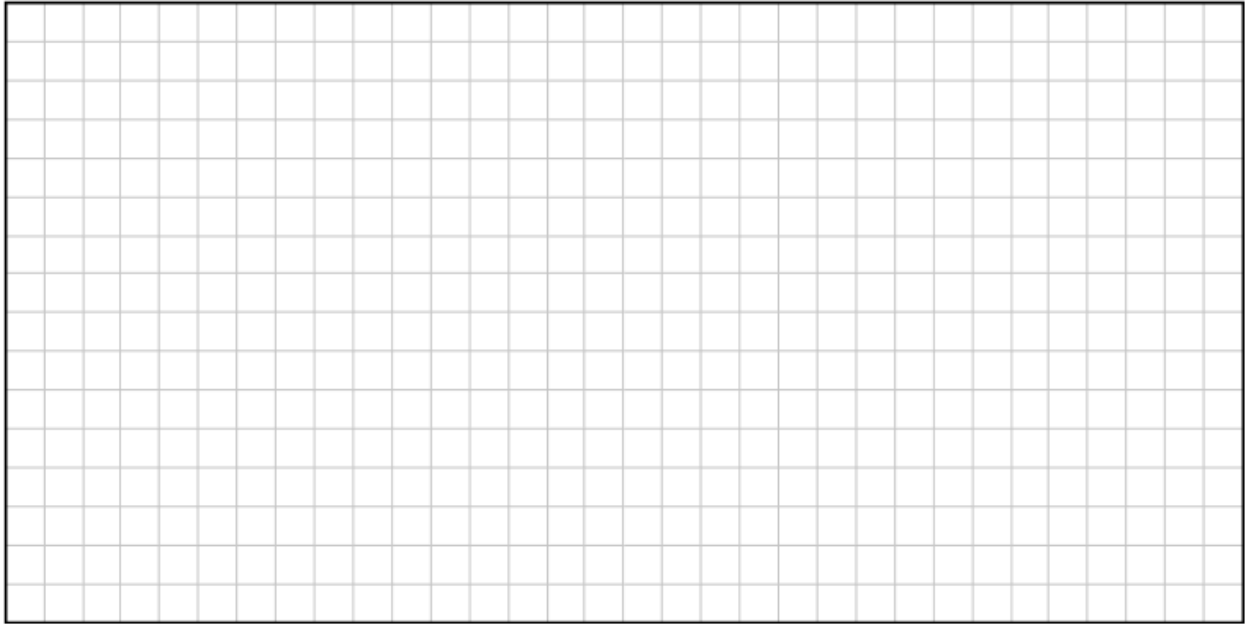


$$2a + 3b = 15$$

$$b + 8 = -5a$$



Find the point of intersection between line $l: 3x + 2y = -7$ and line $m: 5x + 37 = 3y$

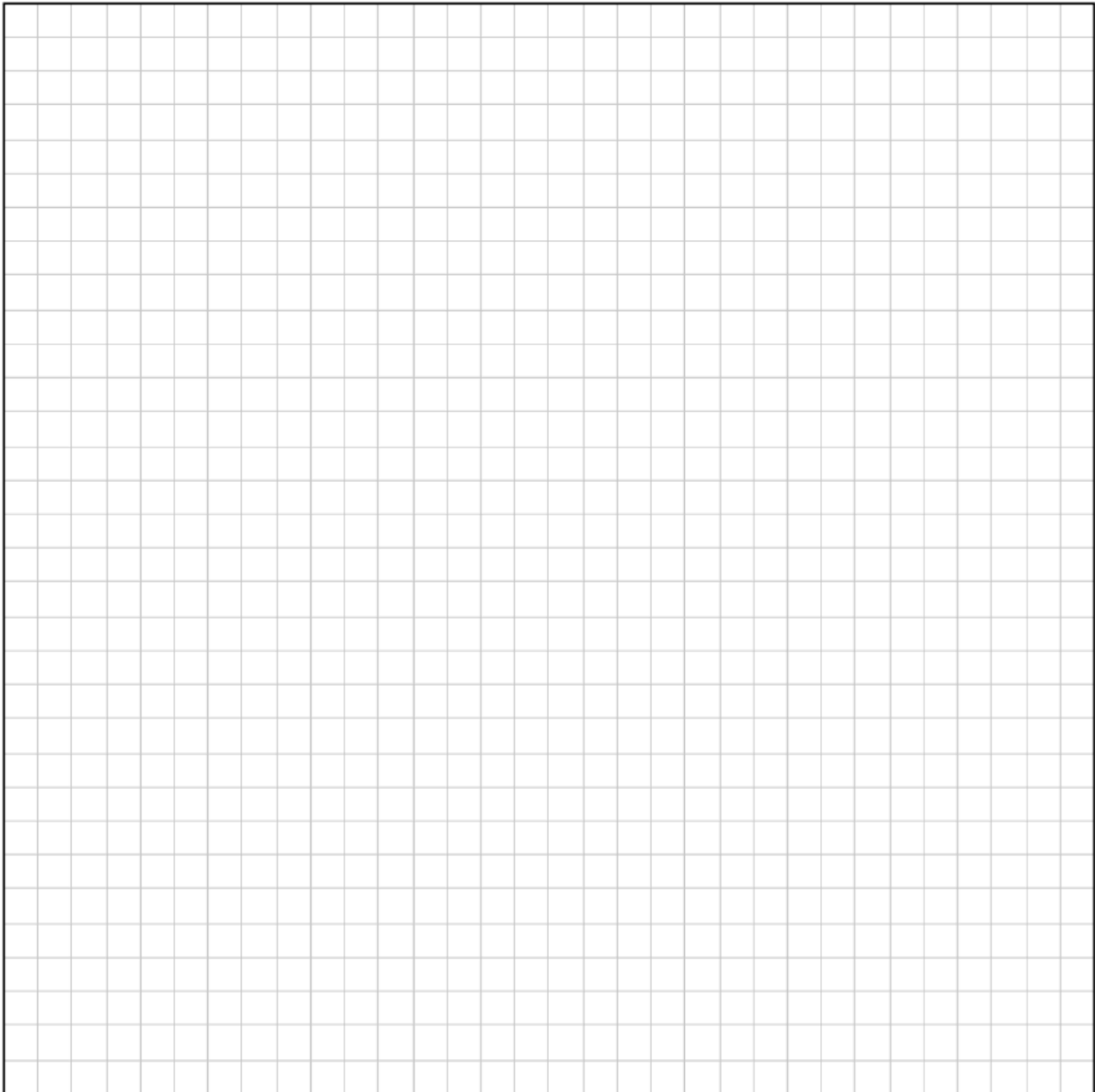


Simultaneous equations: one linear, one non-linear

Solve the following simultaneous equations:

$$2x - y = 7$$

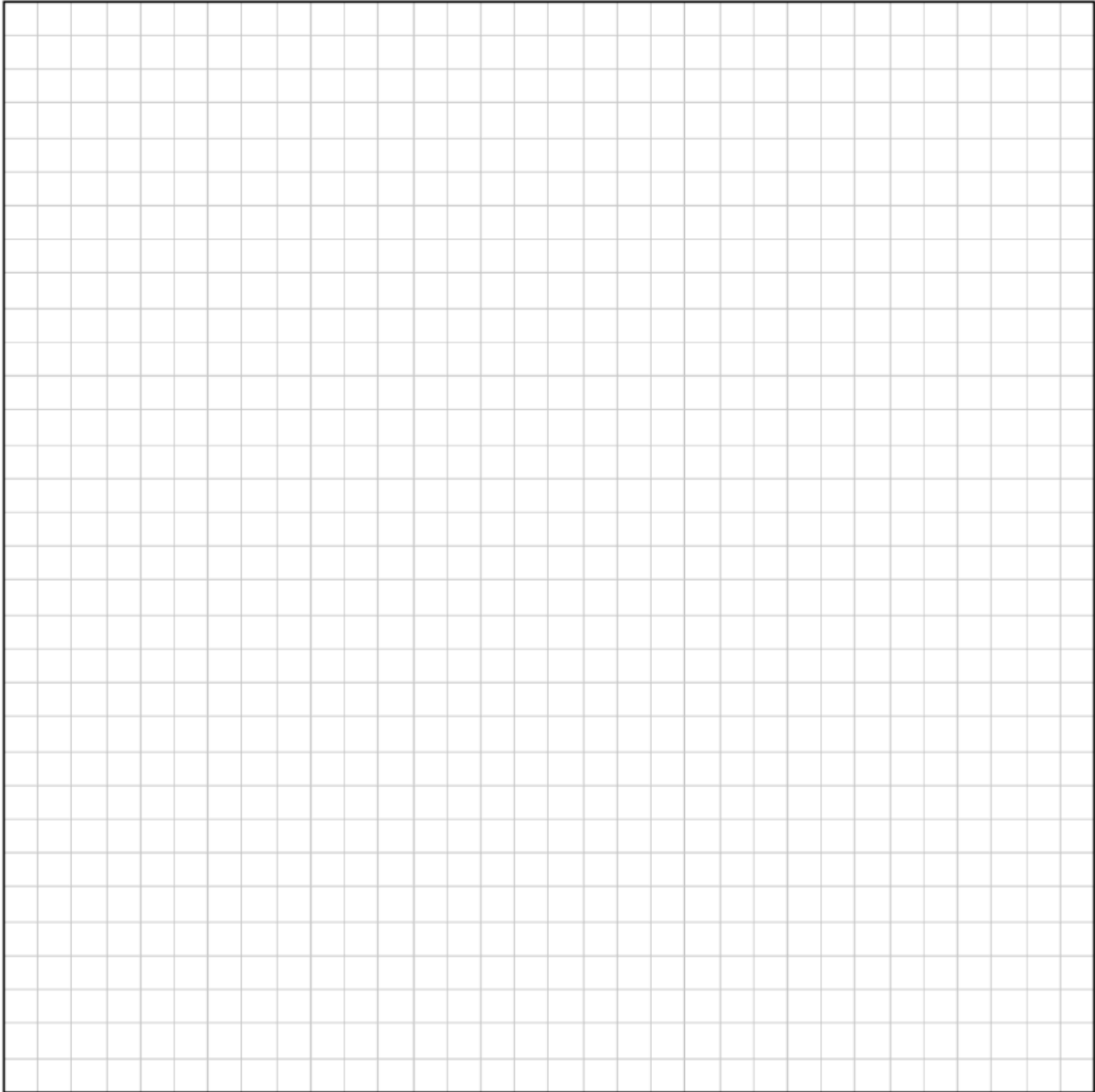
$$x^2 + y^2 = 49$$



Solve the following simultaneous equations:

$$2x - y = 10$$

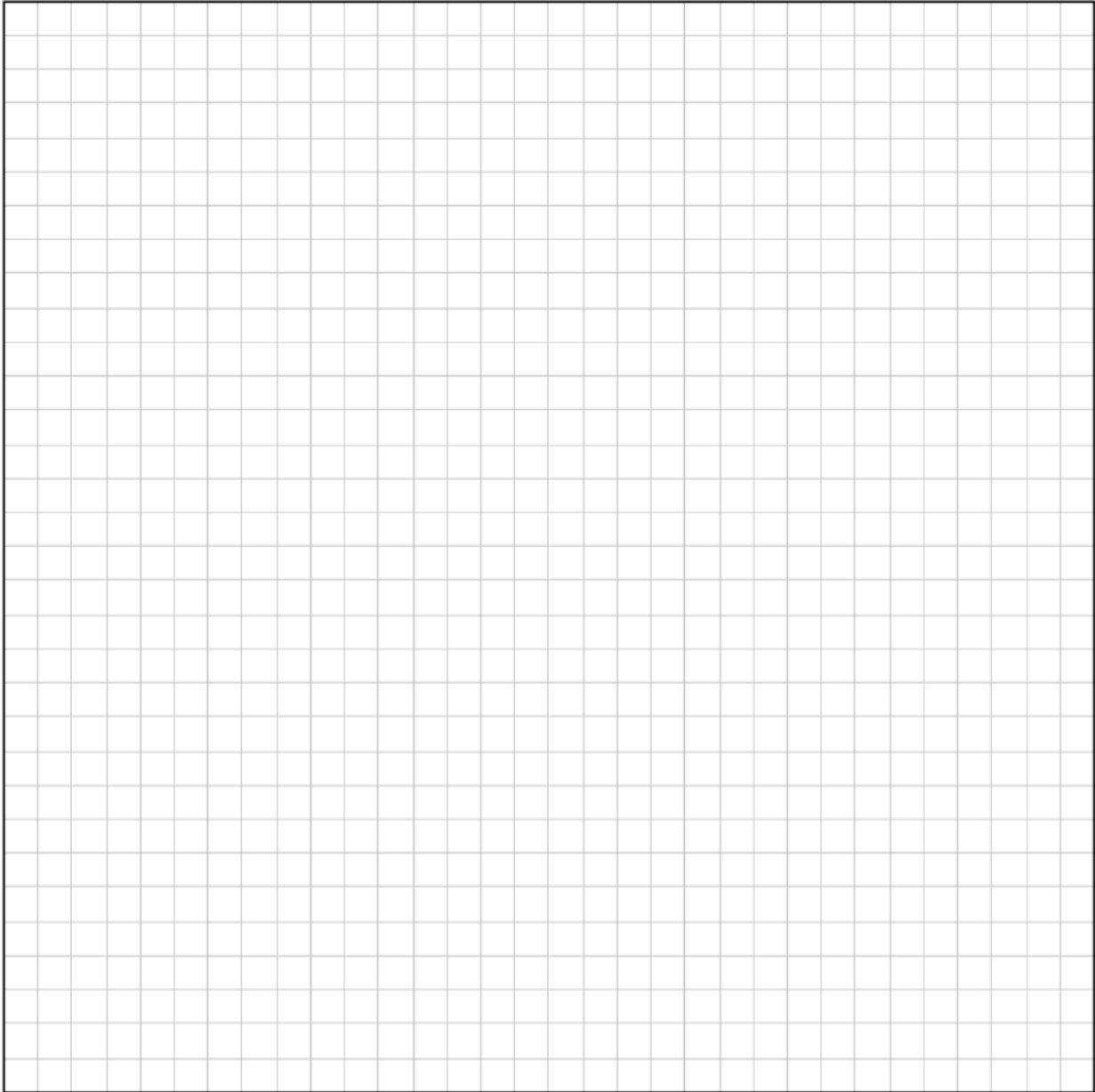
$$x^2 + y^2 = 20$$



Solve the following simultaneous equations:

$$3x - y = 4$$

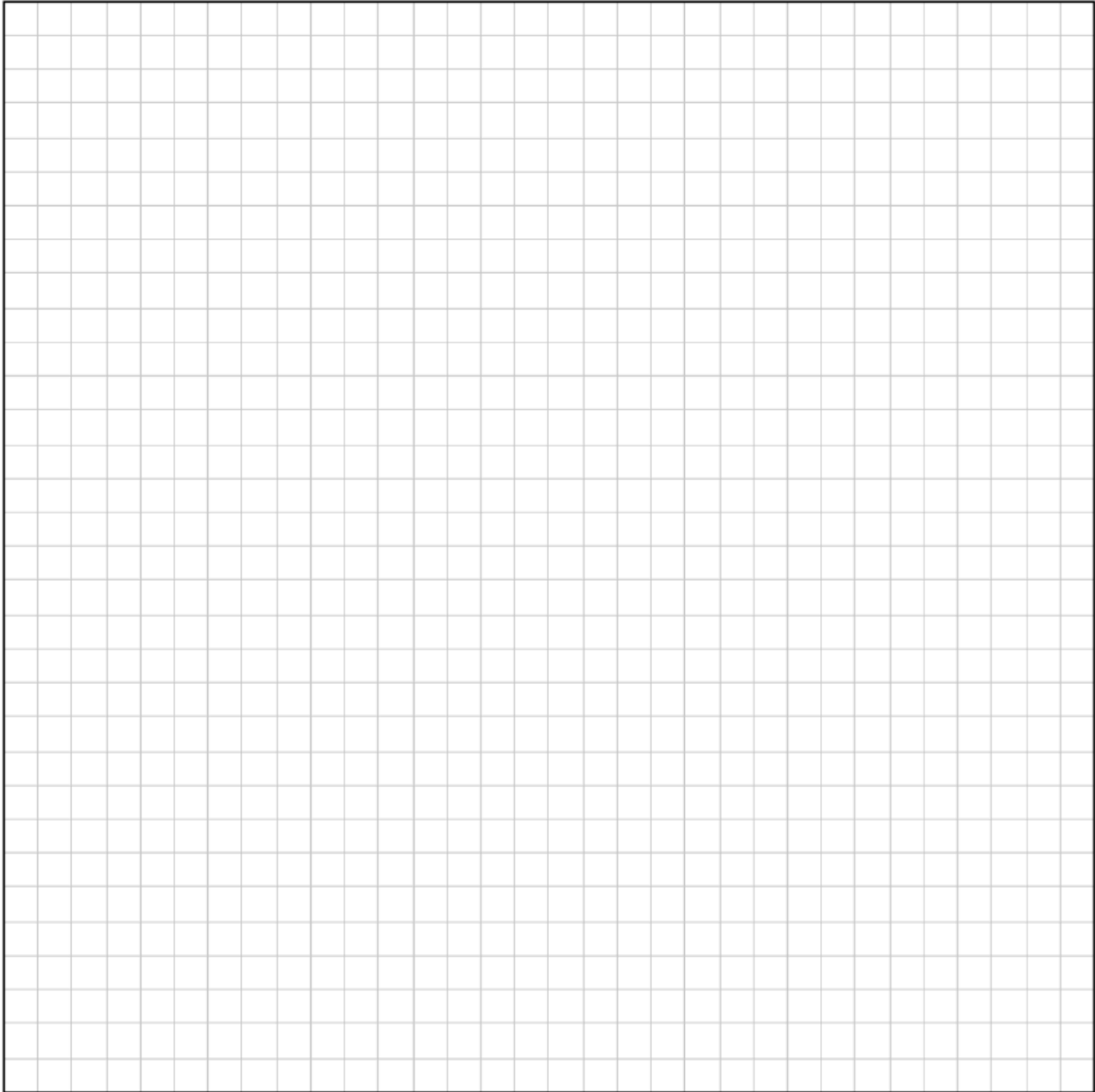
$$4x^2 - 3xy = 4$$



Solve the following simultaneous equations:

$$x + 4y = 5$$

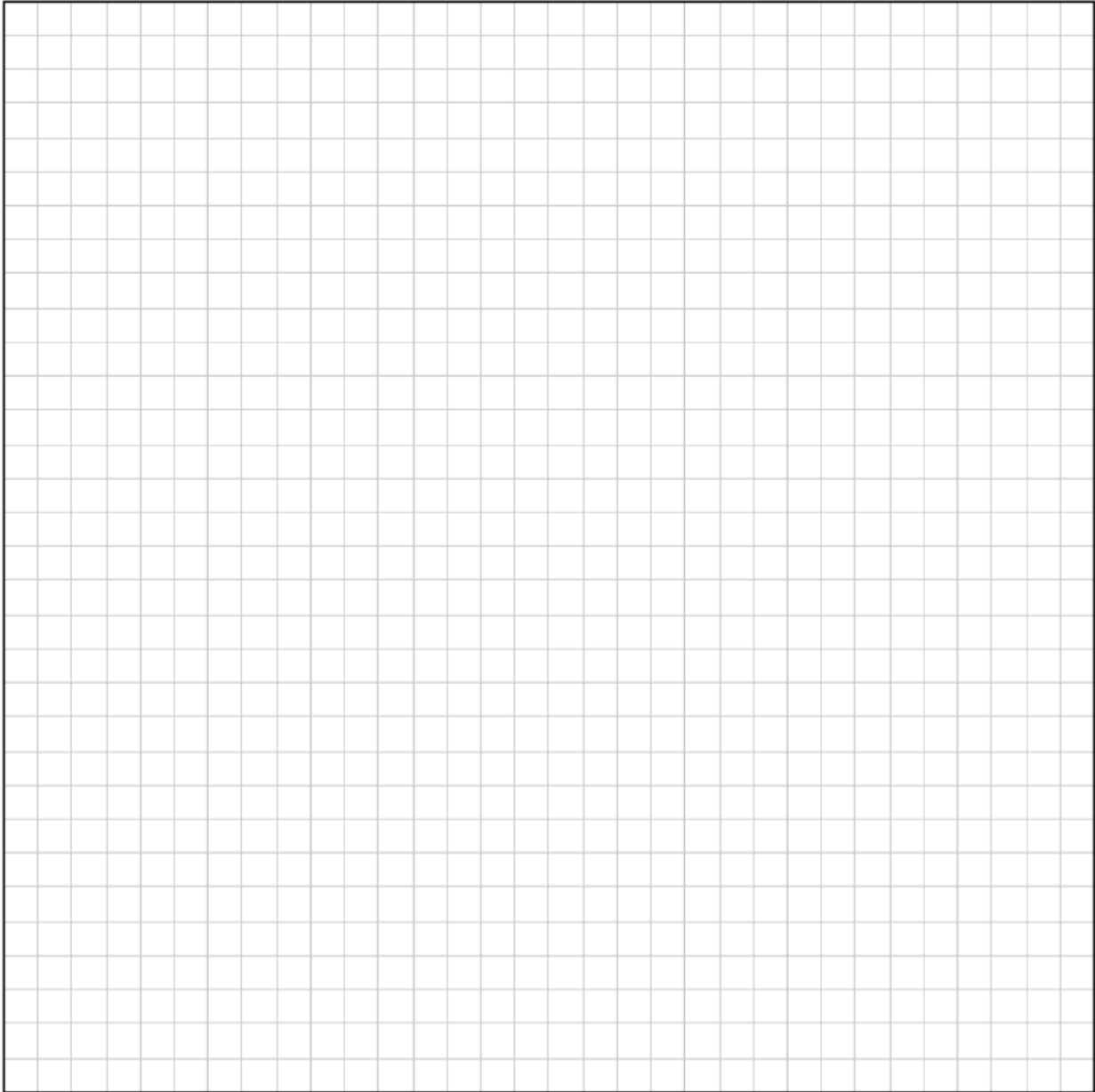
$$x^2 + y^2 = 13$$



Solve the following simultaneous equations:

$$y + 5 = 2x$$

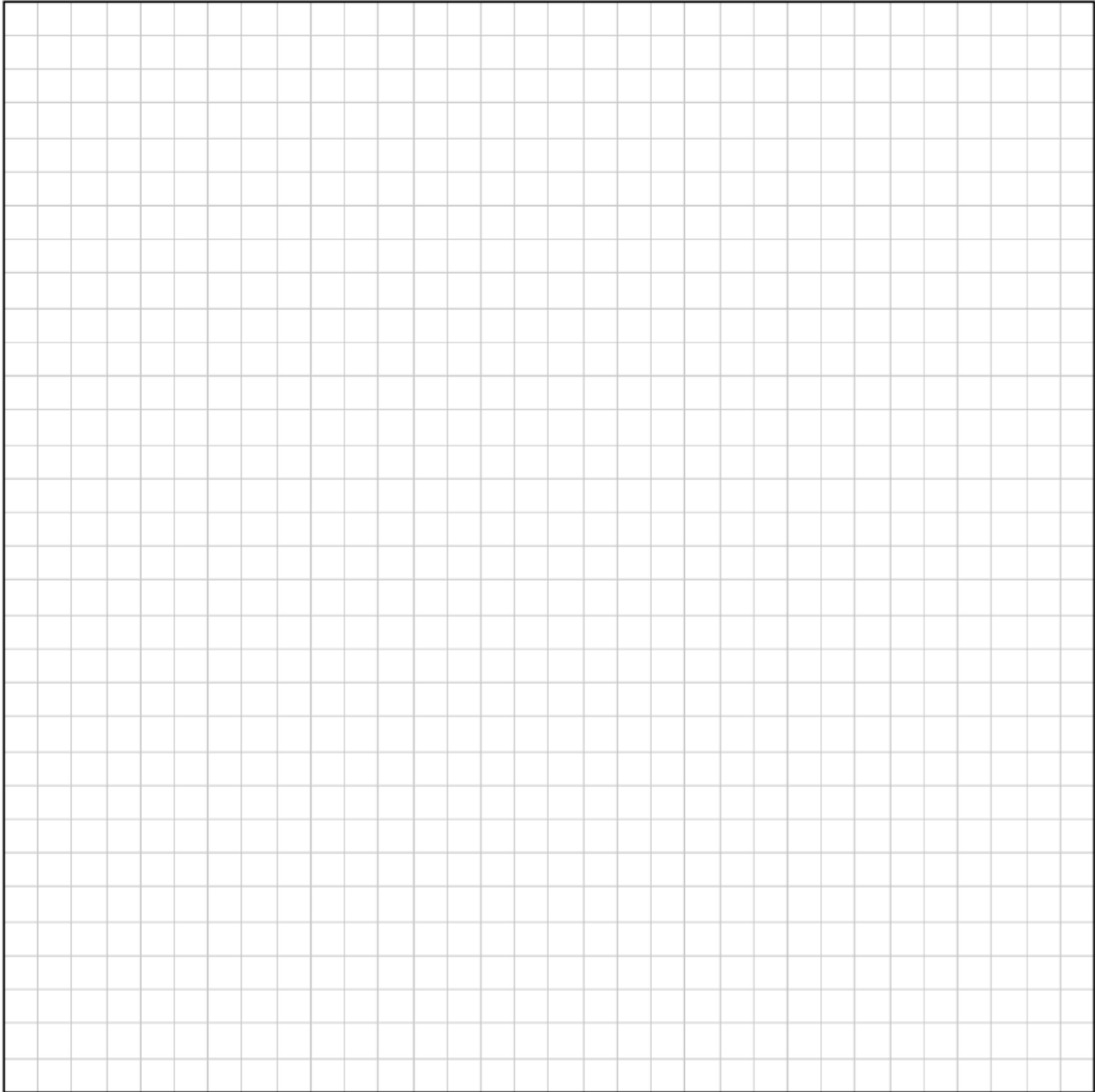
$$x^2 + y^2 = 25$$



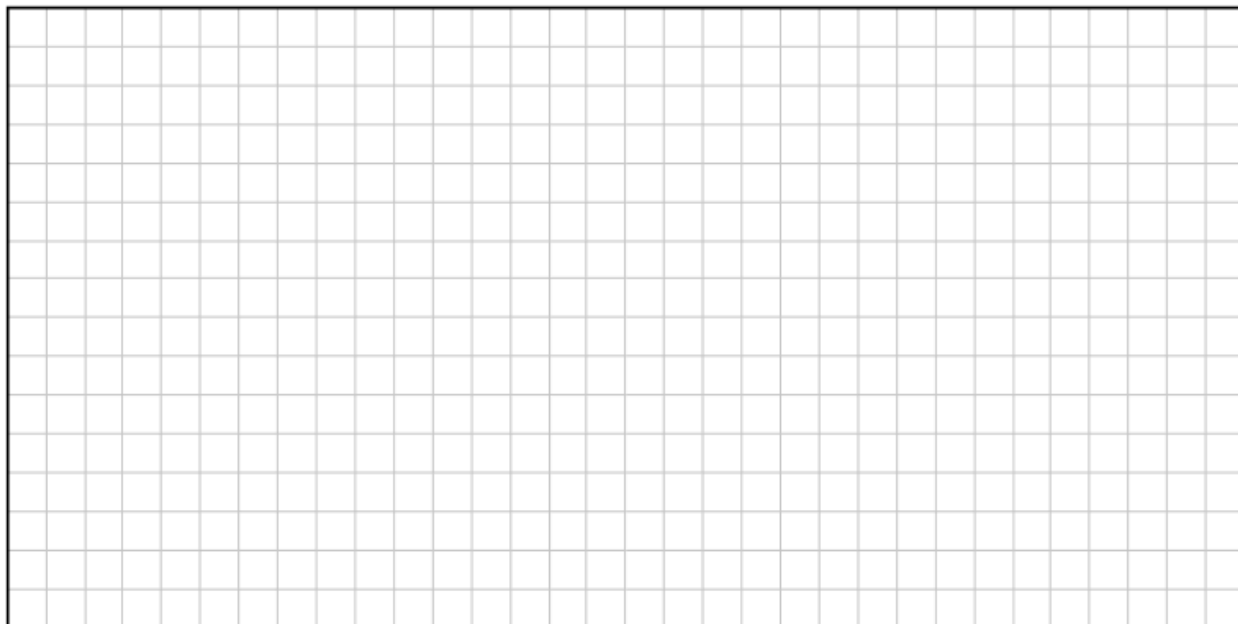
Solve the following simultaneous equations:

$$2r - s = 10$$

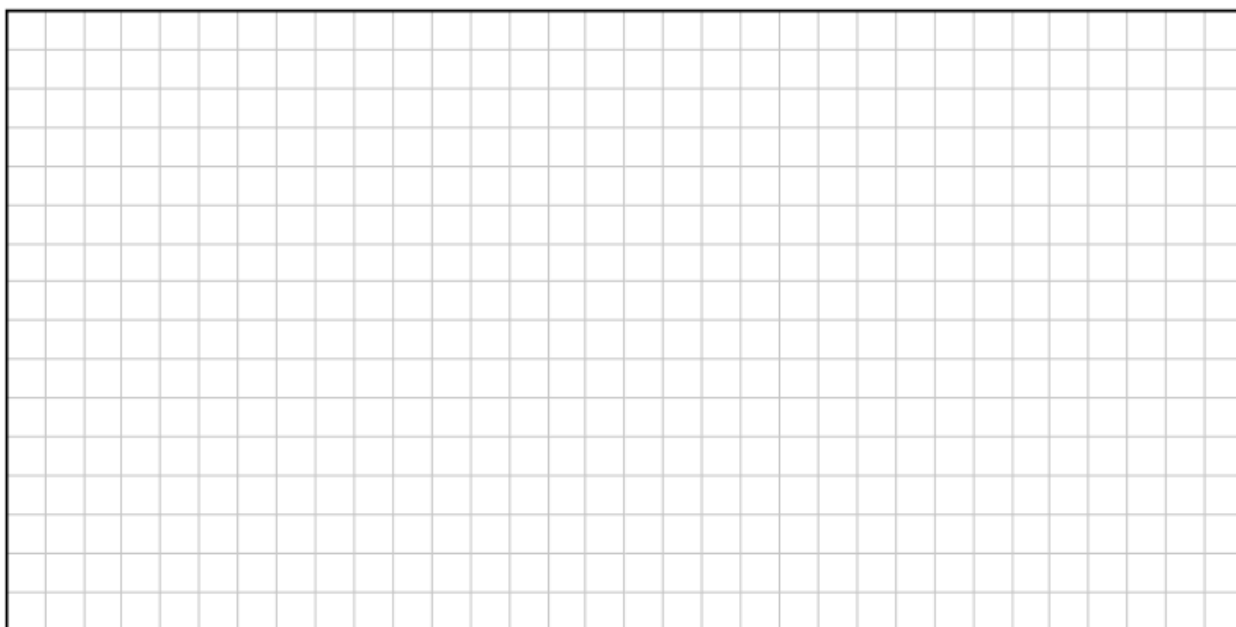
$$rs - s^2 = 12$$



Write $\frac{5^6 \times 25^3}{\sqrt{5}}$ in the form 5^p where $p \in \mathbb{Q}$.

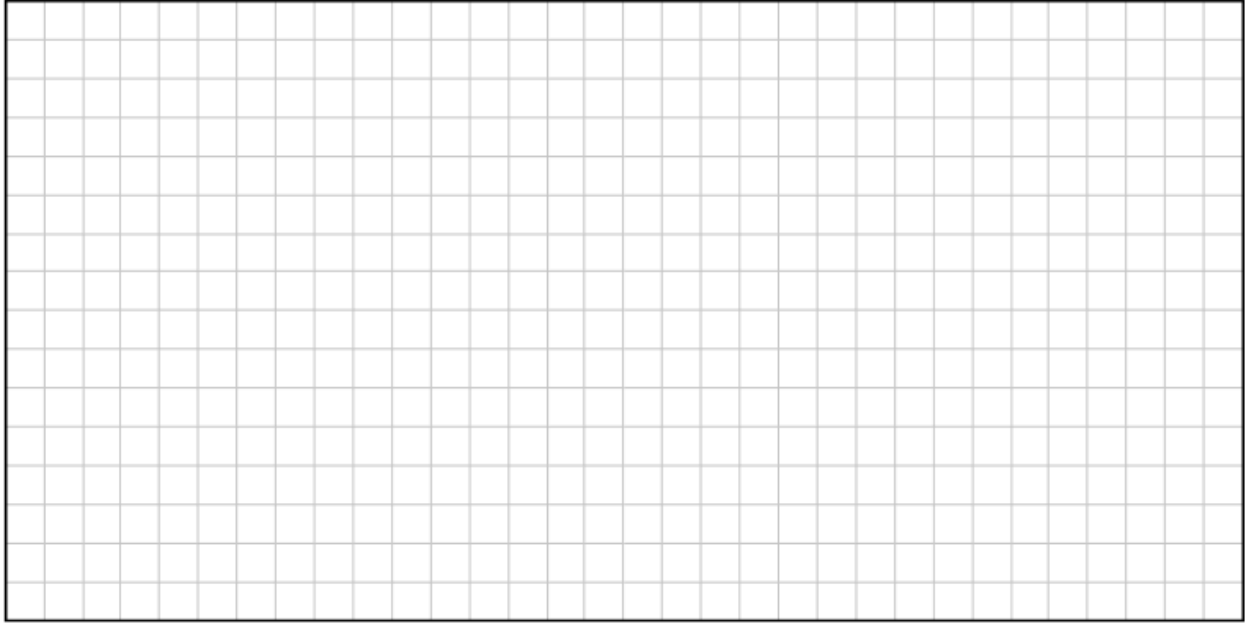


Write $\frac{2^3 \times 32}{\sqrt{2}}$ in the form 2^p where $p \in \mathbb{Q}$.



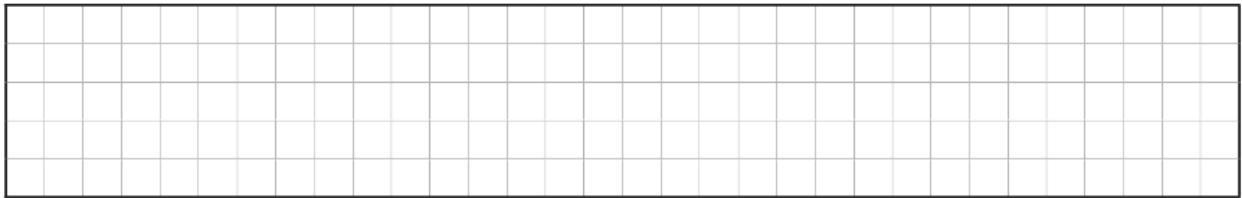
$$a^p \times a^q \times a^n = a^{12}$$

Pick 3 values such that p, q, n are all different, and the above statement remains true.

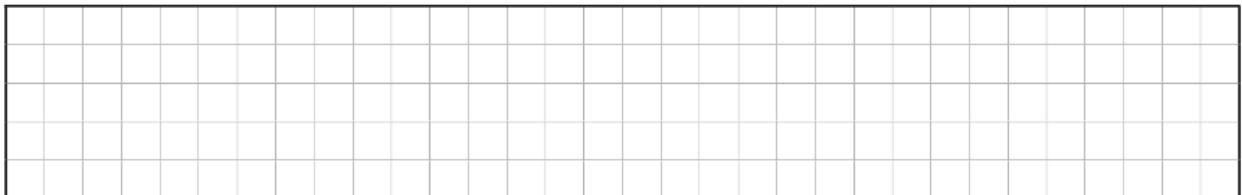


Write the following in the form 2^p where $p \in \mathbb{Q}$.

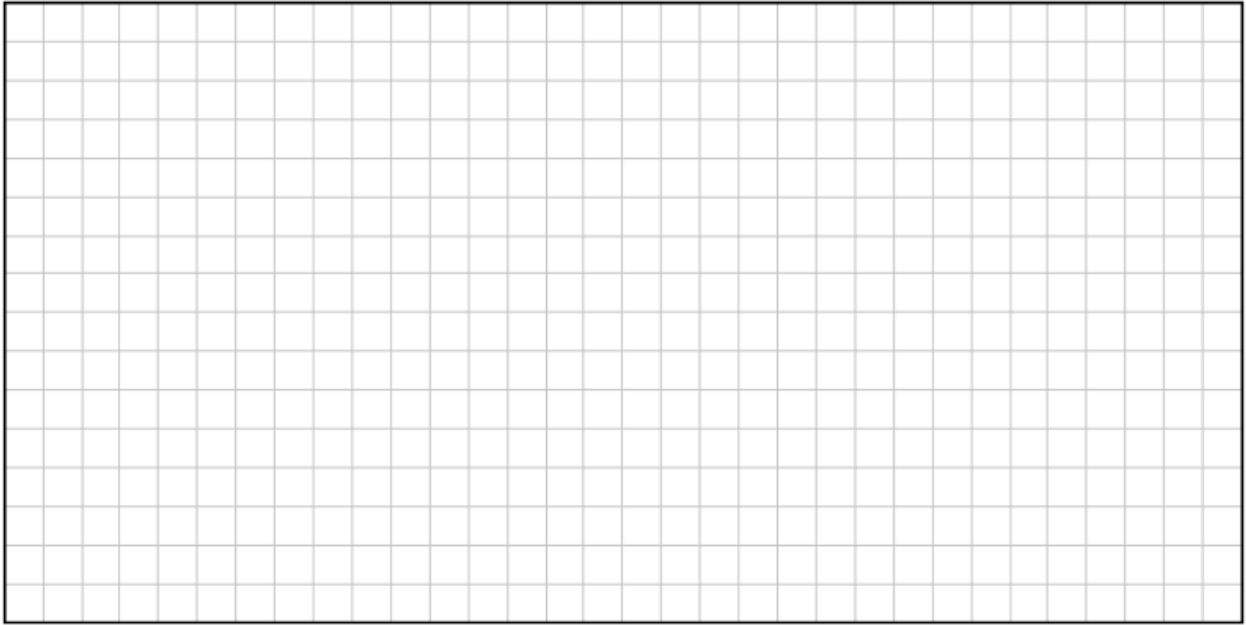
(i) 8^{25}



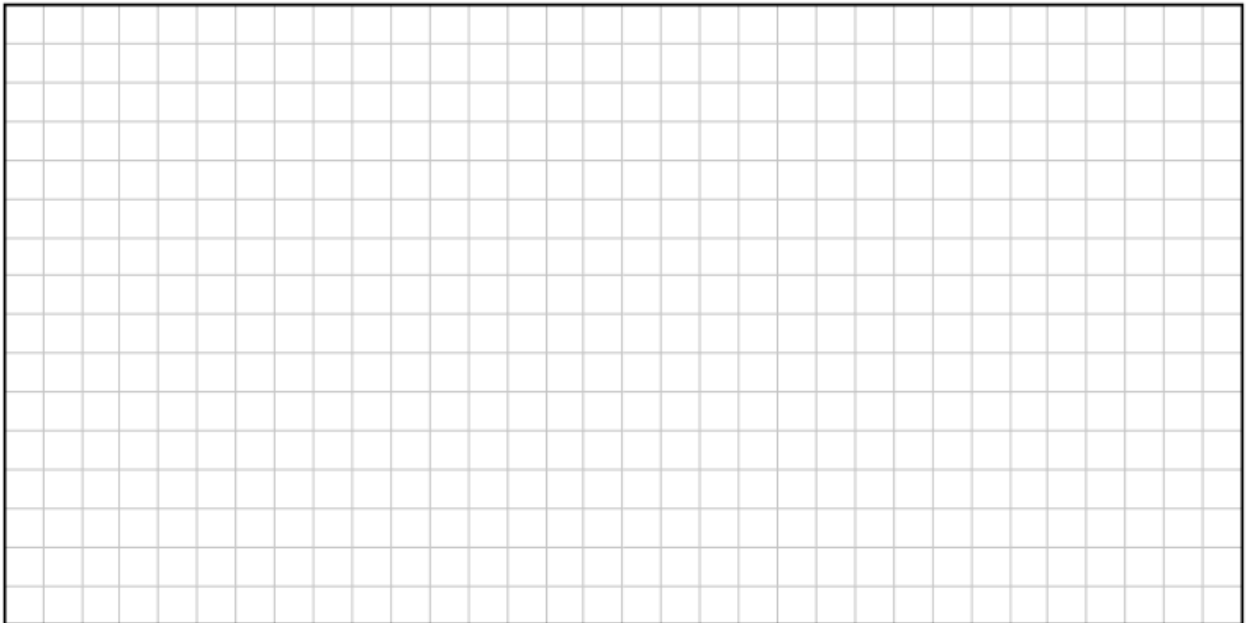
(ii) $\sqrt{32}$



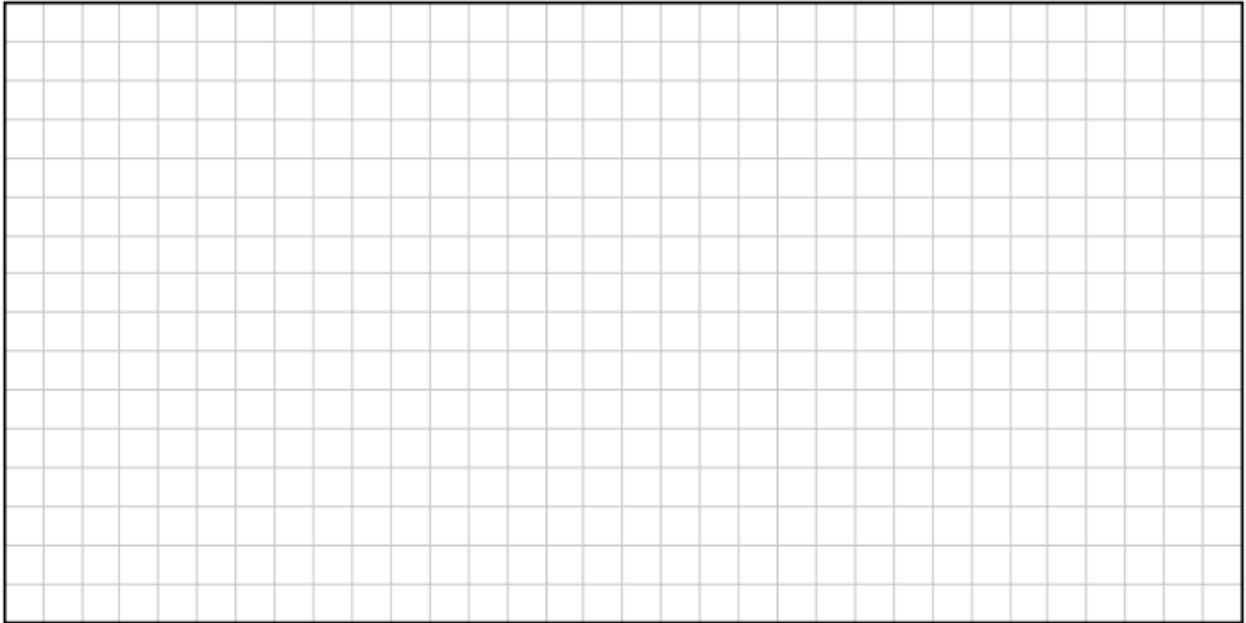
Solve the equation $3^{x-2} = 27^{2x+1}$



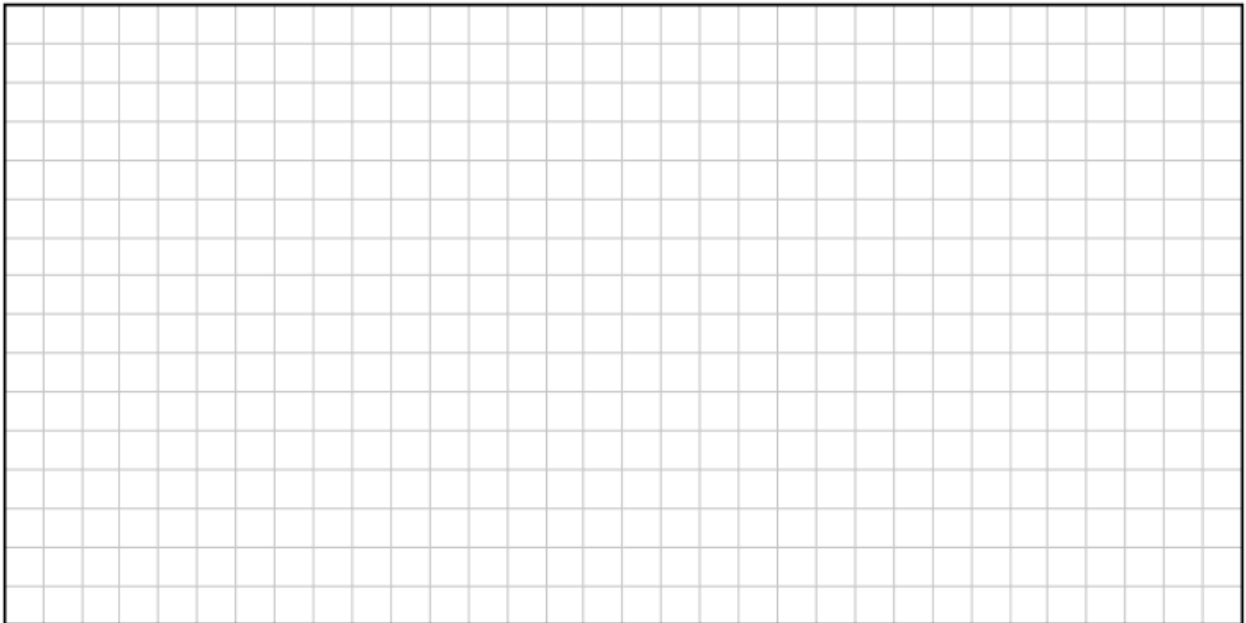
Solve the equation $2^{9x-1} = 8^{2x}$



Solve the equation $2^{2x-1} = 64$.



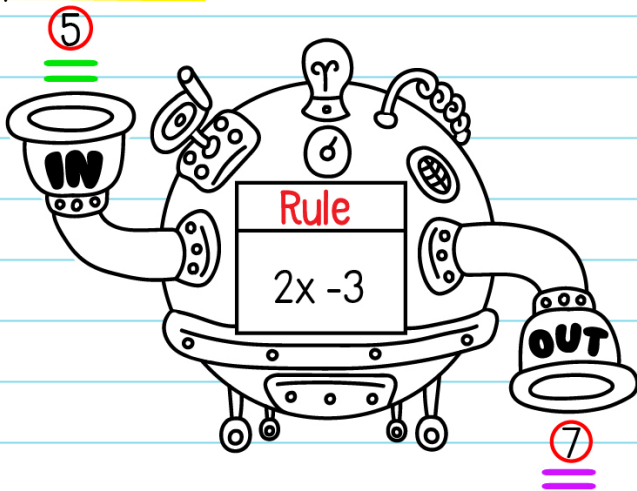
Solve the equation $49^x = 7^{2+x}$ and **verify** your answer



Chapter 2

FUNCTIONS

Input/x-value



$$2(5) - 3 \quad \text{output/y-value}$$

Every input has a valid output

Working :

$$2(5) - 3 = 7$$

Example :

$$f(x) = 2x + 3$$

①

Find $f(2)$

$$\begin{aligned} f(2) &= 2(2) + 3 \\ &= 7 \end{aligned}$$

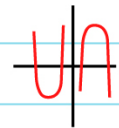
②

If $f(x) = 9$, Solve for x

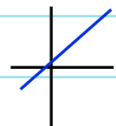
$$\begin{aligned} 2x + 3 &= 9 \\ 2x &= 9 - 3 \\ 2x &= 6 \\ x &= 3 \end{aligned}$$

Shapes of graphs

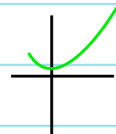
Quadratic



Linear



Exponential



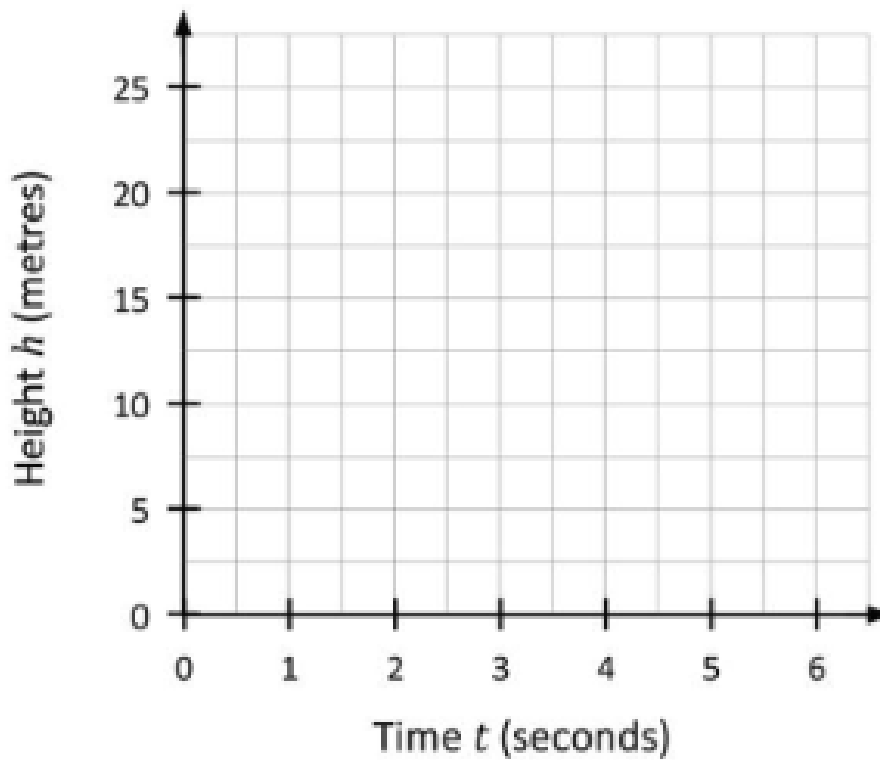
Rory kicks a football while standing on a slope. The height of the ball, in metres, could be modelled by the following quadratic function:

$$h(t) = 5 + 12t - 2t^2$$

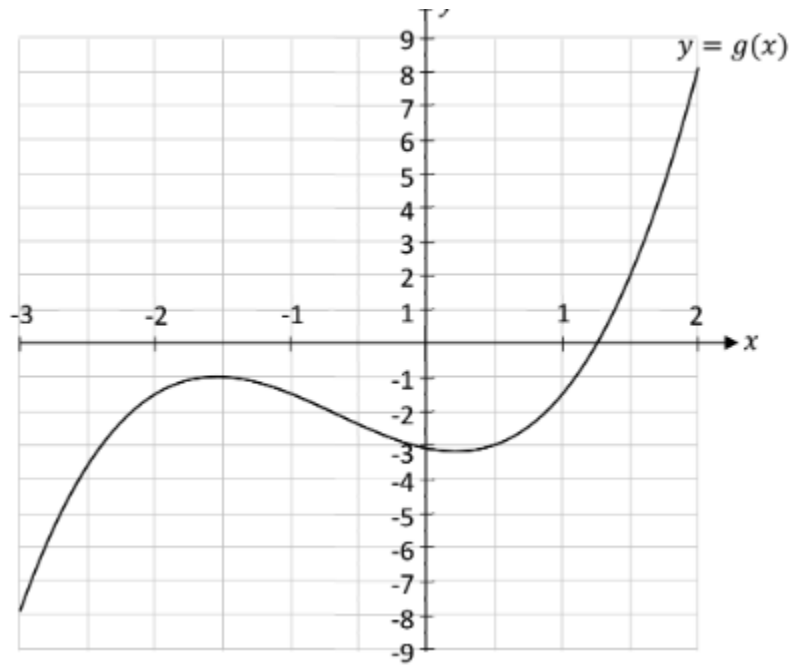
(i) Complete the table below to show the values of $h(t)$ for the given values of t .

Time (seconds)	0	1	2	3	4	5	6
Height (metres)							

(ii) Draw the graph of the function $h(t)$ on the axes below for $0 \leq t \leq 6$, where $t \in R$.



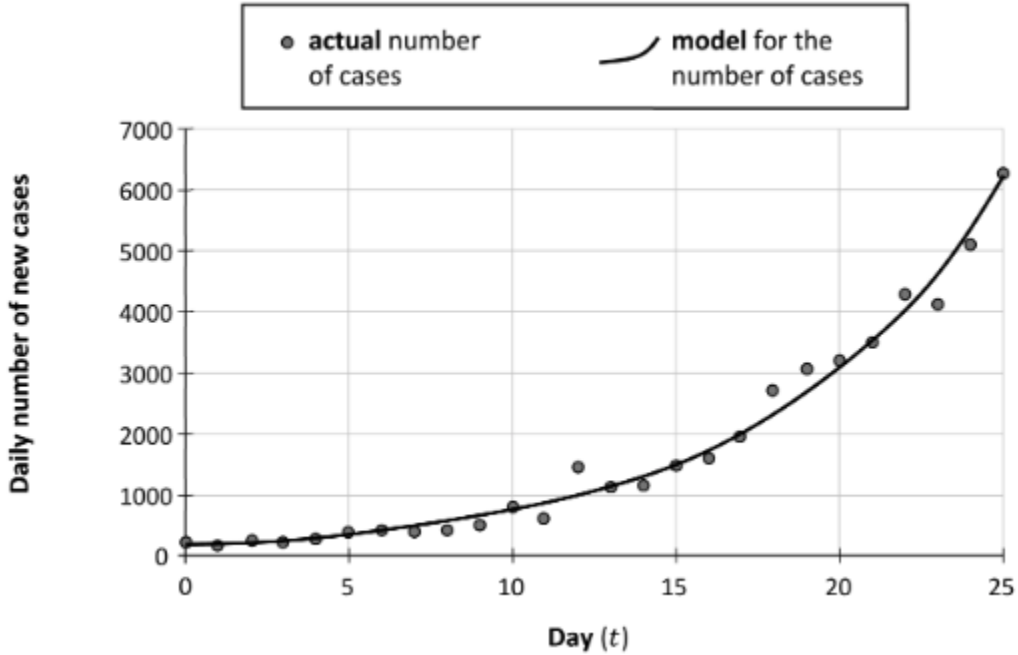
The graph of the function g is shown below for $-3 \leq x \leq 2$, where $x \in \mathbb{R}$. Use the graph to answer the following:



Estimate the value of $g(1.5)$

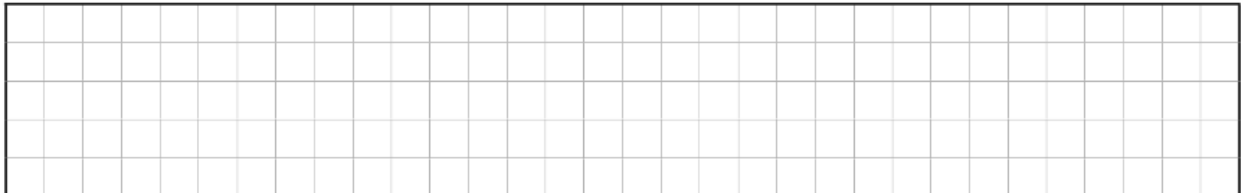
Estimate the value of x for which $g(x) = -6$

The diagram below shows the actual daily number of new cases of a disease, from day $t = 0$ to day $t = 25$. It also shows the number of cases according to a mathematical model over this period of time.



(i) From the diagram, estimate the number of actual new cases there were on day 12, using the relevant dot.

(ii) Show that it takes approximately 5 days for the number of daily new cases to double, according to the curve in the diagram.



(iii) From the model, estimate the day on which the number of new cases was 5,000.

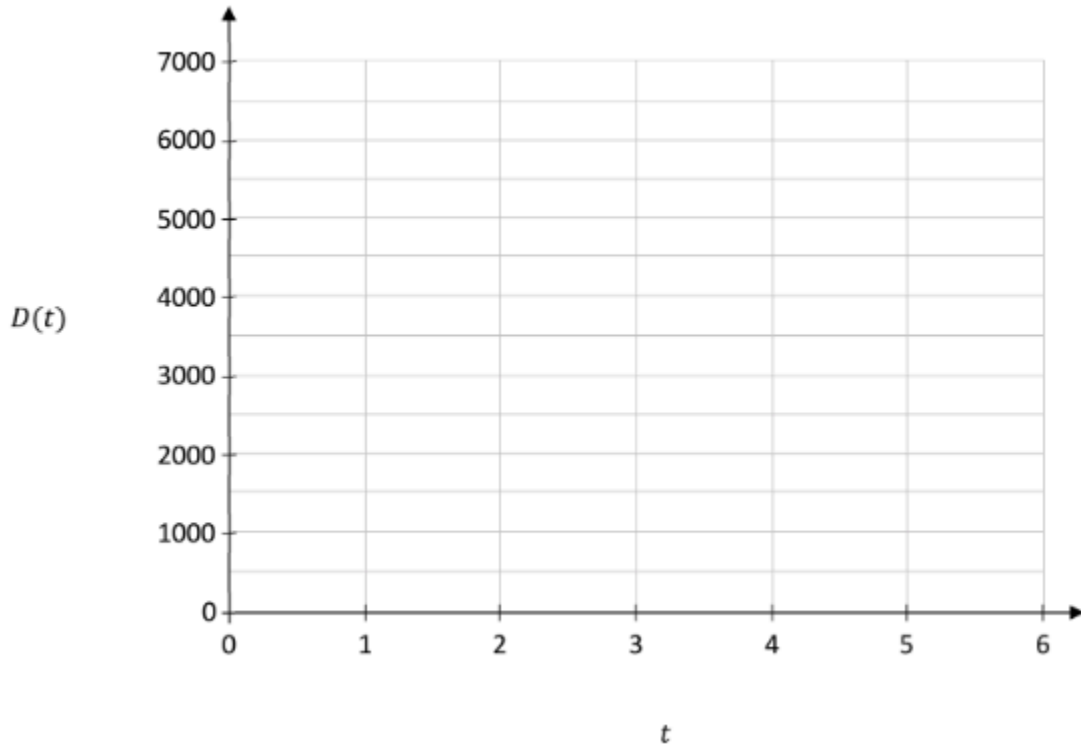
The daily number of new cases of another disease can be modelled using the function

$$D(t) = 2100 \times 1.18^t$$

where D is the daily number of new cases and t is the time in days from $t = 0$ to $t = 14$.

Fill in the table below, leaving each answer correct to the nearest whole number, and hence, draw the associated graph on the given axes.

t	0	1	2	3	4	5	6
$D(t)$			2924				5669



$$g(x) = x^2 + 7x + 12, \text{ where } x \in R$$

(i) Work out the value of $g(6)$.

(ii) Solve for $g(x) = 2$, where $x \neq -5$.

$$h \text{ is defined as } h(x) = -0.38x^3 + 2.6x^2 - 0.13x + 158.$$

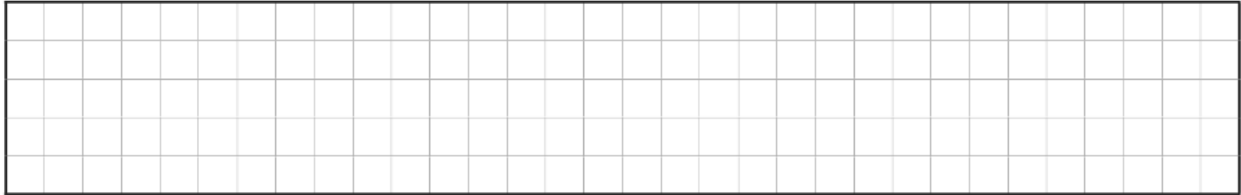
$$c \text{ is defined as } c(x) = 0.1h(x) - 7.$$

Work out the value of $c(6)$.

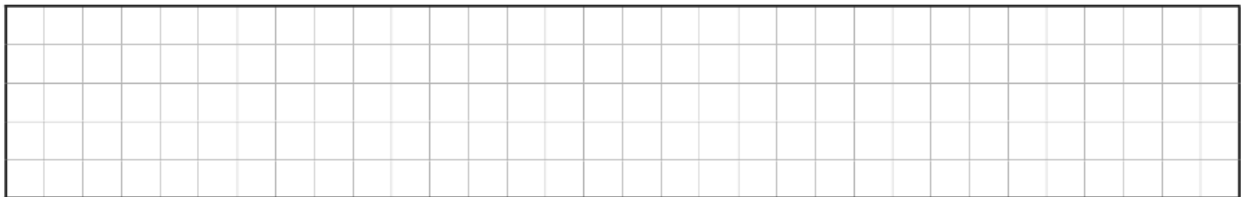
The total number of users on a website can be modelled by the formula:

$$U(m) = 3000(1.8)^m, \text{ where } m \text{ is the number of months since the website was launched.}$$

(i) Estimate the number of user registered 8 months after the website launch



(ii) There are 31 493 users at the end of a particular month. Estimate how many users there will be one month later.

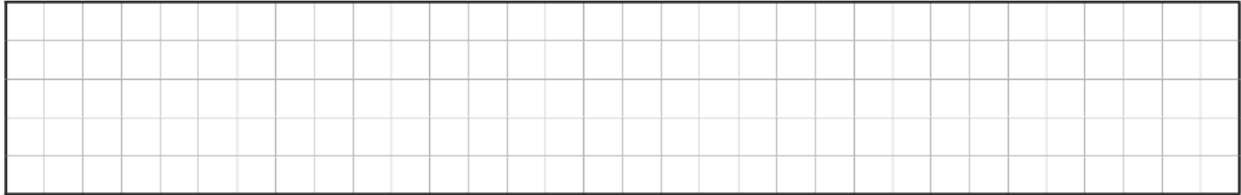


A swimmer is on a starting block at the beginning of a race. When she dives off the block until she resurfaces, the level of the swimmer relative to the level of the water is given by the function:

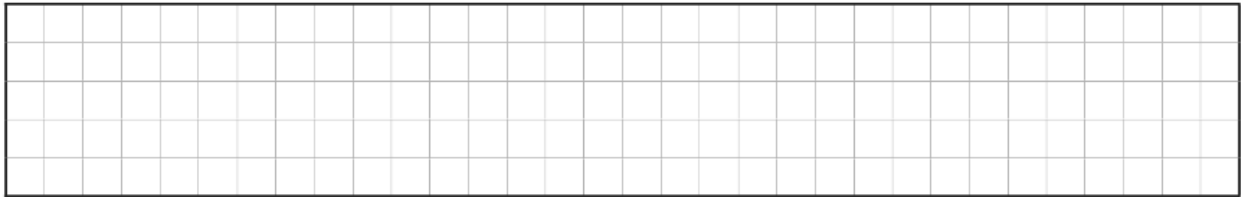
$$h(x) = \frac{1}{60}x^2 - \frac{1}{4}x + \frac{3}{5}$$

In the function, x is the horizontal distance of the swimmer from the starting block in metres, and $h(x)$ is measured in metres.

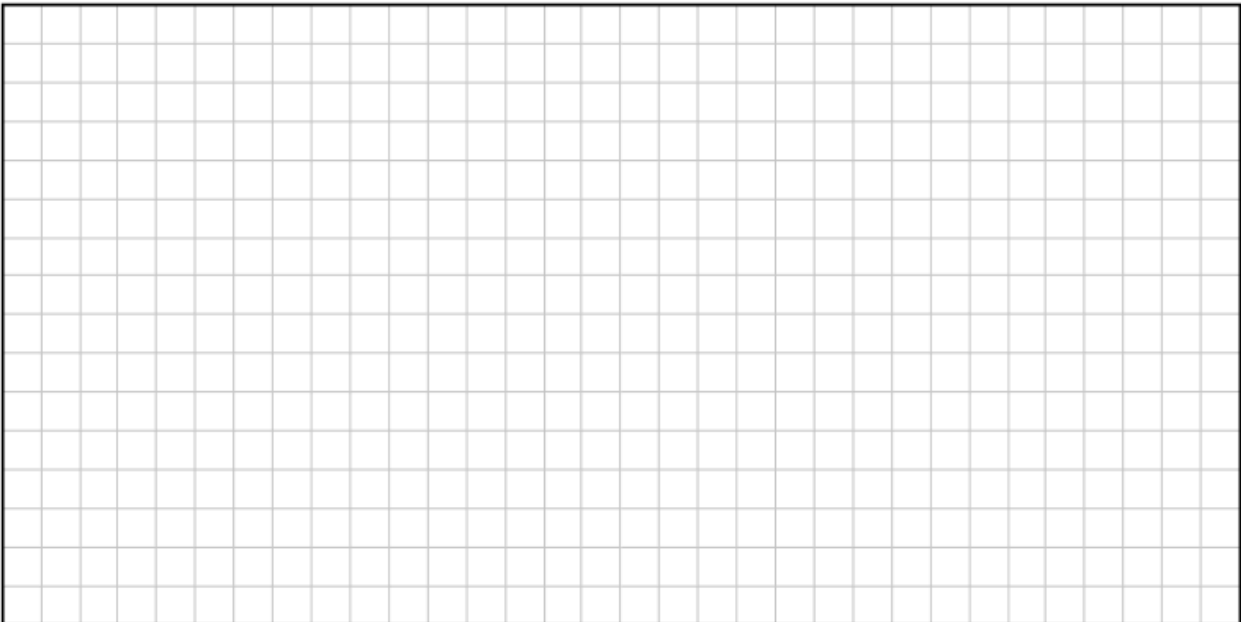
(i) Find the height of the block above the water.



(ii) Show that the swimmer is on the surface of the water when she is 12 metres from the starting block.



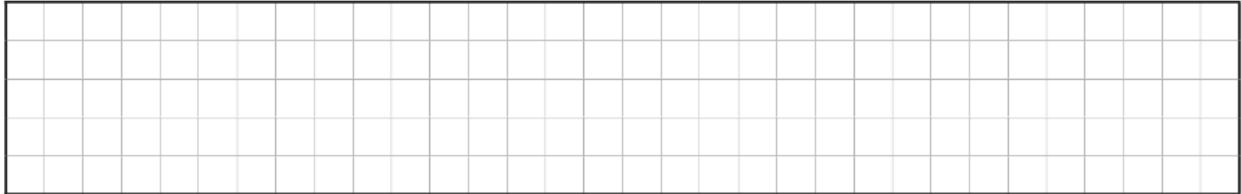
(iii) Find the horizontal distance from the starting block, to the point where the swimmer enters the water



The amount of a certain drug in mg in the bloodstream t hours after being injected can be estimated using:

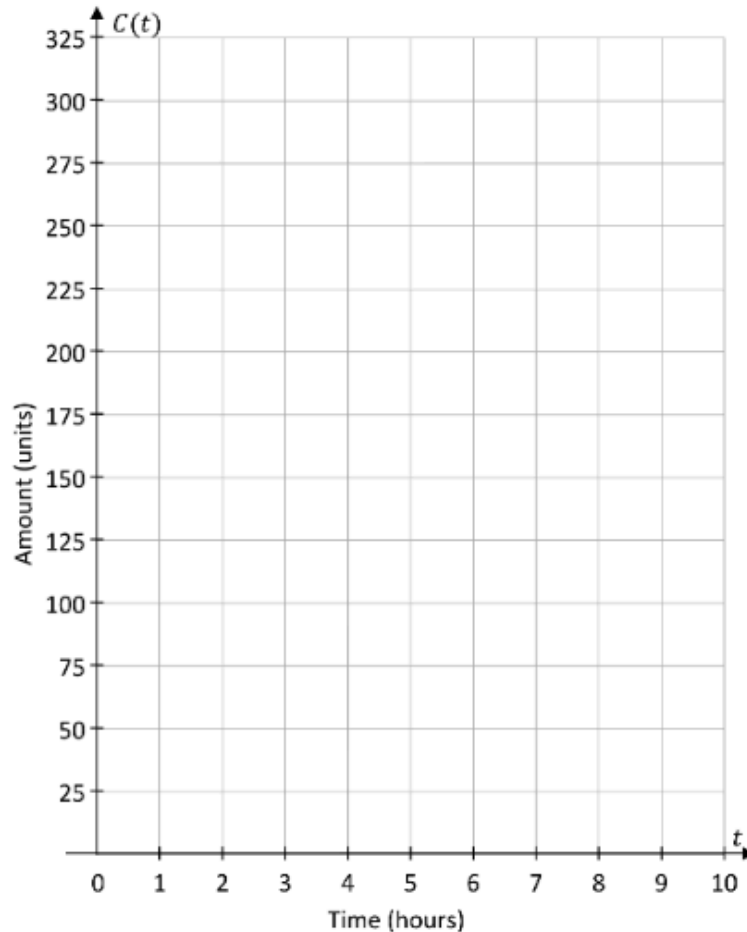
$$C(t) = -t^3 + 4.5t^2 + 54t, \text{ where } 0 \leq t \leq 9, t \in R.$$

(i) Show that the amount of drug in the bloodstream 4 hours after it has been injected is 224 units.



(ii) Use the function $C(t)$ to complete the table below, and hence draw the graph of $C(t)$.

t (Hours)	0	1	2	3	4	5	6	7	8	9
$C(t)$ (Units)	0	57.5			224					



A person's height can be approximated by the length of their femur. One such approximation is given by:

$$H = 2.3x + 65.53,$$

where H is the height in centimetres, and x is the length of the femur in centimetres.

(i) Use the function above to estimate the height of someone with a femur that is 47.54 cm in length. Give your answer correct to two decimal places.

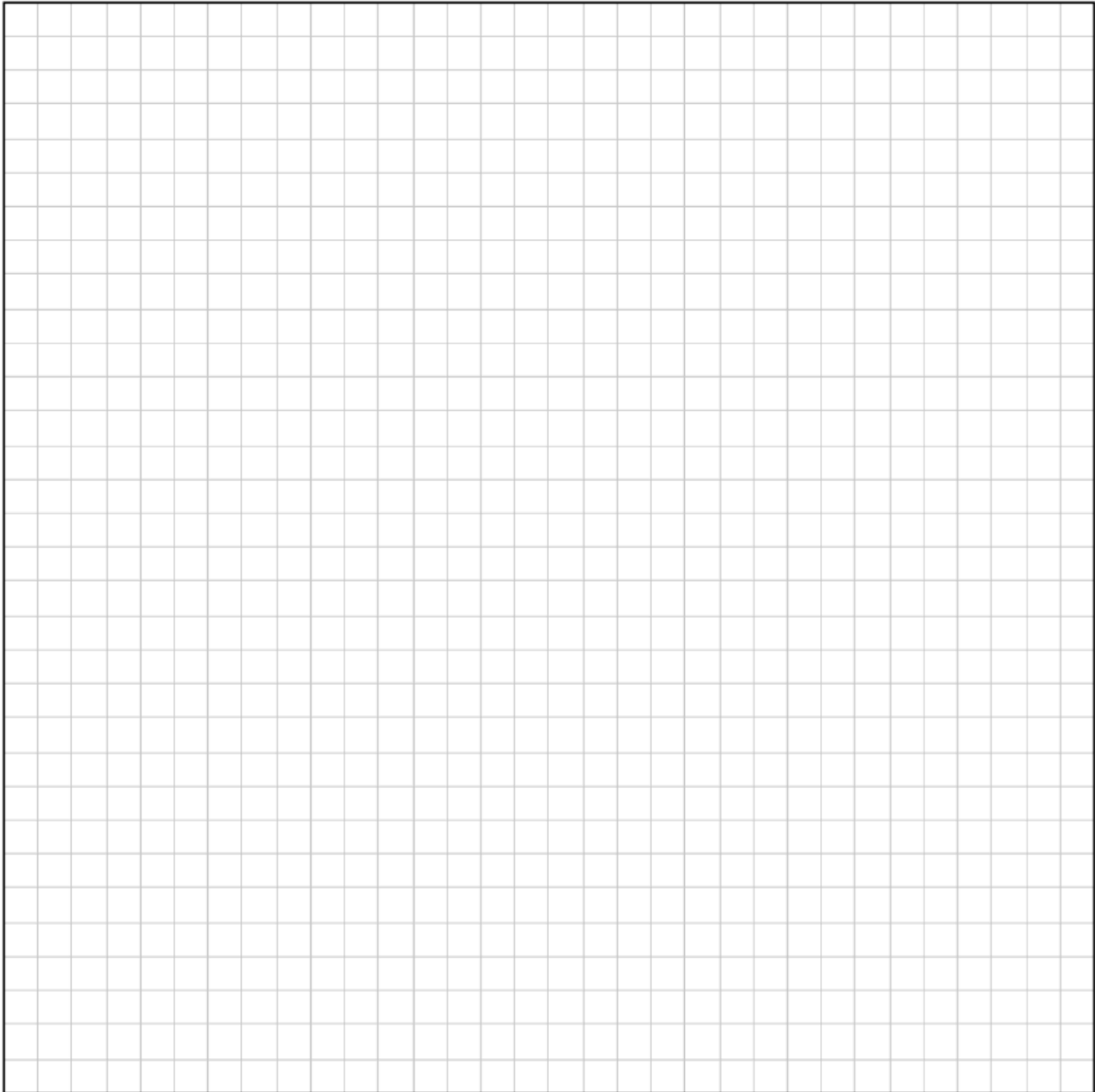
(ii) Use H to estimate the length of a person's femur, given they are 184 cm in height.

The function f is defined as $f(x) = 7x + 3$

Evaluate $f(-3)$ and then subsequently find the value of k if $f(k) = -32$.

The profits of two companies in €0,000s are modelled using two functions, $f(x) = 2x + 5$ and $g(x) = 2^x$, where x is in years.

Using the same scales and axes, draw both functions in the domain $0 \leq x \leq 4$ and hence estimate how many years it will take for the profit of both companies to be the same.



Let $h(x) = x^2 + bx + c$.

(i) Given that $h(0) = 12$, find the value of c .

(ii) Given that $h(3) = 42$, find the value of b .

(iii) Solve $h(x) = 0$

The functions g and t are defined for $x \in \mathbb{R}$ as follows:

$$g: x \rightarrow 4x - 5$$

$$t: x \rightarrow x^2 - 5x + 1$$

(i) Find $t(6)$ and $g(2)$.

(ii) Solve $t(x) = 0$, giving your answers correct to two decimal places.

(iii) Show that $t(g(x)) = 16x^2 - 60x + 51$.

Chapter 3

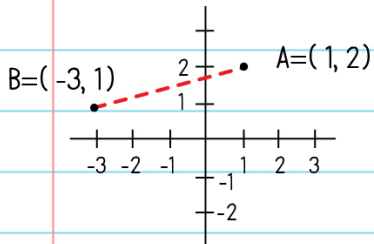
(x, y)

CO-ORDINATE GEOMETRY: THE LINE

• The formulae

• The equation

• Area of a triangle



A line crosses the x-axis when $y=0$

A line crosses the y-axis when $x=0$

Exercise ① $y = \frac{2}{3}x + 7$

② $3x - 4y + 24 = 0$

Find the x and y intercepts, and the slope of the above lines


• The formulae

- 1) Slope
- 2) Length/Distance
- 3) Midpoint
- 4) Equation

Pg 18 Log tables

Perpendicular Slope: flip the fraction, and change the sign.

Example:

L is \perp to j
 means perpendicular

• The equation

We need (i) The slope
 (ii) A point

Answer can be left in the form:

- ① $ax + by + c = 0$
 where slope = $-\frac{a}{b}$
- ② $y = mx + c$
 where slope = m

Slope of L	Slope of j
$\frac{2}{3}$ →	$-\frac{3}{2}$
$\frac{4}{5}$ →	$-\frac{5}{4}$
-8 →	$\frac{1}{8}$
$-\frac{6}{4}$ →	$\frac{4}{6}$
$\frac{1}{7}$ →	-7

Area of a triangle

$$\frac{1}{2} |x_1 y_2 - x_2 y_1|$$

Tip Translate one point to (0, 0)

Prove (2,3) is on $4x+2y=14$
and prove (4, 1) isn't

$$4(2)+2(3)=14$$

$$8+6=14$$

$$14=14$$



$$4(4)+2(1)=14$$

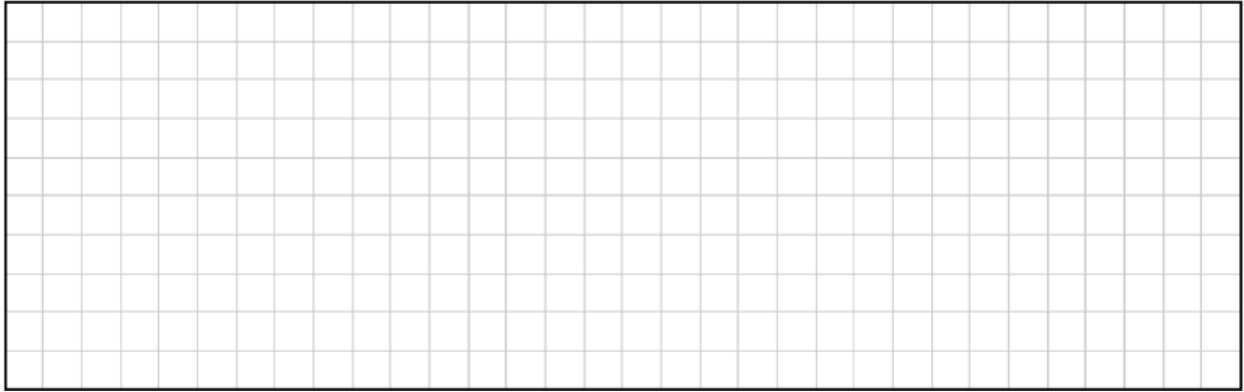
$$16+2=14$$

$$18 \neq 14$$

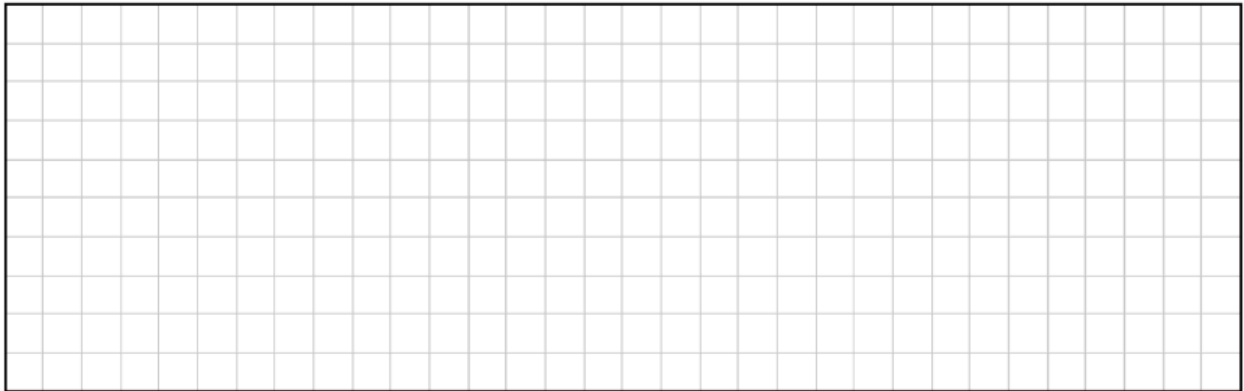


$A(1, -2)$, $B(0, 4)$, $C(3, 1)$ are three points on a coordinate plane.

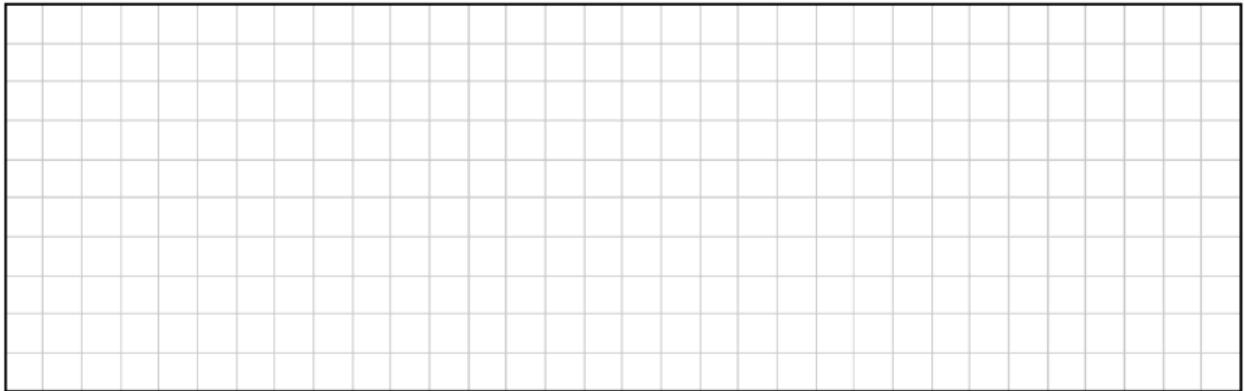
(i) Find $|AC|$



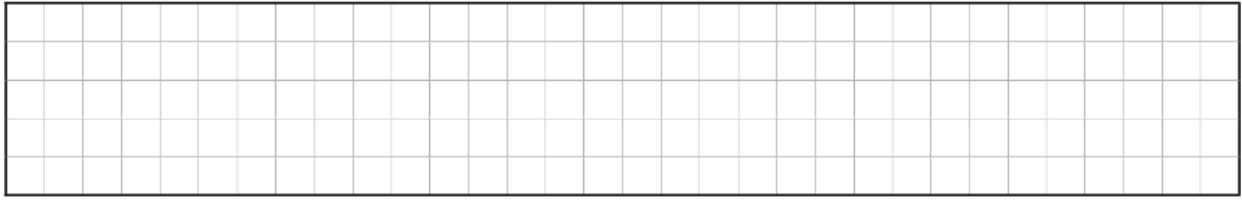
(ii) Find the slope of AB



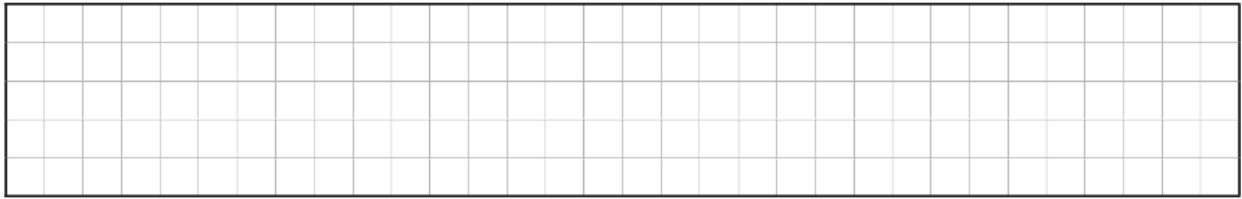
(iii) Find the midpoint of BC



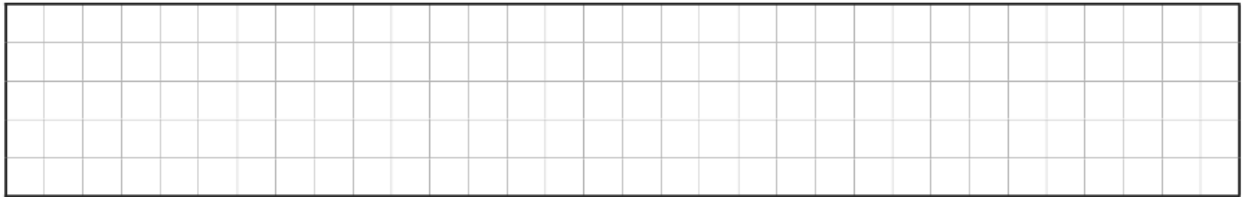
(iii) Find C , the midpoint of OB



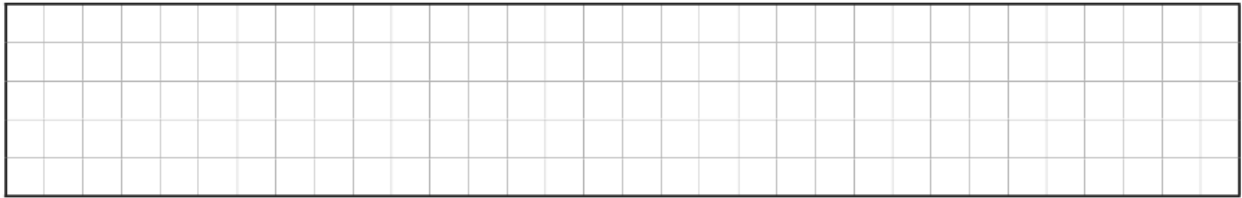
(iv) Find the equation of OB in the form $ax + by + c = 0$.

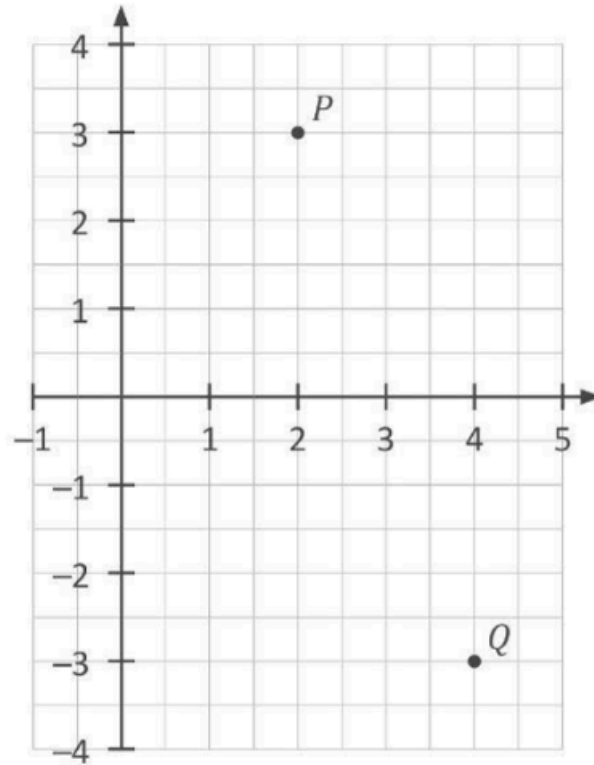


(v) Find the distance from point A to point C

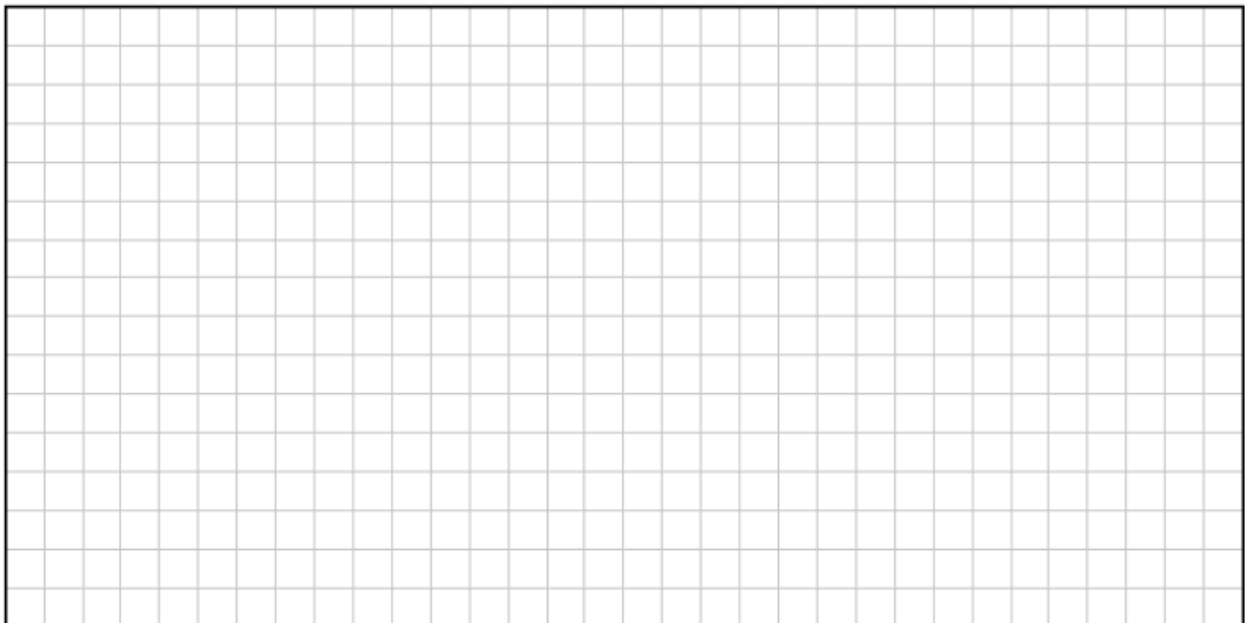


(vi) Find the slope of the line from point A to point C





Find the slope, midpoint, length and equation of PQ .

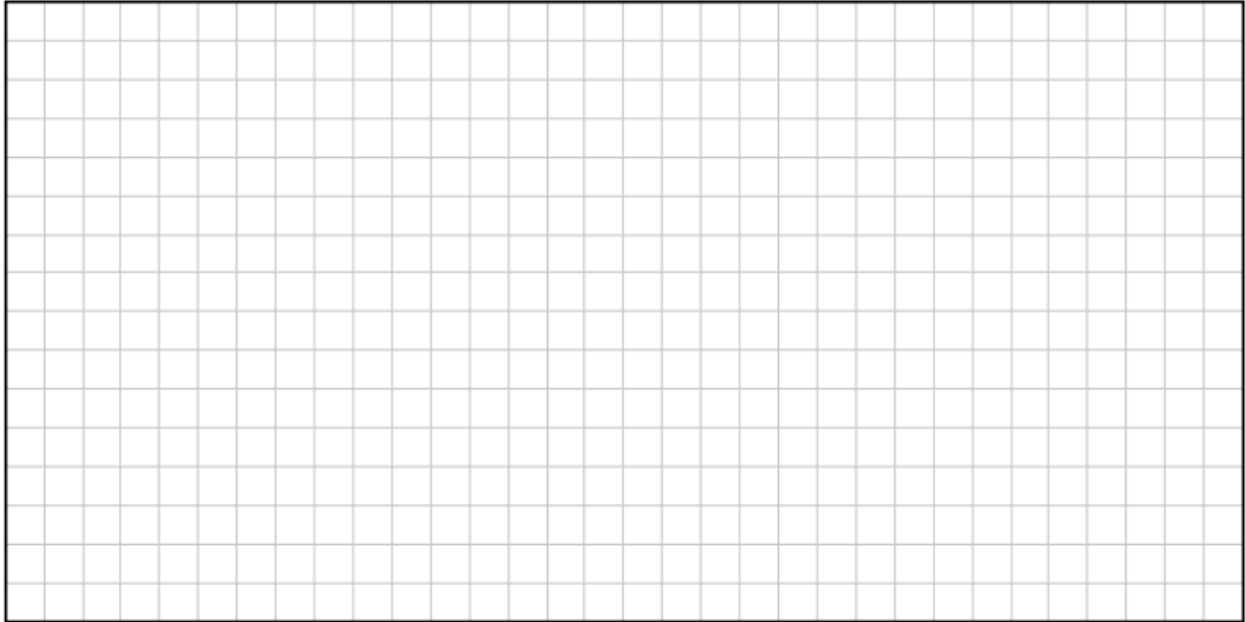


(i) The equations of two lines are:

$$g: 7x + 2y = -2$$

$$h: x - y = -8$$

Use algebra to find the point of intersection between the two lines.

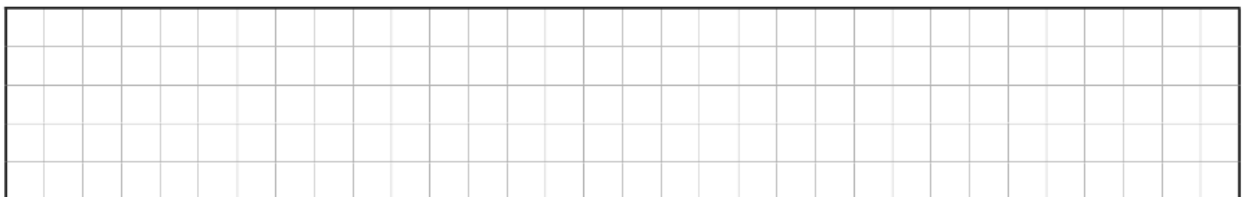


(ii) Find the slope of g and the slope of h

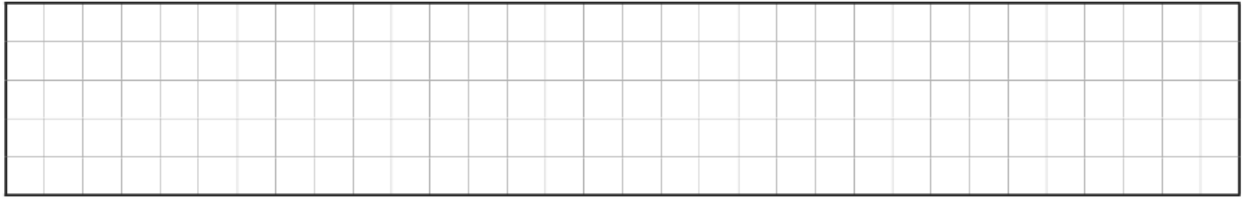
Slope of g

Slope of h

(iii) Verify that the two lines are **NOT** perpendicular.

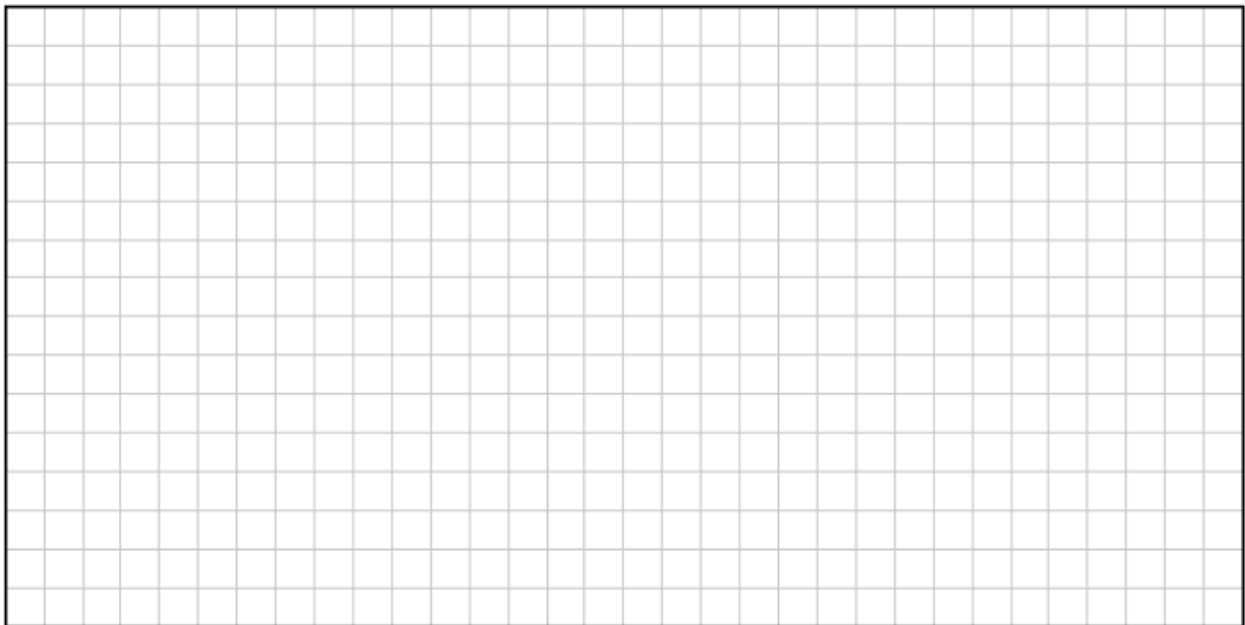


(i) Given the equation of a line $l: 2x - y + 5 = 0$, find its slope

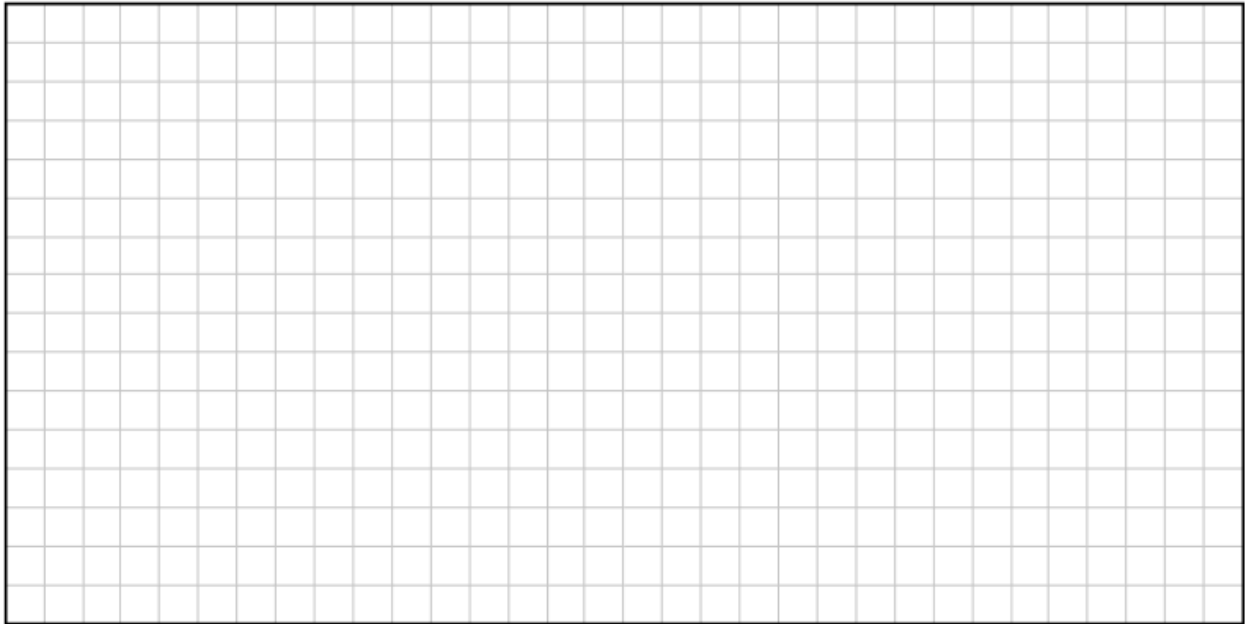


(ii) $A = (-1, -3)$ and $B = (k, 3)$.

If the line segment $[AB]$ is parallel to l , find the value of k .

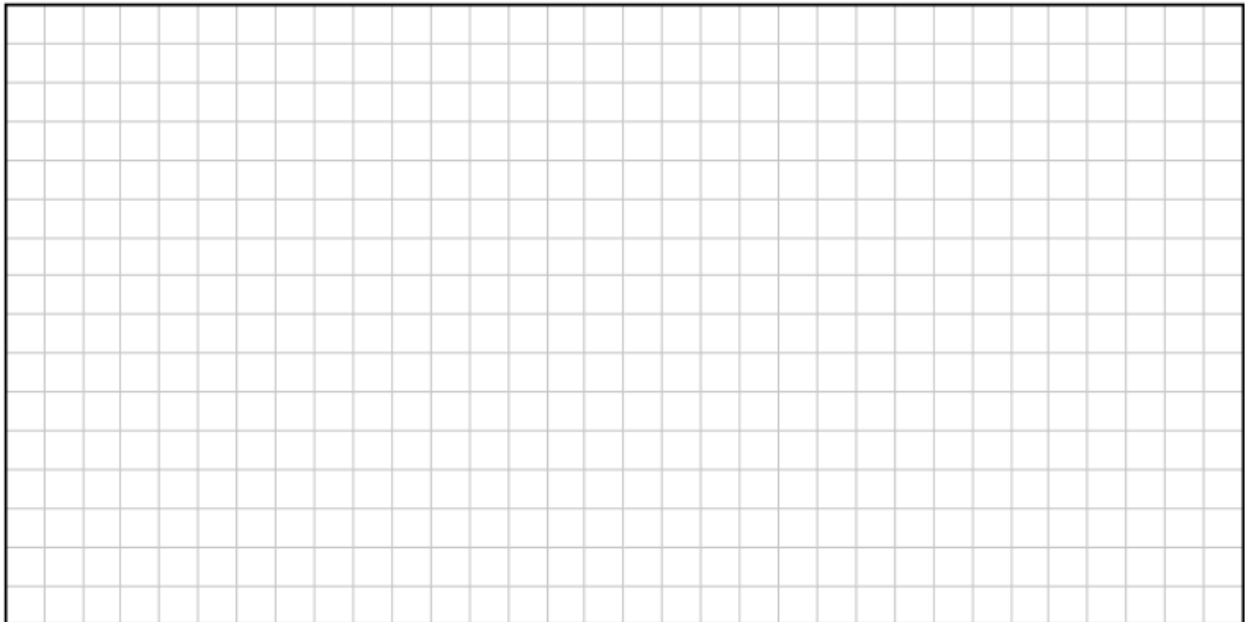


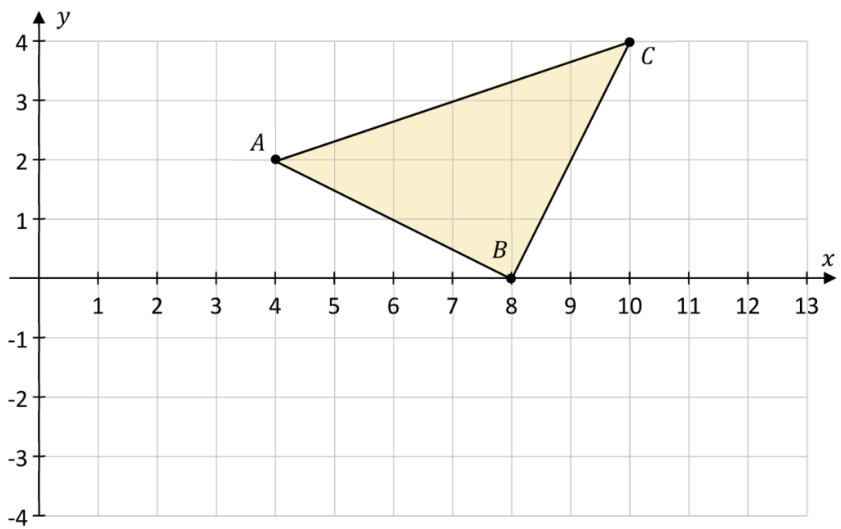
The line h has a slope of 4, and passes through the point $(20, 12)$. Find the coordinates of two other points on h , other than $(20, 12)$.



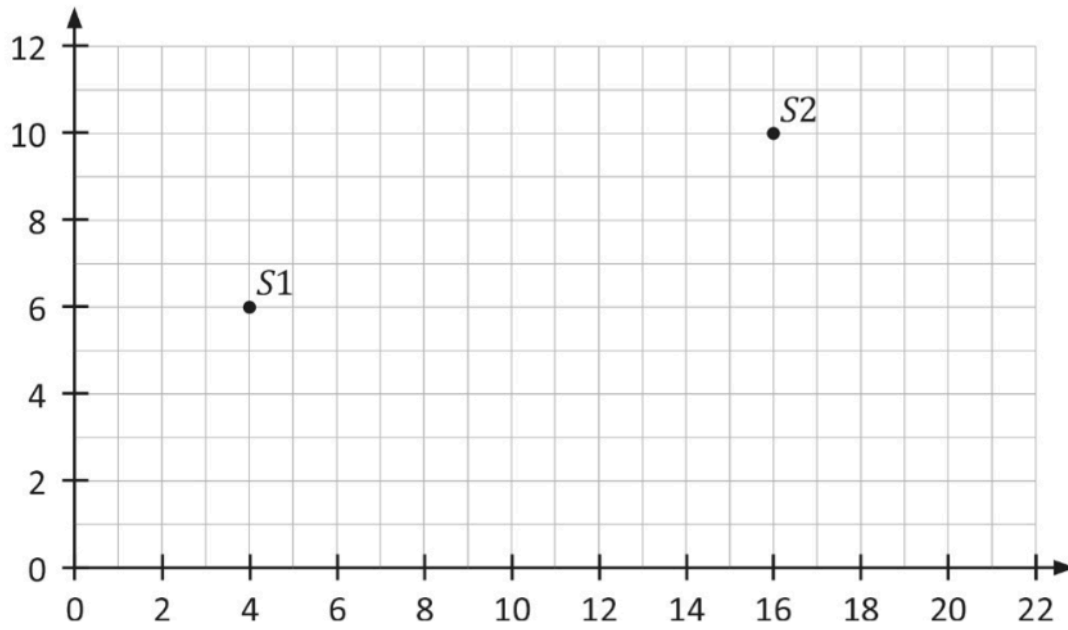
The equation of PQ is $x + 7y = 20$. Find the coordinates of the point where PQ crosses the y -axis and the x -axis.

Another line passes through Q , and is perpendicular to PQ . Find the equation of this line.

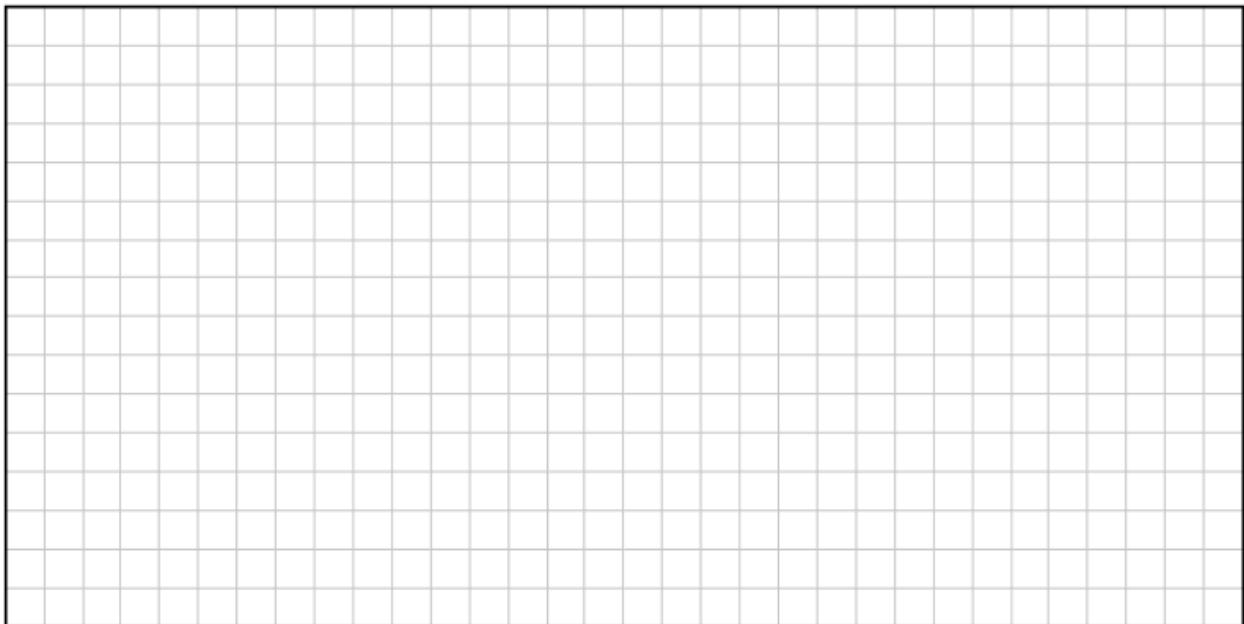


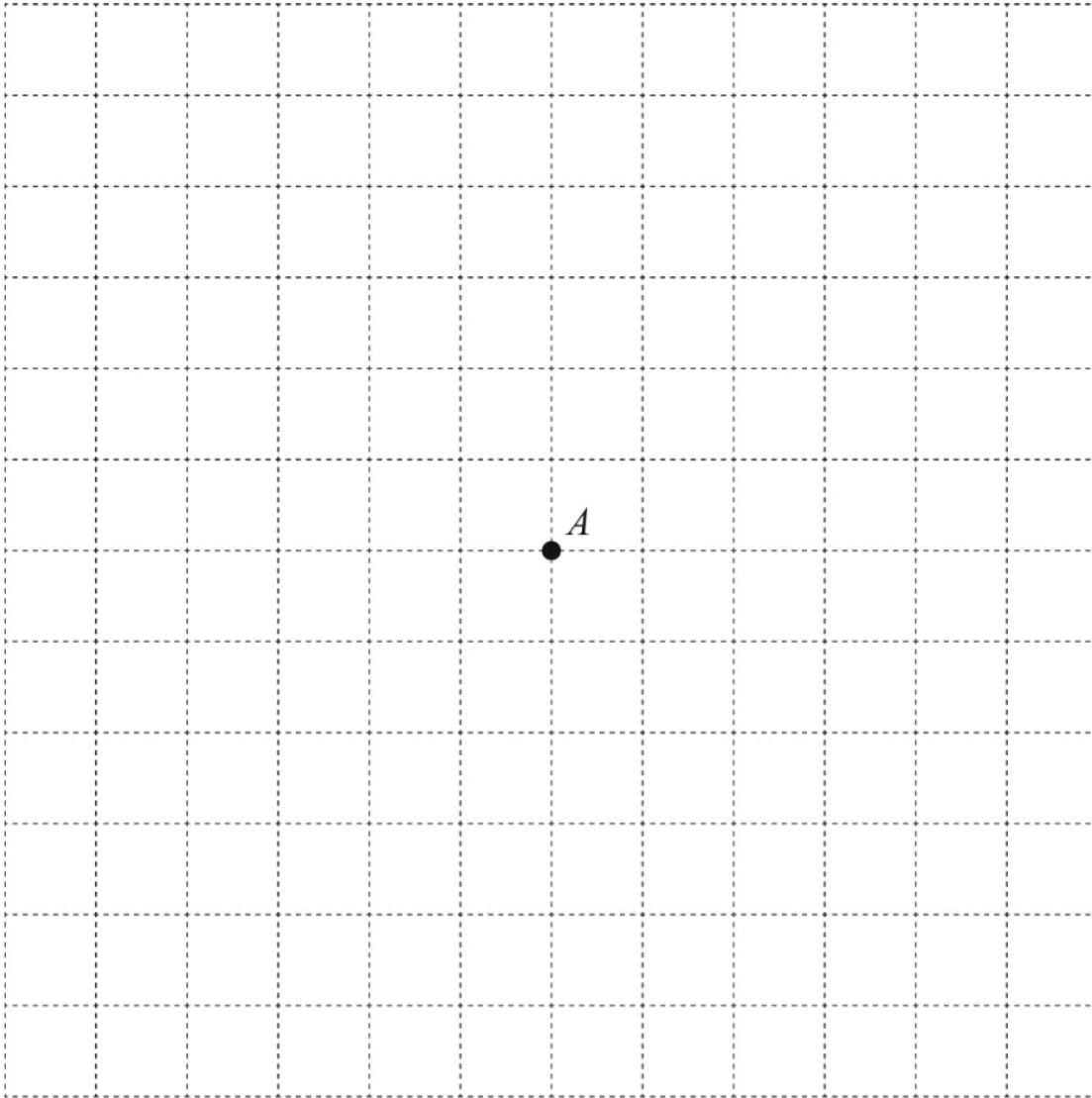


Line AC	Line AB
Equation: $y = mx + \frac{2}{3}$	Equation: $y = -\frac{1}{2}x + k$
Answer: $m = \underline{\hspace{2cm}}$ <div style="border: 1px solid black; height: 100px; width: 100%;"></div>	Answer: $k = \underline{\hspace{2cm}}$ <div style="border: 1px solid black; height: 100px; width: 100%;"></div>



A farmer has sprinklers set around his field to water his crops. The main water feed connects at a right angle midway along the line joining S_1 and S_2 . Find the equation of the main water feed.

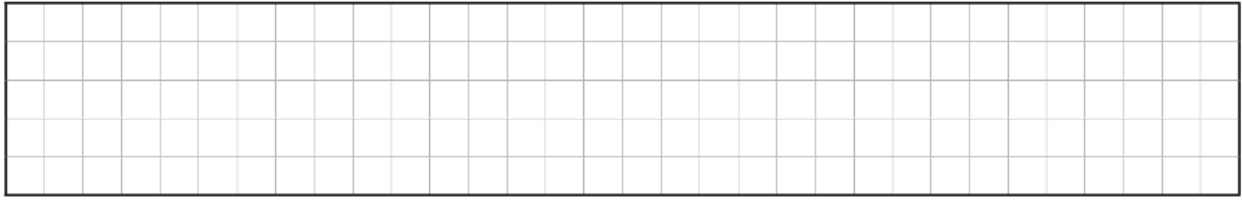




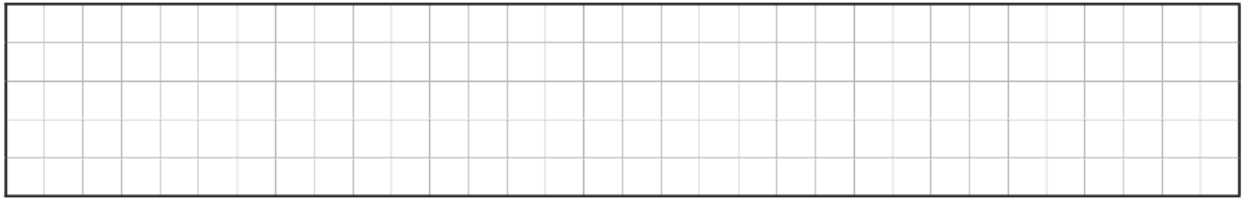
(i) On the diagram above, through the point A, draw a line with a slope of $\frac{3}{4}$.

(ii) On the diagram above, through the point A, draw another line with a slope of $-\frac{4}{3}$.

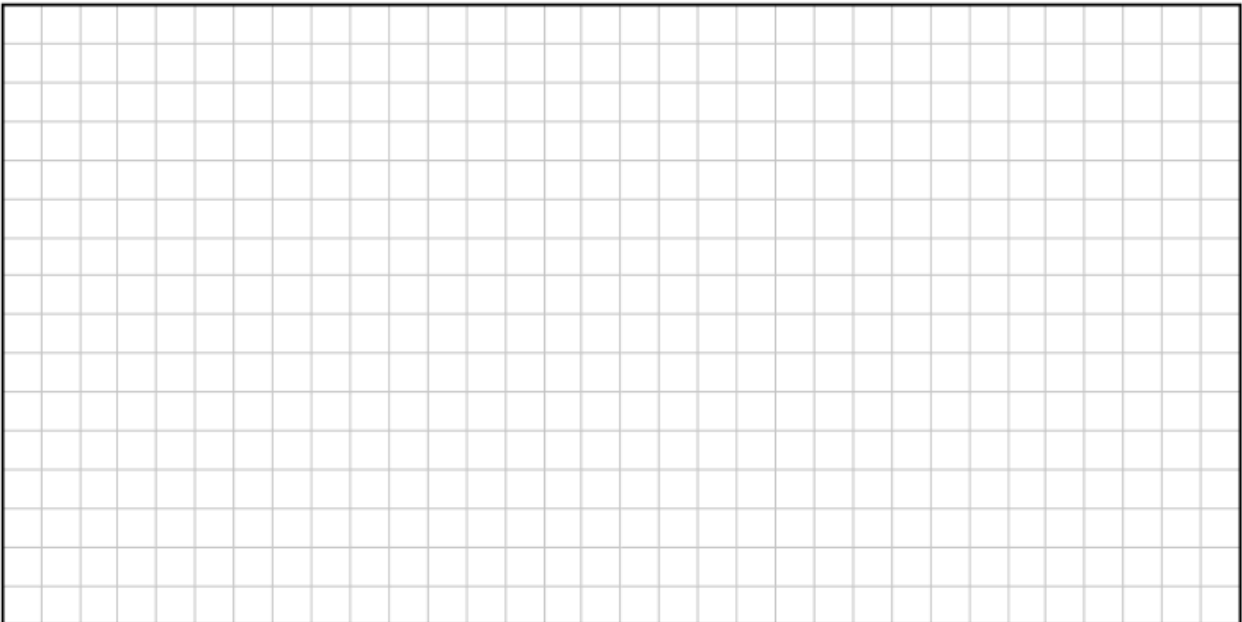
Show that $(1, -2)$ is not on the line $x - 3y - 6 = 0$.



Show that $(3, 4)$ is on the line $2x + 3y = 18$.

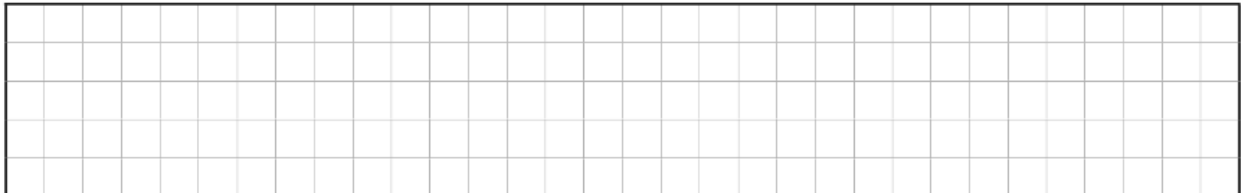


Find the coordinates where $4x - 6y - 24 = 0$ crosses the x -axis and the y -axis.



Line 1	$y = 3x - 6$
Line 2	$y = 3x + 12$
Line 3	$y = 5x + 20$
Line 4	$y = x - 7$
Line 5	$y = -2x + 4$
Line 6	$y = 4x - 16$

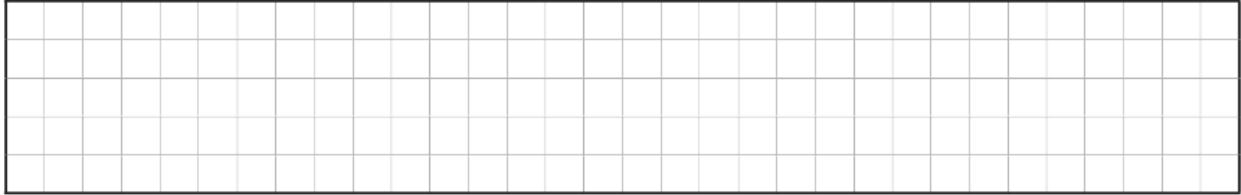
Based on the above table, which two lines are parallel?



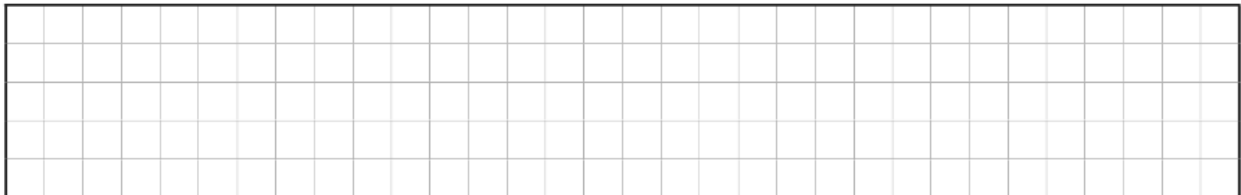
The line l_1 has equation $2x - 3y + k = 0$, where k is a constant.

The point $A(2, 3)$ lies on l_1 . Find:

(i) The value of k .



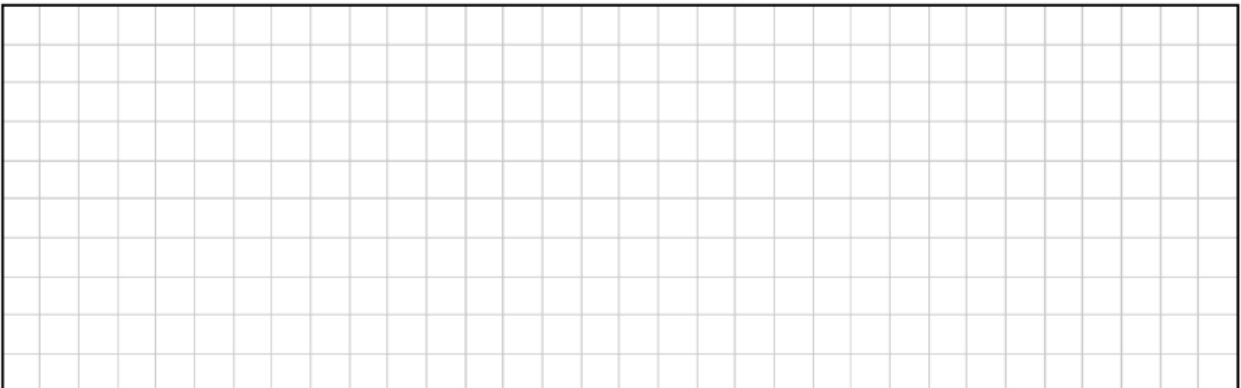
(ii) The slope of l_1 .



(iii) The line l_2 passes through the point A and is perpendicular to l_1 . Find the equation of l_2 , giving your answer in the form $ax + by + c = 0$.



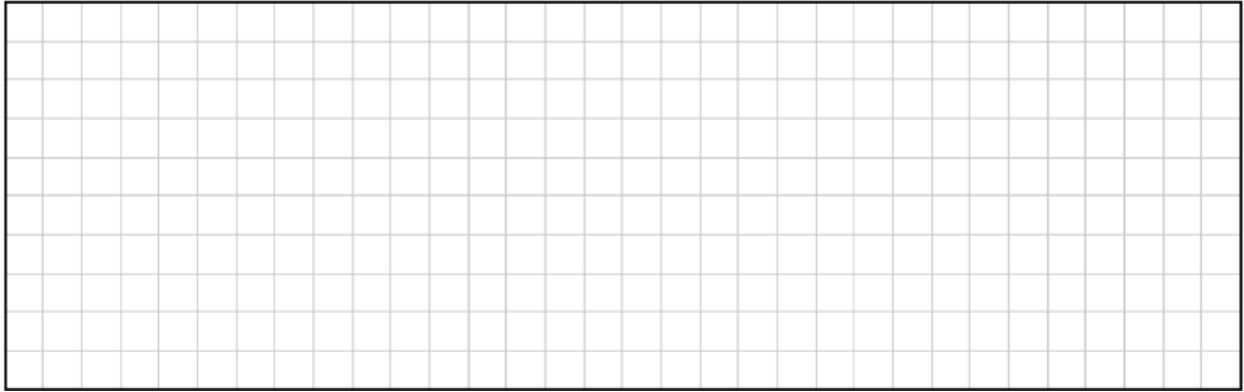
(iv) The line l_1 crosses the x -axis at the point B and the y -axis at the point C . Find the coordinates of B and C .



Area of a triangle

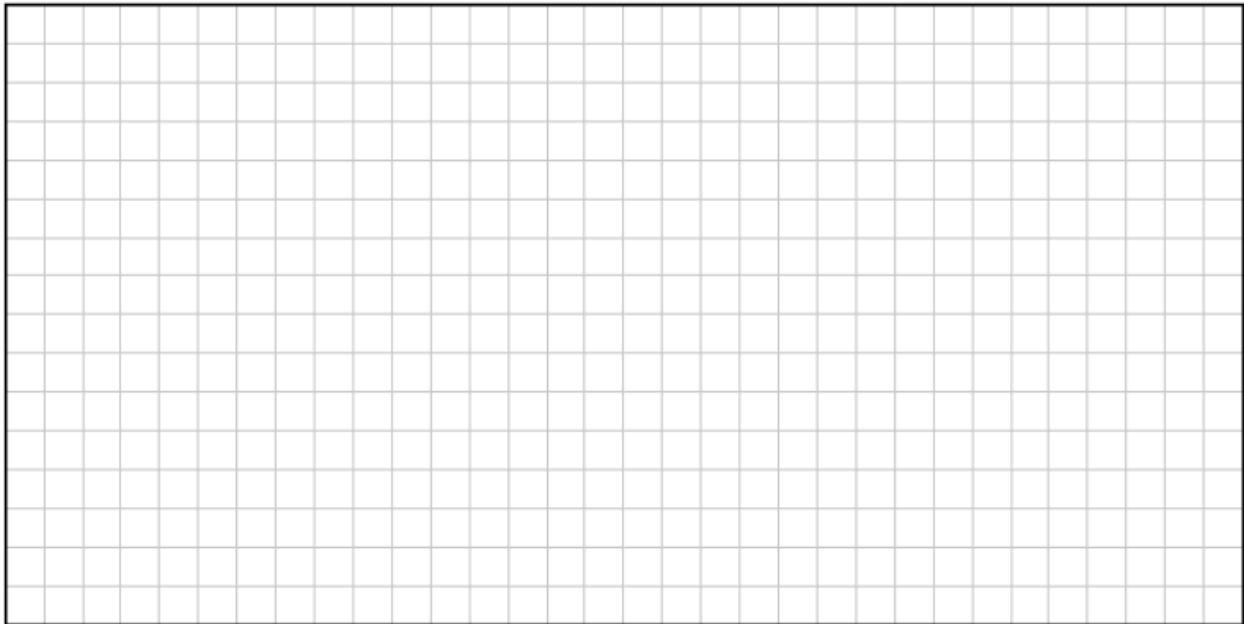
$A(3, 8)$ $B(6, 4)$ and $C(0, 0)$.

Find the area of $\triangle ABC$.



$P(6, 3)$, $S(3, - 1)$, $Q(0, 5.5)$

Find the area of triangle PQS



Chapter 4

CO-ORDINATE GEOMETRY: THE CIRCLE

• Equations

• Tangents

• Equations

We need (i) Centre
(ii) Radius

- 1st form : $x^2 + y^2 = r^2$
Centre (0,0) radius = r

e.g. $x^2 + y^2 = 25$
Centre (0,0), radius = 5

- 2nd form : $(x-h)^2 + (y-k)^2 = r^2$
Centre (h,k) radius = r

e.g. $(x-3)^2 + (y+2)^2 = 49$
Centre (3,-2), radius = 7

• Points inside, outside, on

• Constructions

• Points inside, outside, on

(5, 0) is **on** $x^2 + y^2 = 25$ because $(5)^2 + (0)^2 = 25$
 $25 = 25$

(6, 1) is **outside** $x^2 + y^2 = 25$ because $(6)^2 + (1)^2 > 25$
 $37 > 25$

(2, 1) is **inside** $x^2 + y^2 = 25$ because $(2)^2 + (1)^2 < 25$
 $5 < 25$

• Tangents

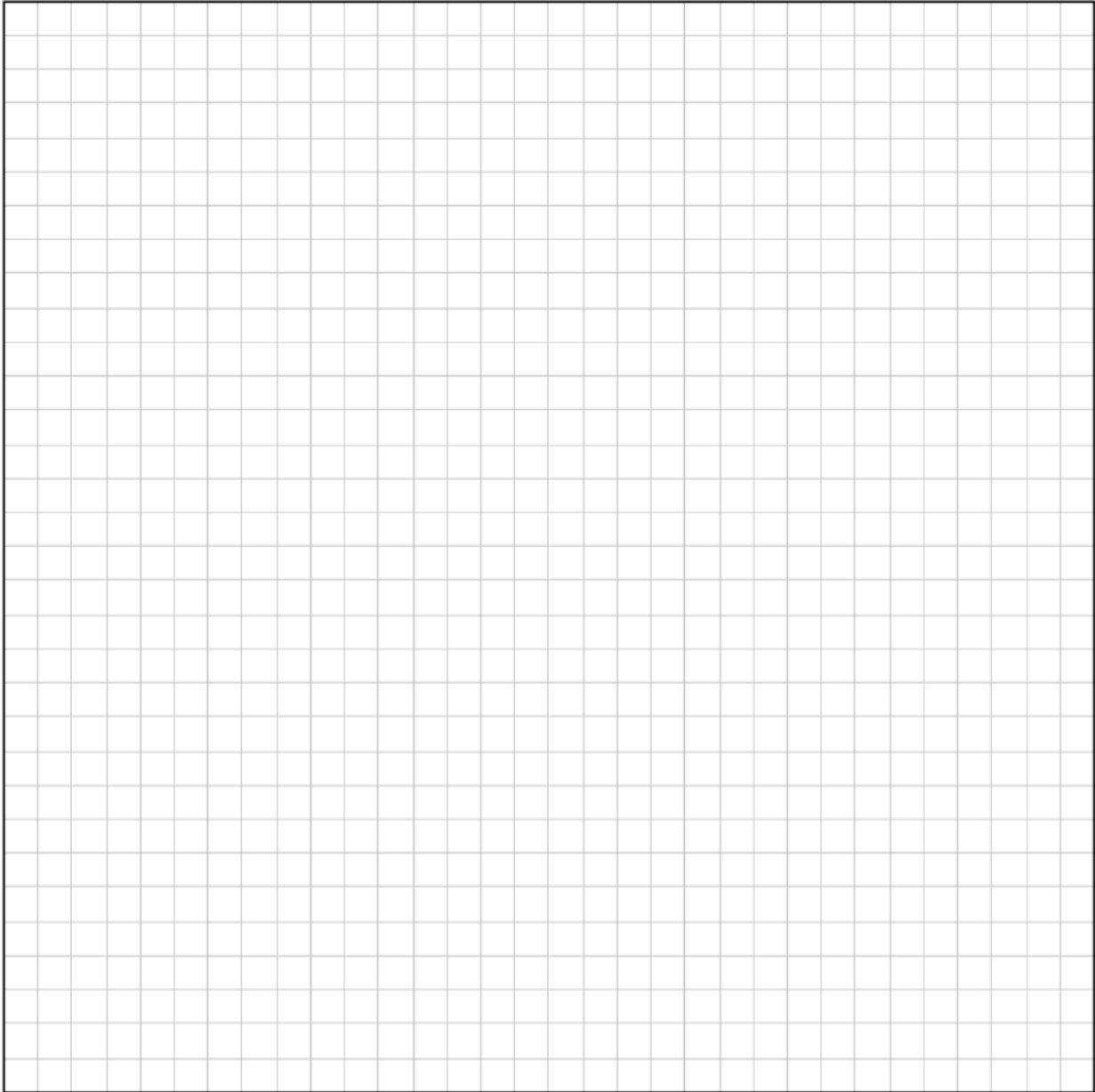


Tangents are always perpendicular to the line connecting the tangent point and the centre

Find the points of intersection between circle k and the line l where:

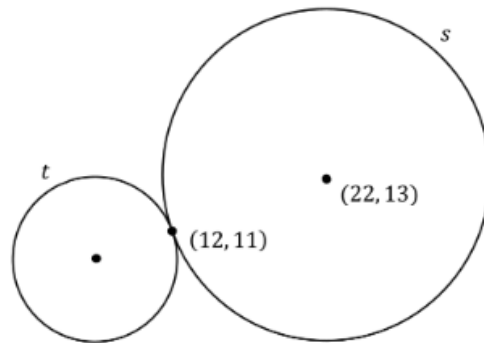
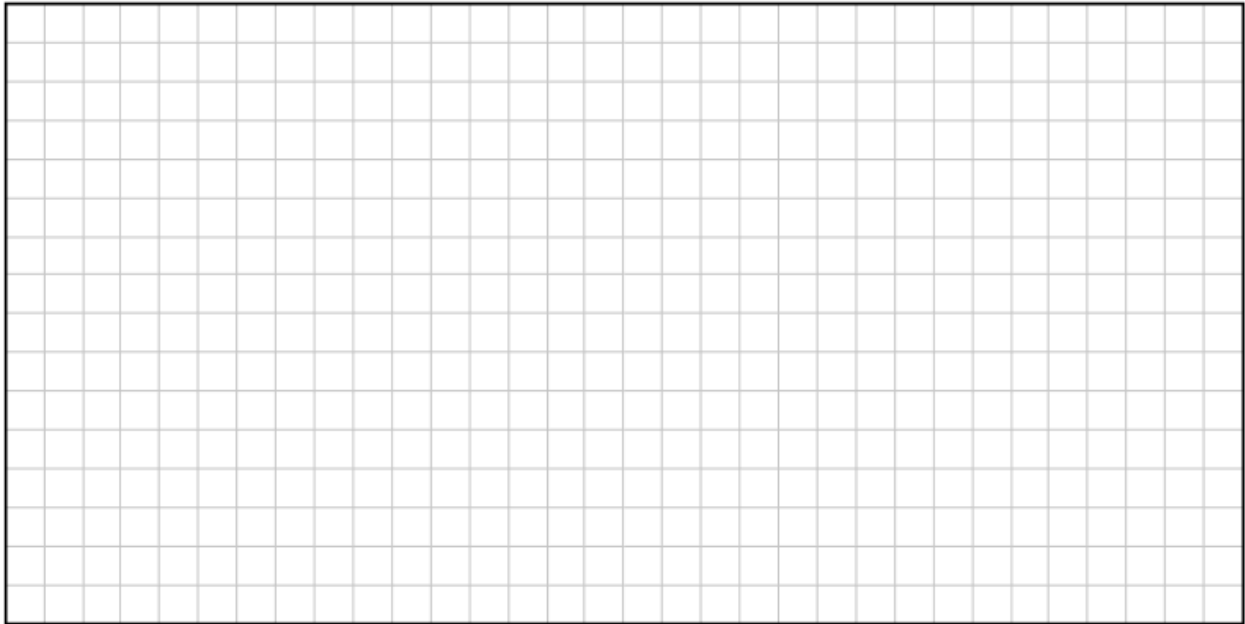
$$l: 5x - y - 13 = 0$$

$$k: x^2 + y^2 = 13$$

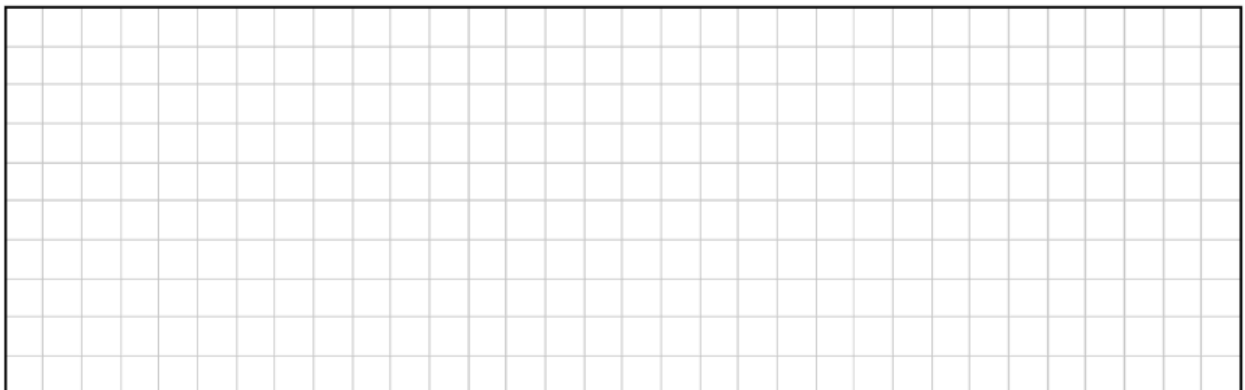


The circle k has the points $A(-2, 1)$ and $B(4, 7)$ as the endpoints of a diameter.

Find the equation of circle k .

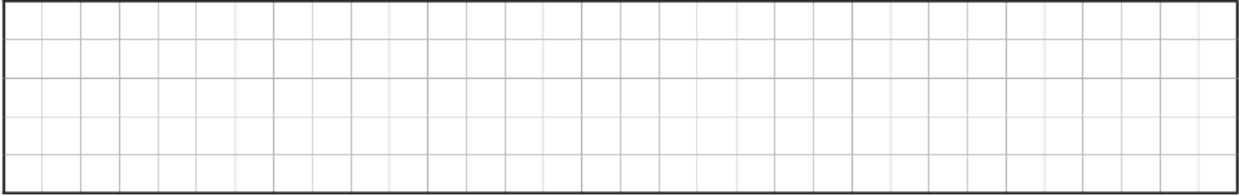


Find the coordinates of another point on the circle s other than $(12, 11)$. The radius of circle t is half the radius of circle s . Find the equation of circle t .

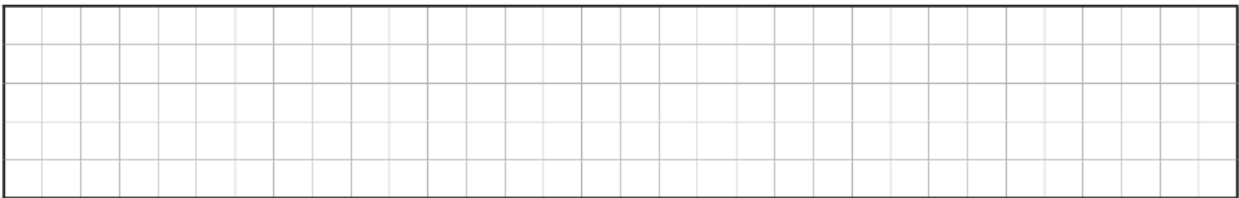


Points inside, outside, and on a circle

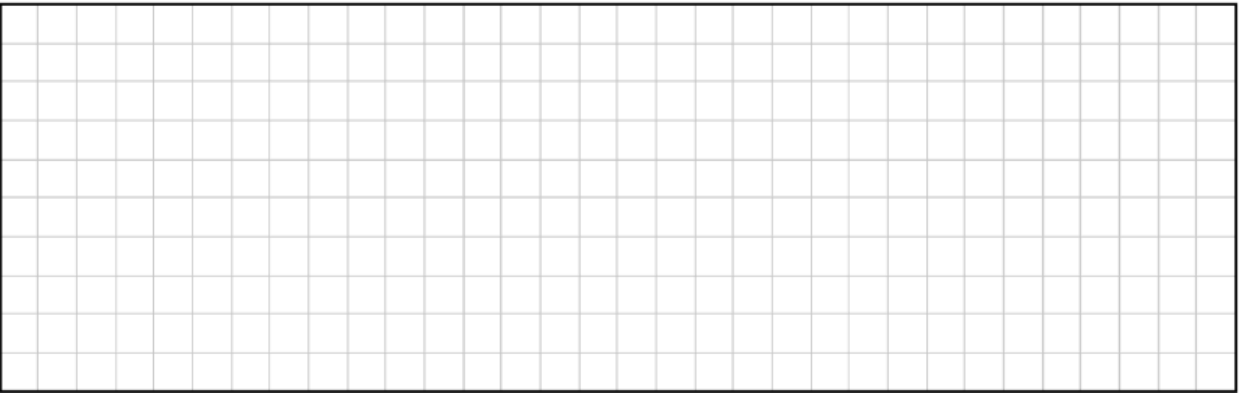
A circle c has equation $x^2 + y^2 = 10$. Is the point $(3, 1)$ inside, outside or on the circle c .



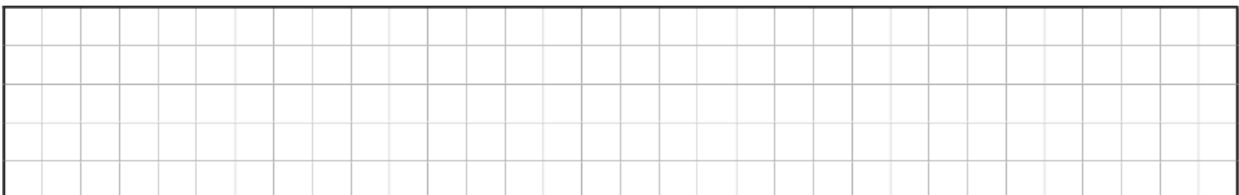
Show that the point $(3, -4)$ is on the circle $x^2 + y^2 = 25$.



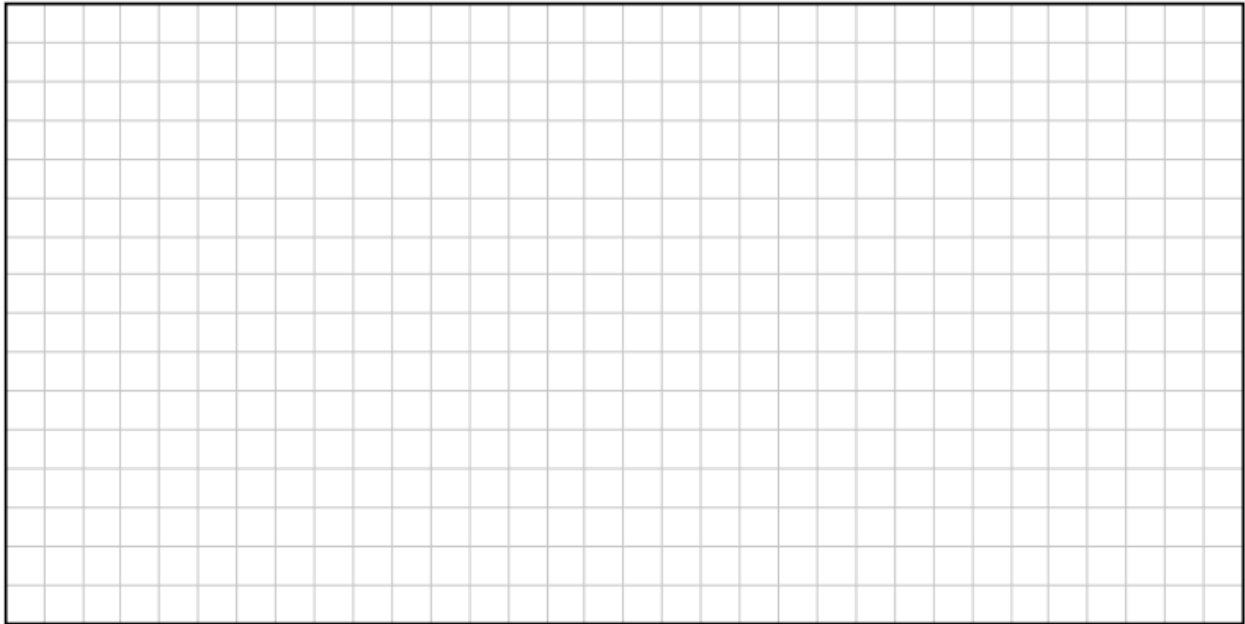
Circle C has centre $(1, -2)$ and radius 4. Is the point $(4, -5)$ inside, outside or on the circle C .



Show that the point $(2, 3)$ lies inside the circle with centre $(0, 0)$ and radius 5.

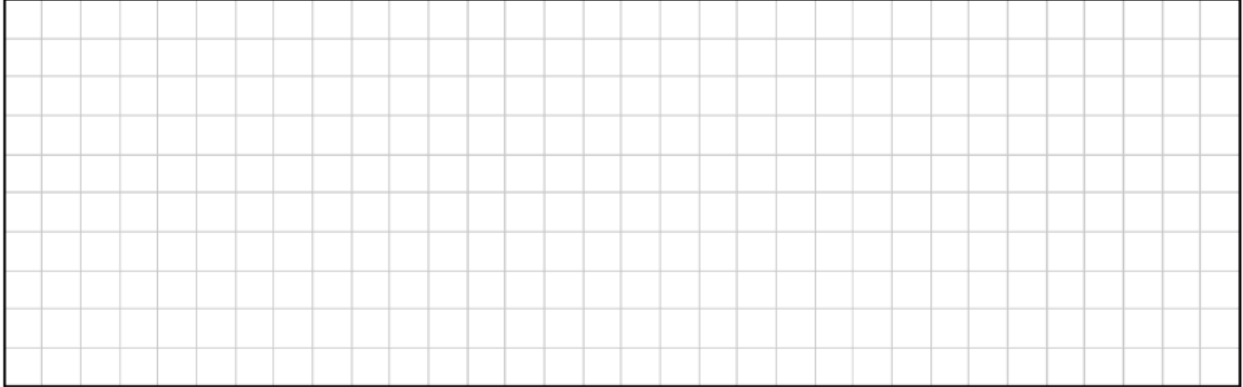


The circle C_2 has centre $(2, -1)$ and radius length $\sqrt{32}$. The point $(2p, p)$ is on C_2 . Find the value of p , where $p \in \mathbb{Z}$.

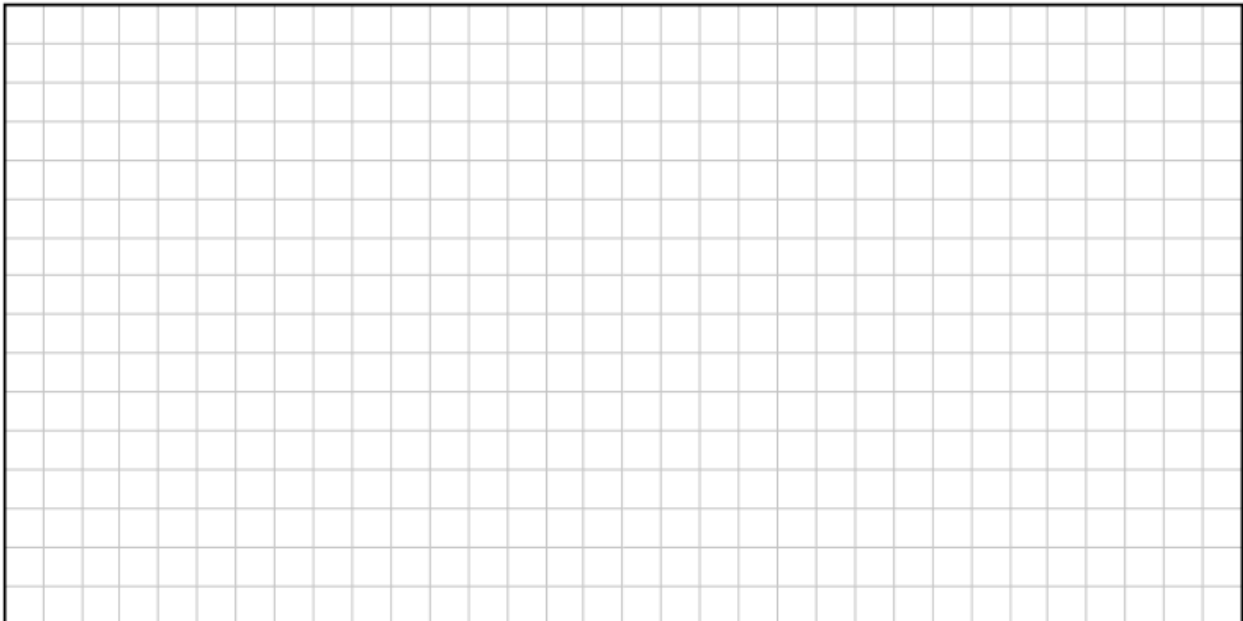


Tangents

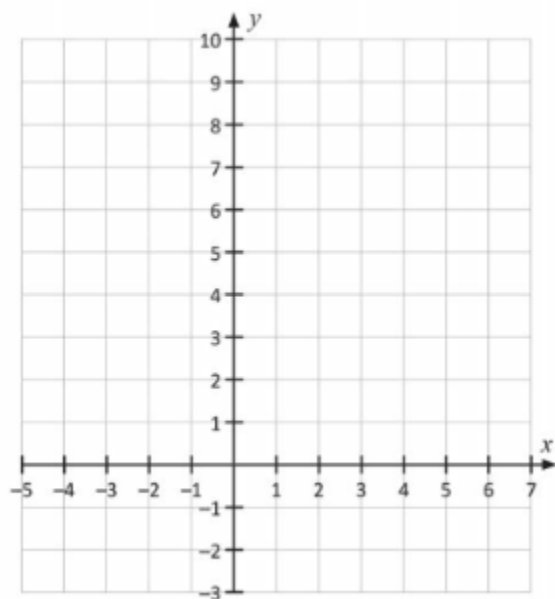
The line l is a tangent to the circle $x^2 + y^2 = 25$ at the point $A(-3, 4)$. Find the equation of l .



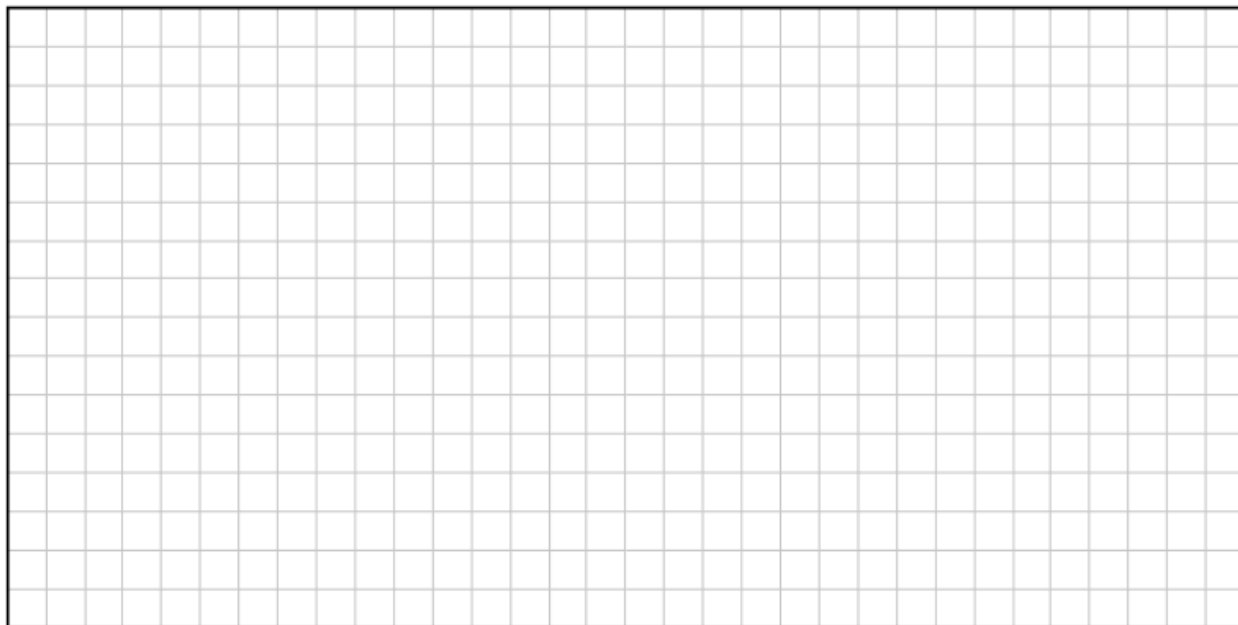
Prove that the line $3x + y - 10 = 0$ is a tangent to $x^2 + y^2 = 10$.



Draw the circle $(x - 1)^2 + (y - 4)^2 = 18$, and a tangent to the circle at $(4, 7)$ on the diagram below.

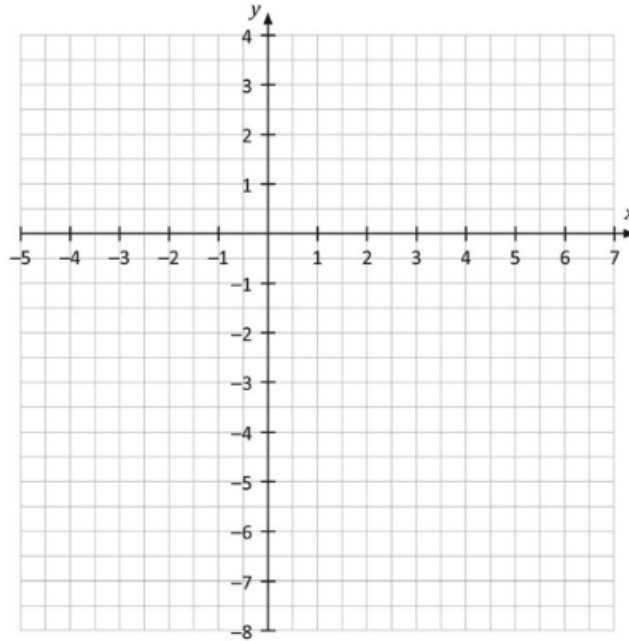


The circle s has equation $x^2 + y^2 = 13$. The point $A(3, -2)$ is on s . Find the equation of the tangent to the circle at the point A . Leave your answer in the form $ax + by + c = 0$, where $a, b, c \in \mathbb{Z}$.

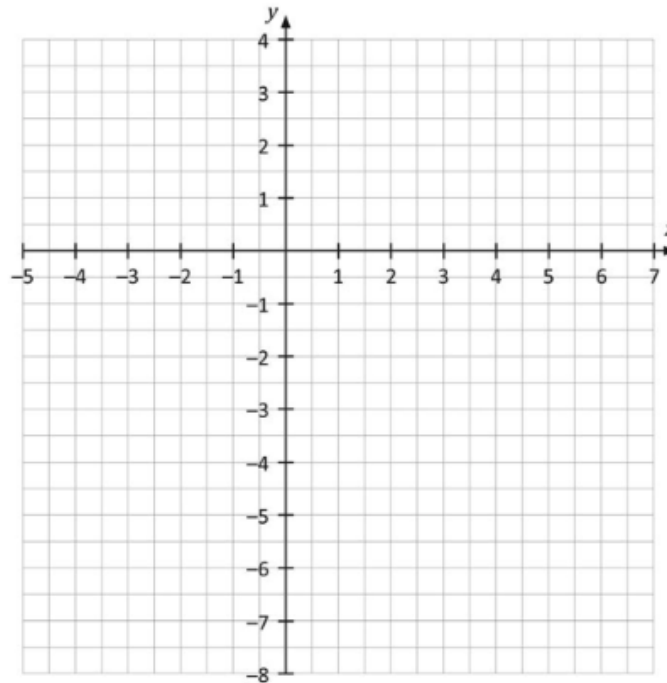


Constructions

Circle C has centre $(1, -2)$ and radius 4. Using a compass, construct circle C on the coordinate plane below.



Circle S has equation $x^2 + y^2 = 16$. Using a compass, construct circle S on the coordinate plane below.



Chapter 5

SOH
CAH
TOA

TRIGONOMETRY



Extra information:

Area → If right angled: $\frac{1}{2}$ base x height

If not: $\frac{1}{2} ab \sin C$ [Sandwich]

All 3 angles in a triangle add to 180°

A straight line adds to 180°



Is it right angled?

YES

Pythagoras

SOH
CAH
TOA

$$c^2 = a^2 + b^2$$

Always the hypotenuse

Use if given 2 sides, and asked to find 3rd

Use otherwise

NO

Cosine

Sine

$$a^2 = b^2 + c^2 - 2bc \cos A$$

a or A can be unknown

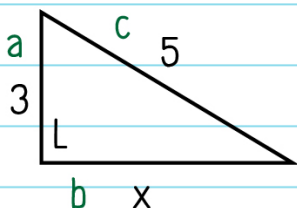
Use if given 2 sides that sandwich an angle.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

②

Use otherwise

Examples:



①

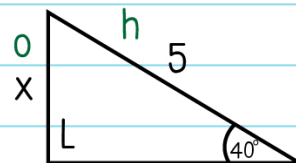
$$3^2 + x^2 = 5^2$$

$$9 + x^2 = 25$$

$$x^2 = 25 - 9$$

$$x^2 = 16$$

$$x = 4$$



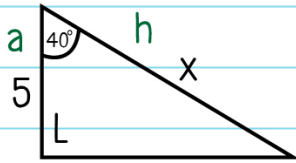
$$\sin(40) = \frac{x}{5}$$

$$5 \sin(40) = x$$

$$x = 3.21$$

Big letters = Angles,
Small letters = Sides

Sin(angle) = opp/hyp
Cos(angle) = adj/hyp
Tan(angle) = opp/adj

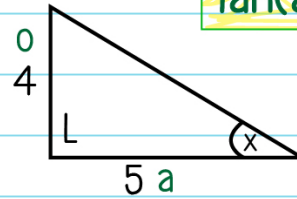


③

$$\cos(40) = \frac{5}{x}$$

$$x \cos(40) = 5 \rightarrow x = \frac{5}{\cos(40)}$$

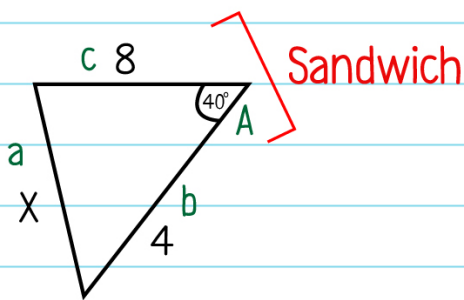
$$x = 6.53$$



④

$$\tan(x) = \frac{4}{5}$$

$$x = \tan^{-1} \frac{4}{5} \rightarrow x = 38.7^\circ$$

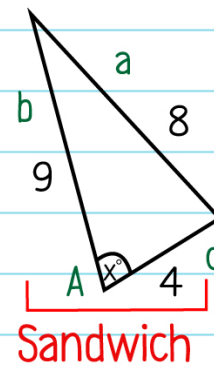


⑤

$$x^2 = 4^2 + 8^2 - 2(4)(8)(\cos(40))$$

$$x^2 = 31$$

$$x = \sqrt{31} \rightarrow x = 5.57$$



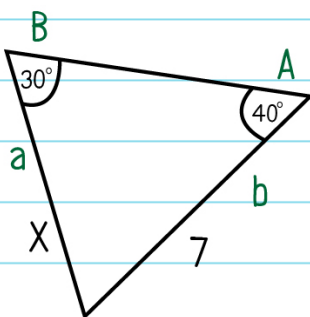
⑥

$$8^2 = 9^2 + 4^2 - 2(9)(4)(\cos(x))$$

$$8^2 - 9^2 - 4^2 = -72 \cos(x)$$

$$\frac{-33}{-72} = \cos(x) \rightarrow x = \cos^{-1}(33/72)$$

$$x = 62.7^\circ$$

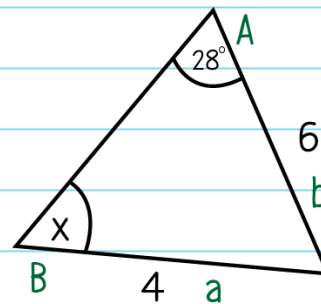


⑦

$$\frac{x}{\sin(40)} = \frac{7}{\sin(30)}$$

$$x \sin(30) = 7 \sin(40)$$

$$x = \frac{7 \sin(40)}{\sin(30)} \quad x = 9$$



⑧

$$\frac{4}{\sin(28)} = \frac{6}{\sin(x)}$$

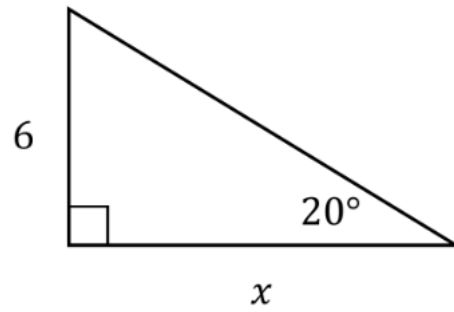
$$4 \sin(x) = 6 \sin(28)$$

$$\sin(x) = \frac{6 \sin(28)}{4}$$

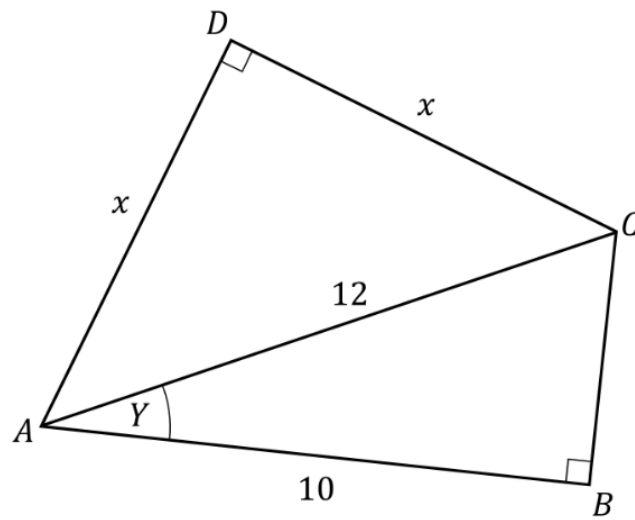
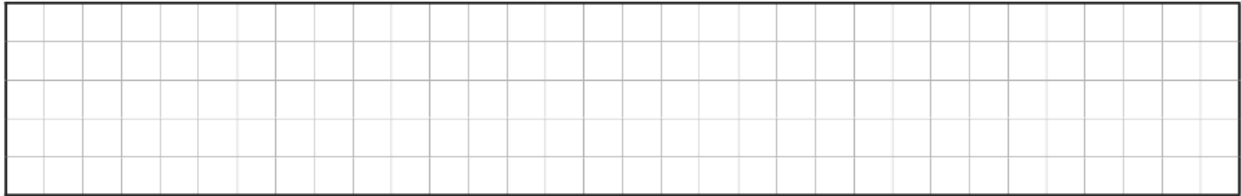
$$x = \sin^{-1}\left(\frac{6 \sin(28)}{4}\right)$$

$$x = 69.9^\circ$$

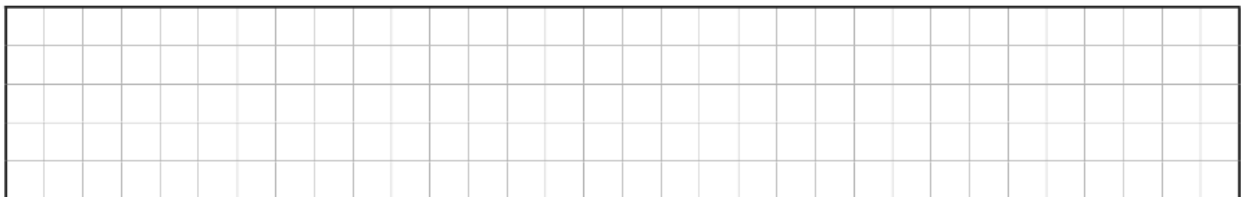
Right angled triangles

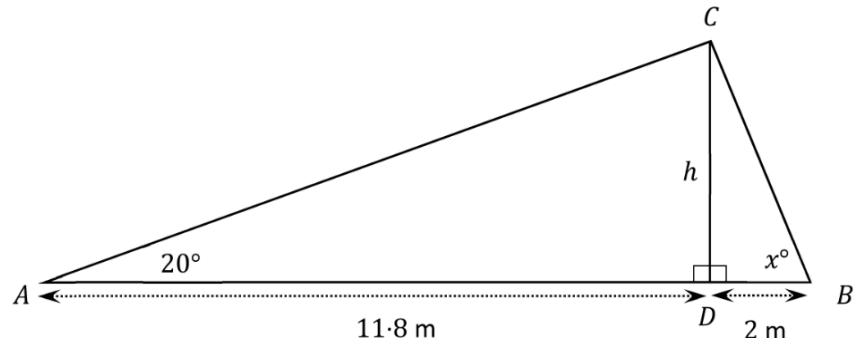


Use trigonometry to work out the value of x .

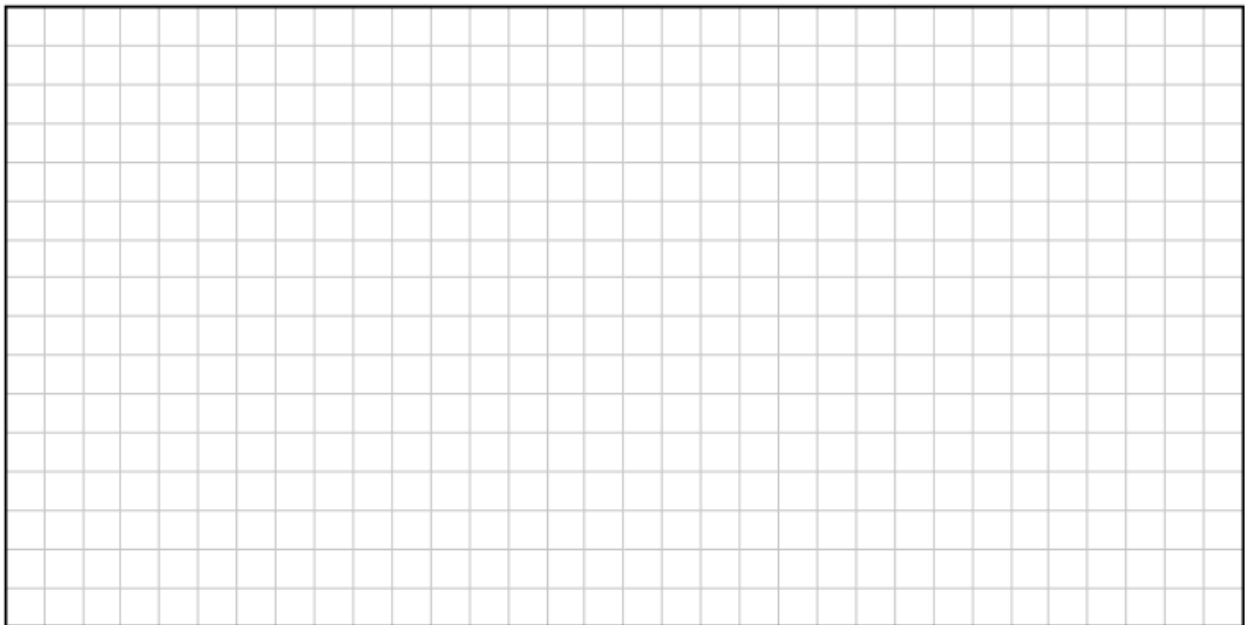


Use trigonometry to work out the size of the angle marked Y , and the value of side x .



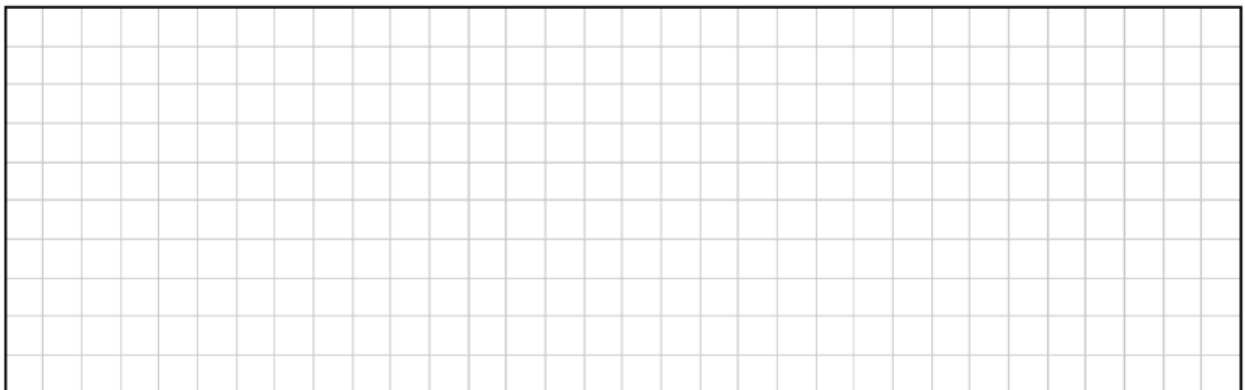


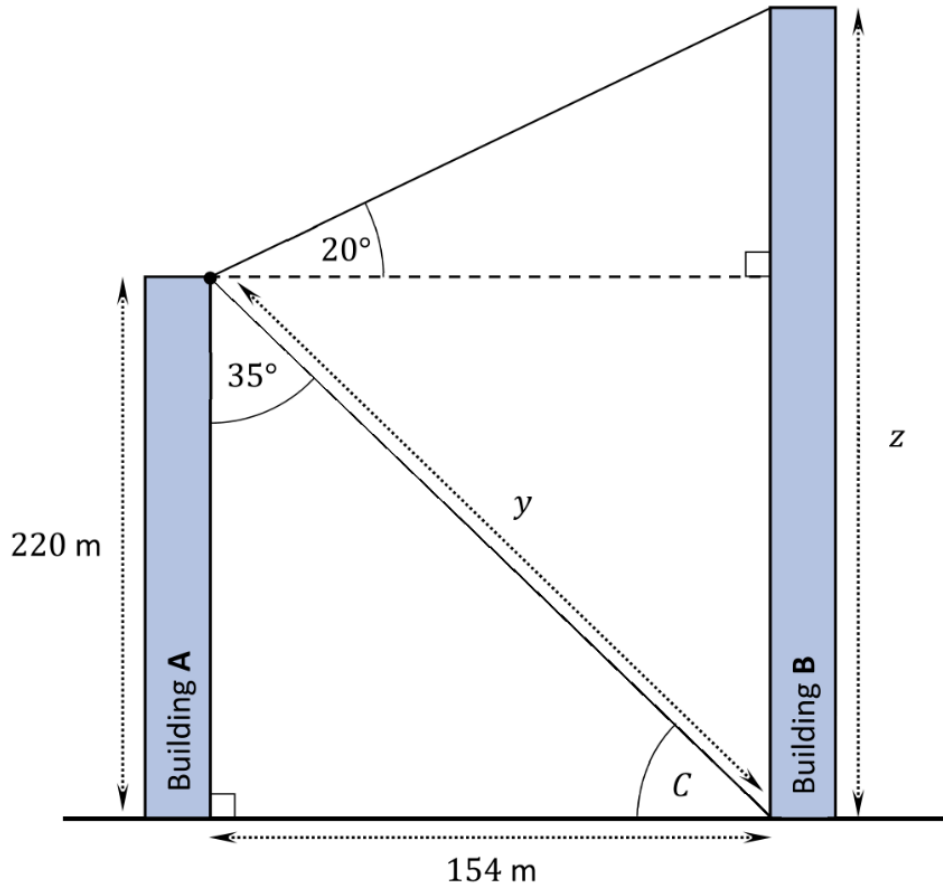
Find the value of x . Give your answer correct to the nearest whole number.



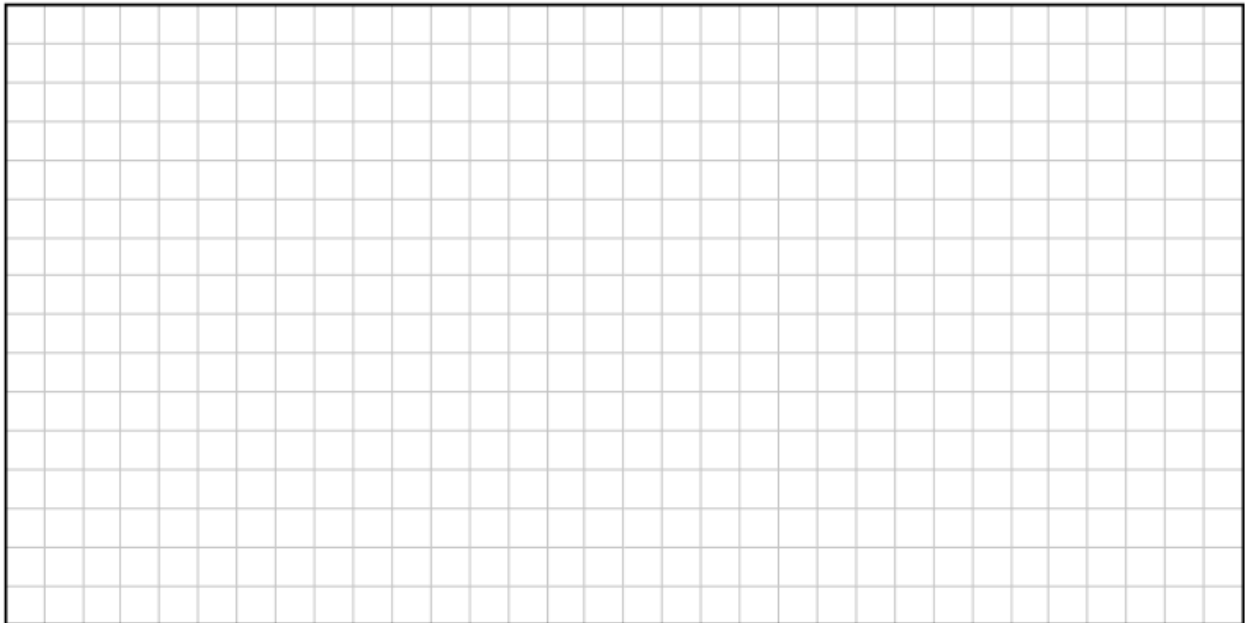
F is an angle in a right angled triangle such that $\cos F = \frac{6}{11}$.

Find the value of $\sin F$. Give your answer in surd form.



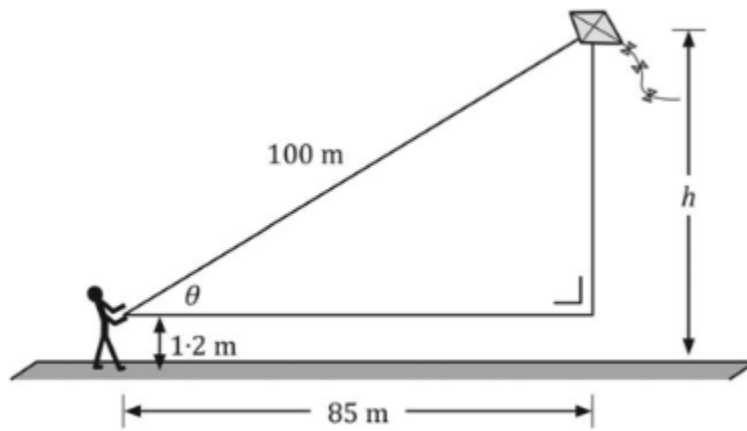
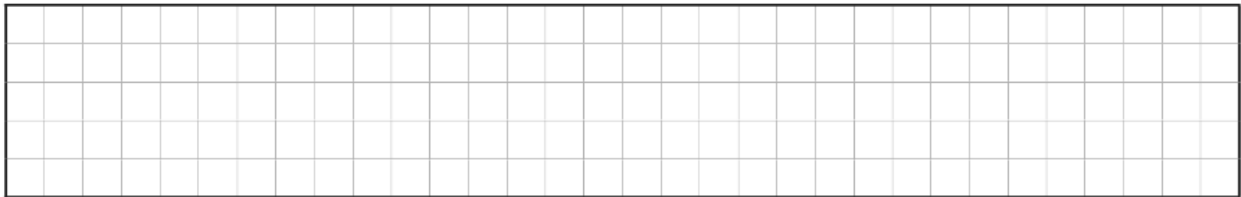
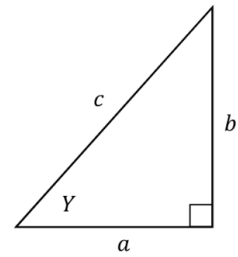


Work out the size of angle C , the length of side y , and the height of building B (z).

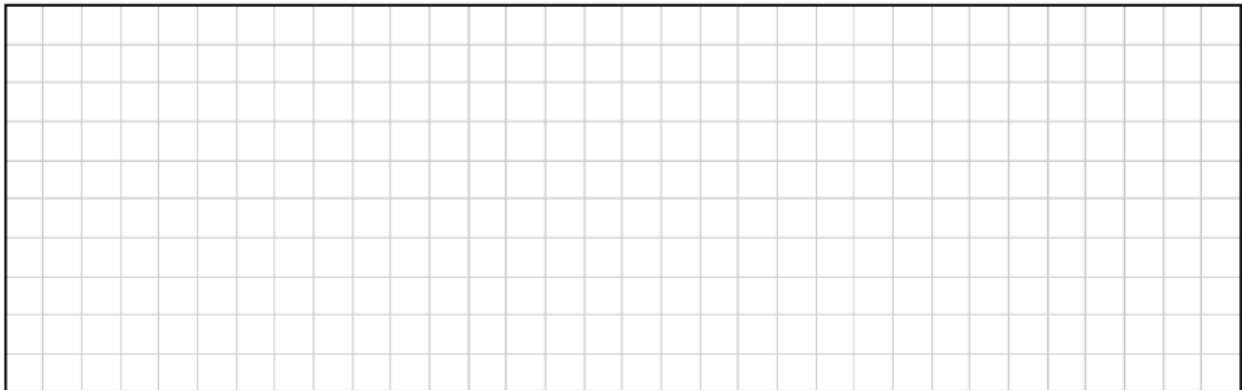


In this triangle, $a + b > c$.

Use this to prove that $\cos Y + \sin Y > 1$

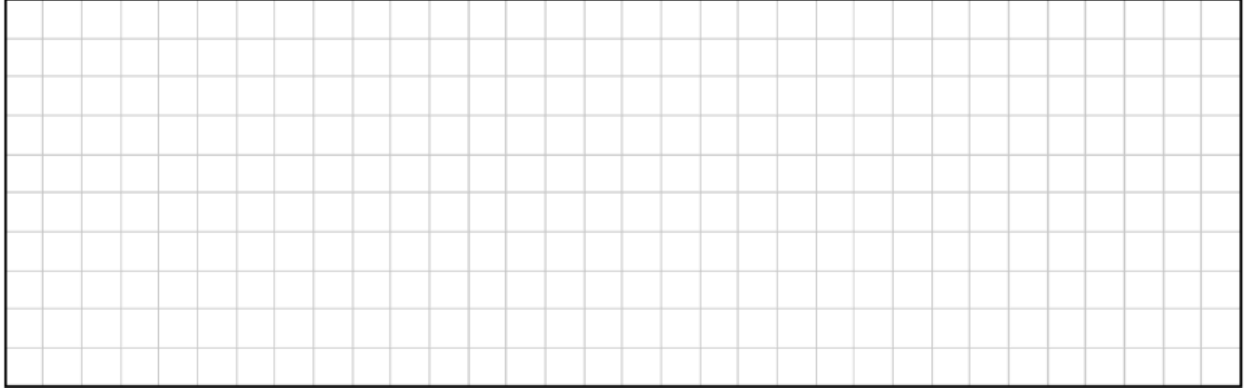


Find the size of angle θ and h , the height of the kite above the ground.

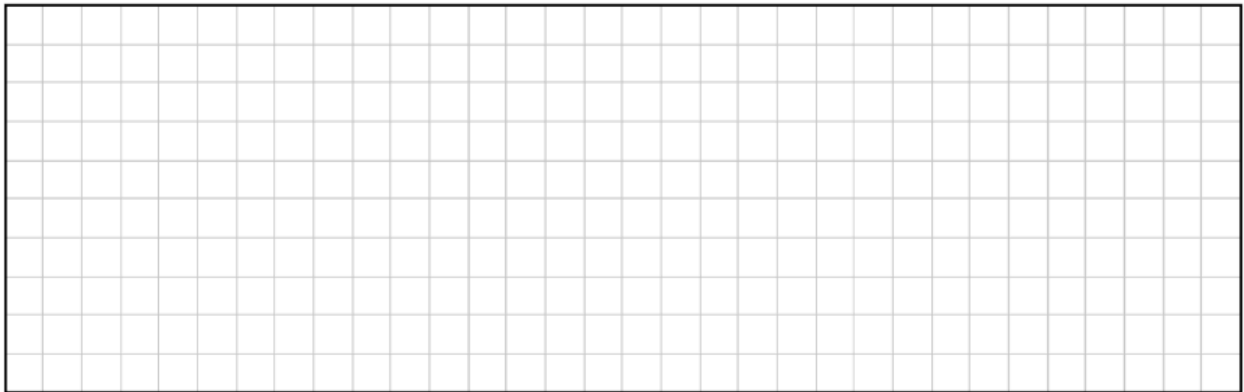


Phillipe, who is 1.65 metres tall, walks 50 metres away from the base of the Eiffel Tower. He then uses a clinometer and finds that the angle of elevation to the top of the monument is 81° .

Find the height of the eiffel tower correct to the nearest metre.

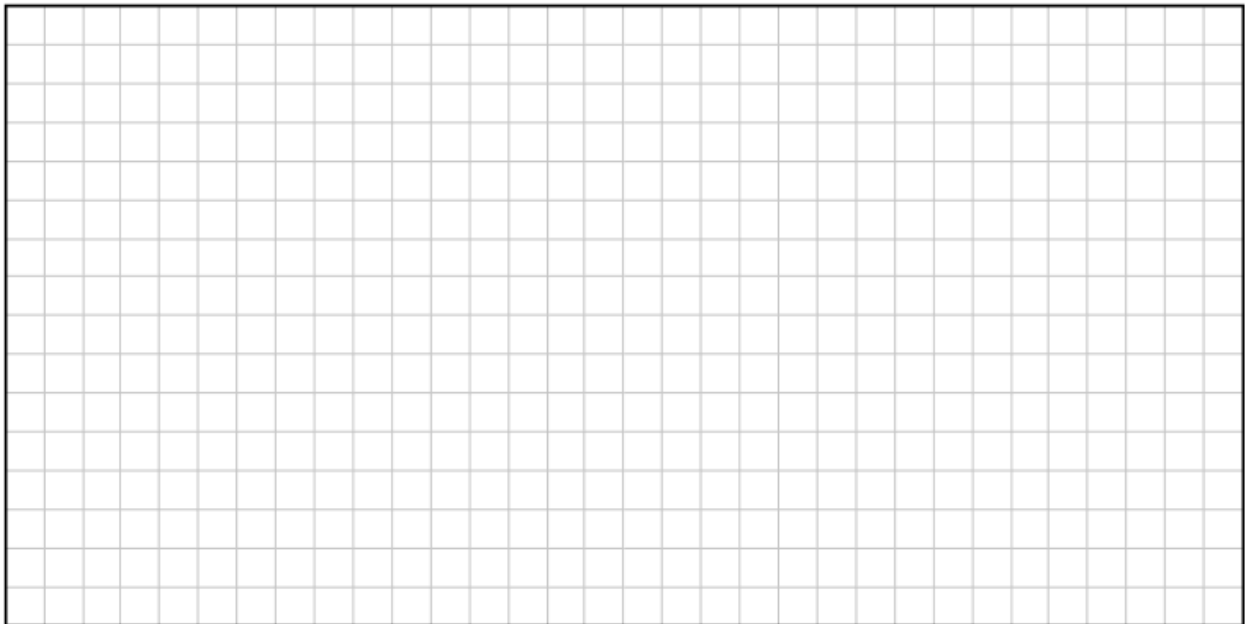
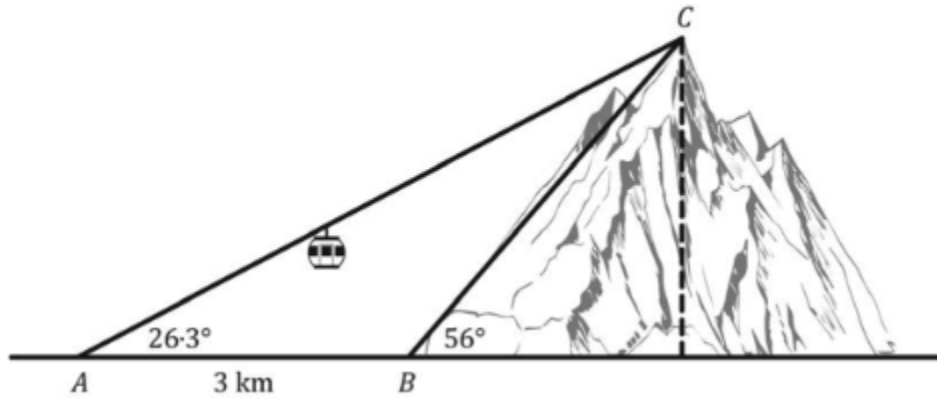


A right angled triangle has sides of length 7 cm, 24 cm and 25 cm. Find the size of the smallest angle in this triangle. Give your answer correct to one decimal place.

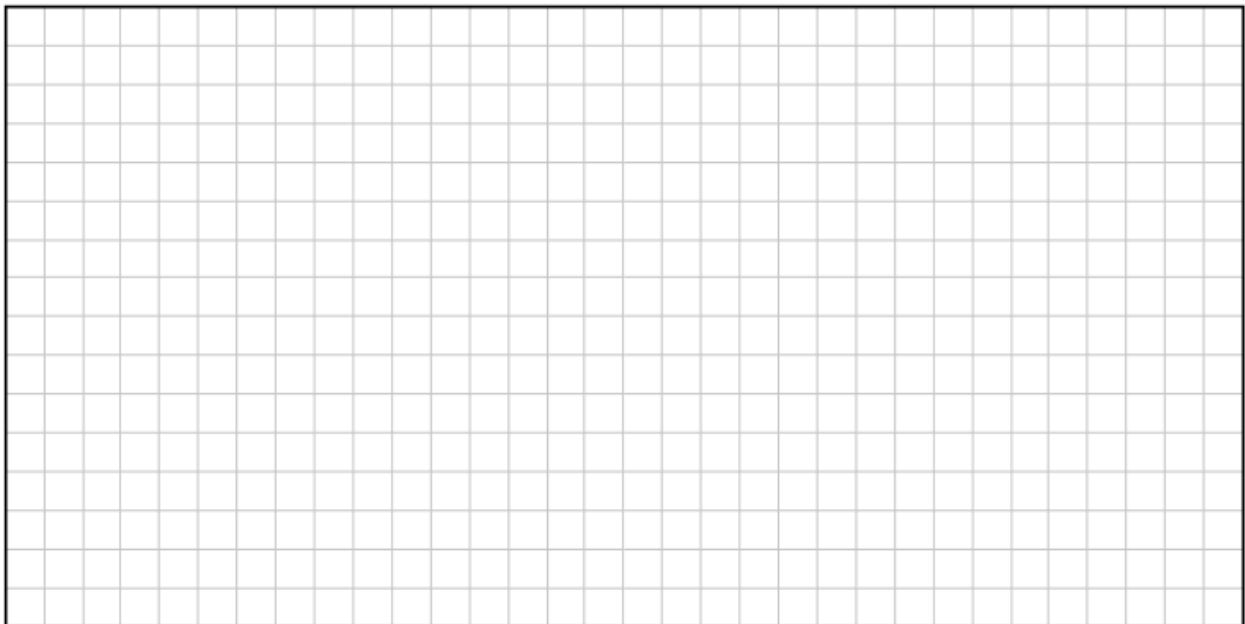
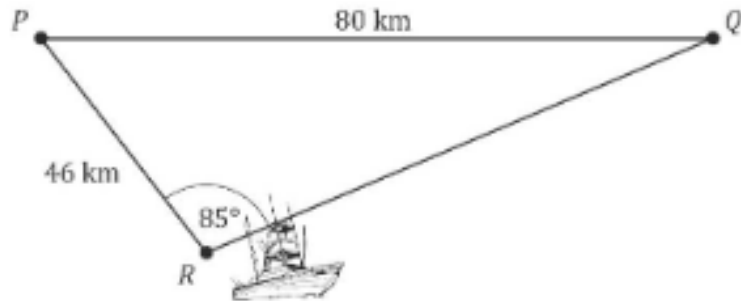


Non right-angled triangles

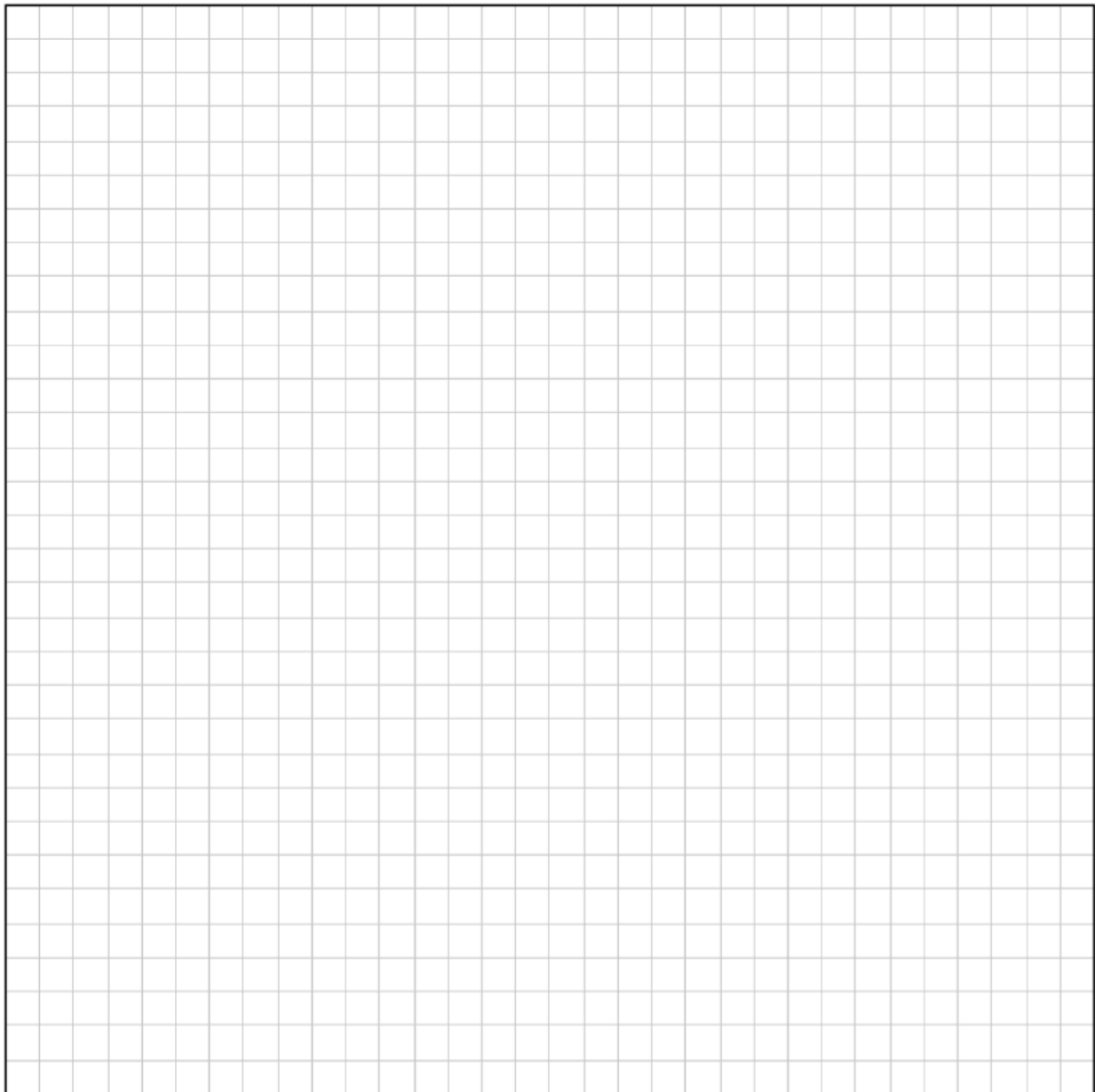
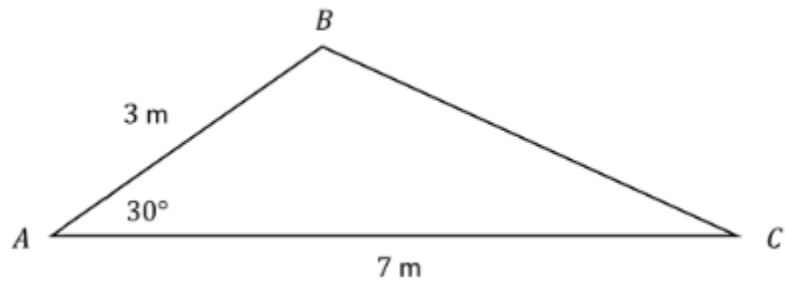
A cable car carries passengers from Point A, which is 3 km from point B, at the base of a mountain, as shown below. Find the $|\angle ACB|$, and hence, find $|BC|$.



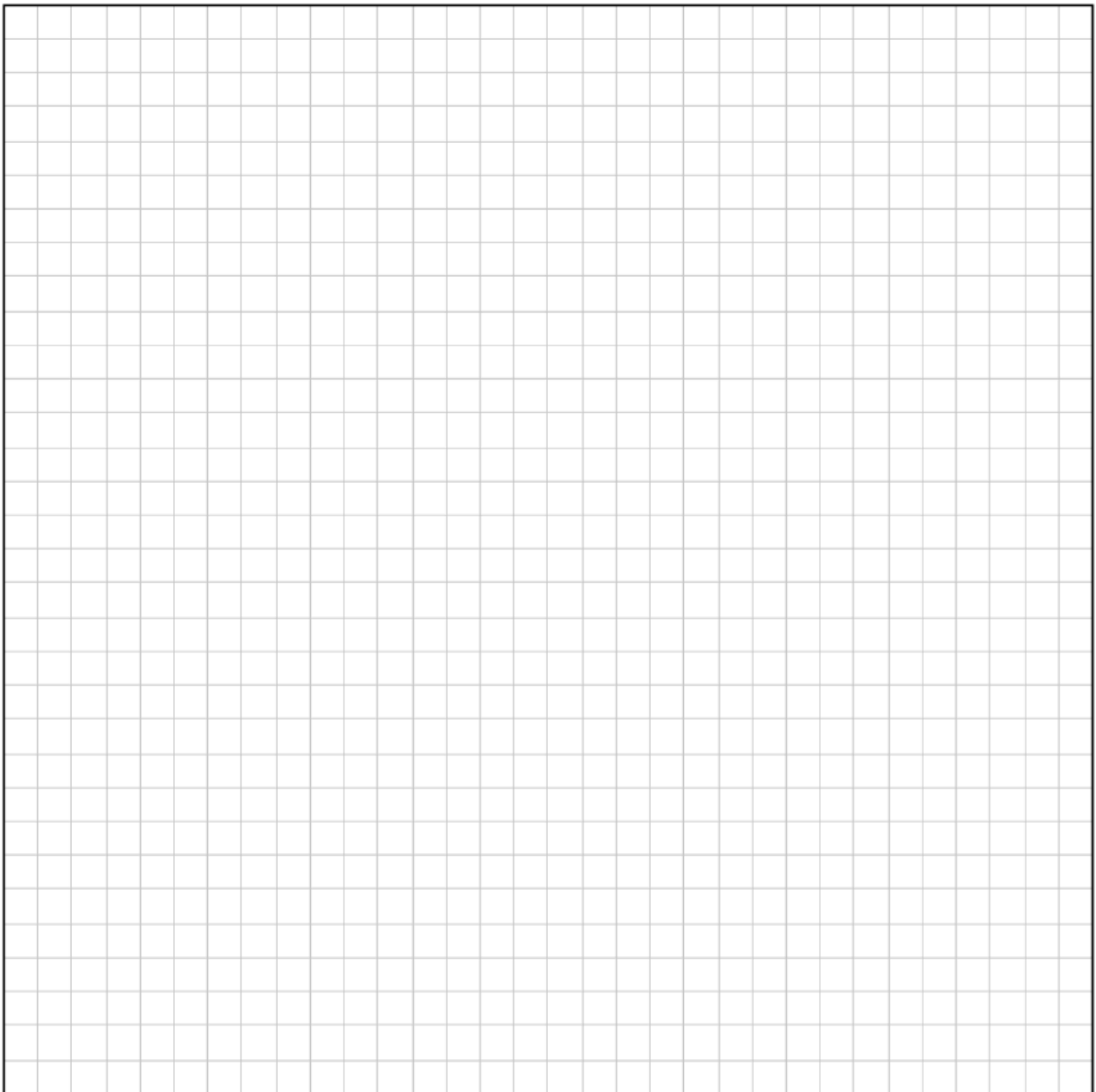
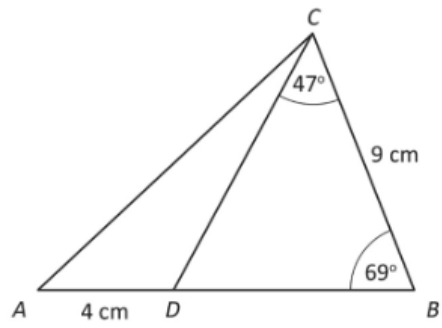
Two coast guard bases, P and Q , are located 80 km apart along a coastline, with Q due east of P . Find $|\angle PQR|$. Give your answer correct to the nearest degree.



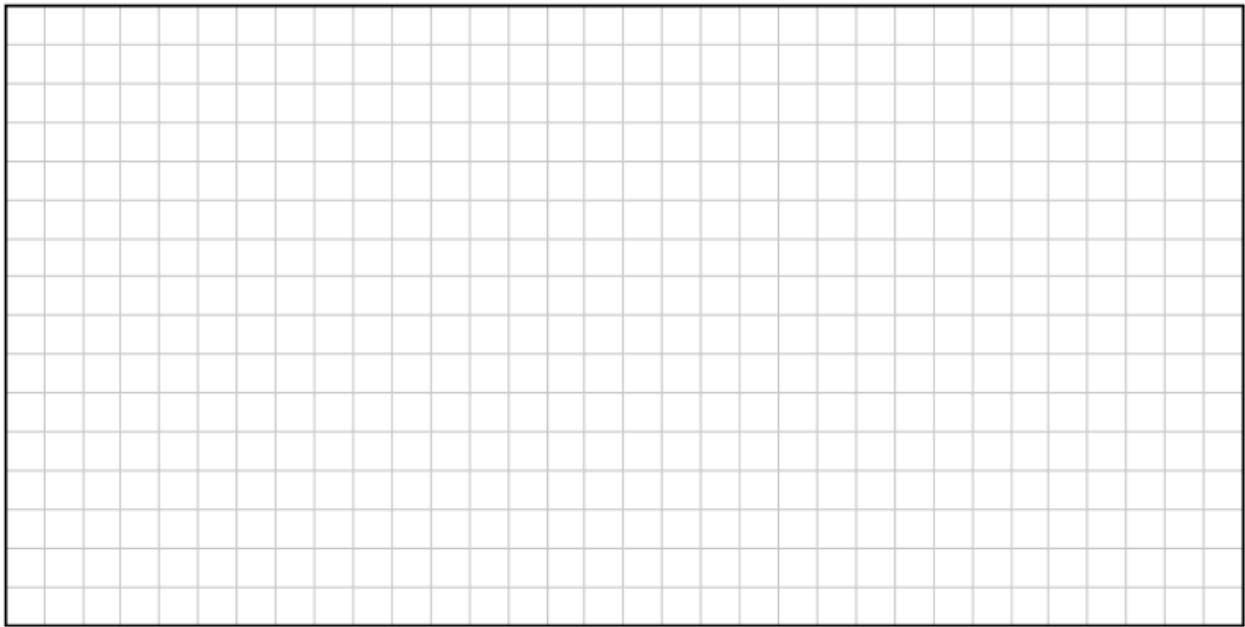
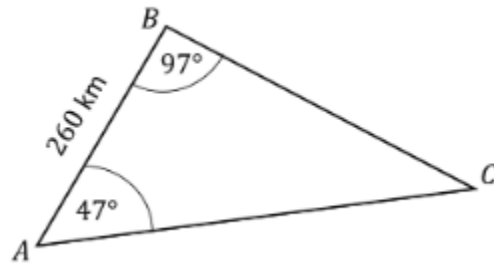
Find $|BC|$, correct to two decimal places and find $|\angle ACB|$



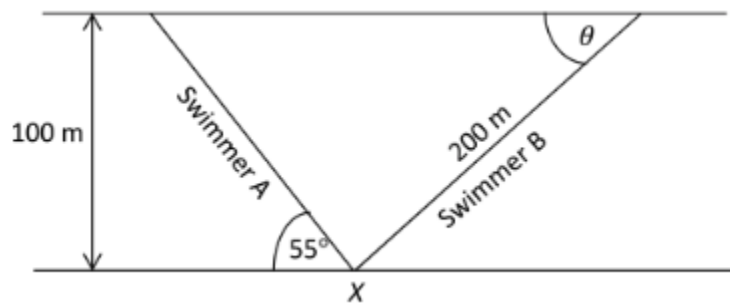
Find $|DB|$ and hence find $|AC|$.



Find the distance from point A to point C . Leave your answer correct to the nearest km.

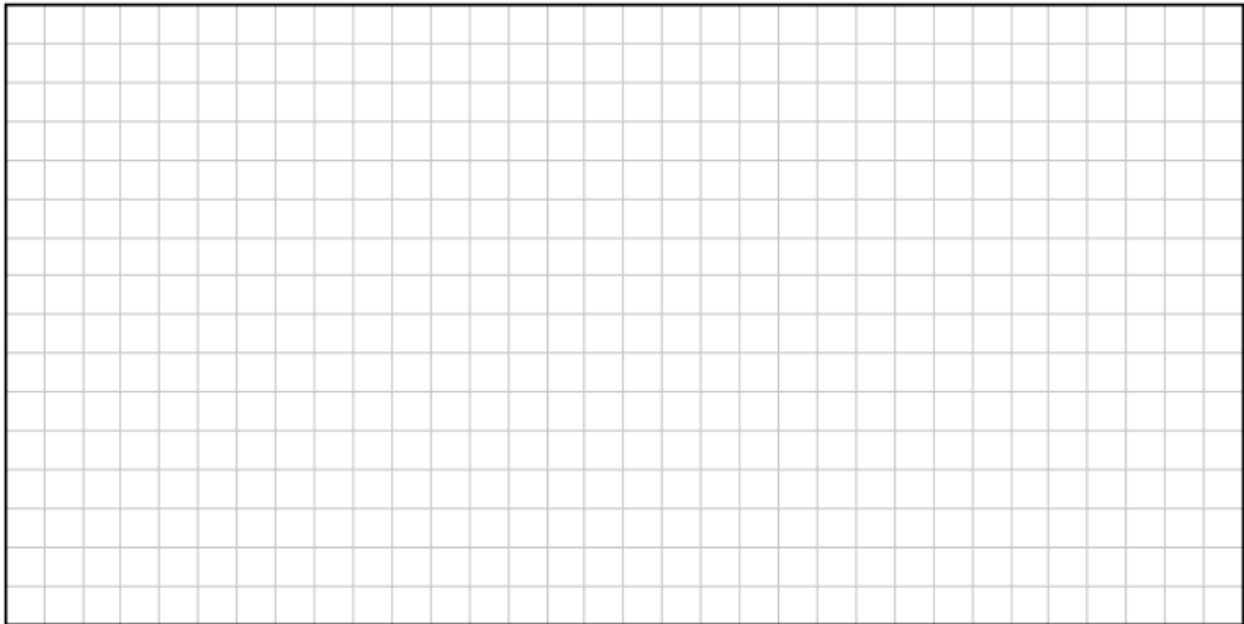


Two swimmers A and B stand at the same point X , on one shore of a lake. Both swim to the opposite side of the lake.

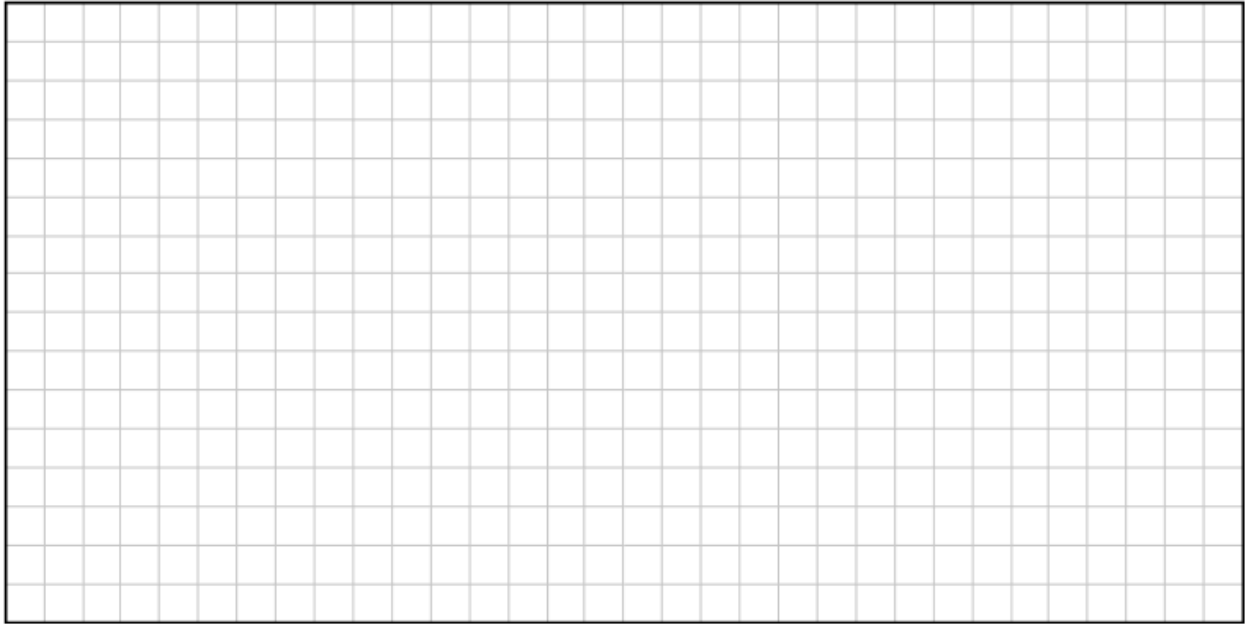


Find:

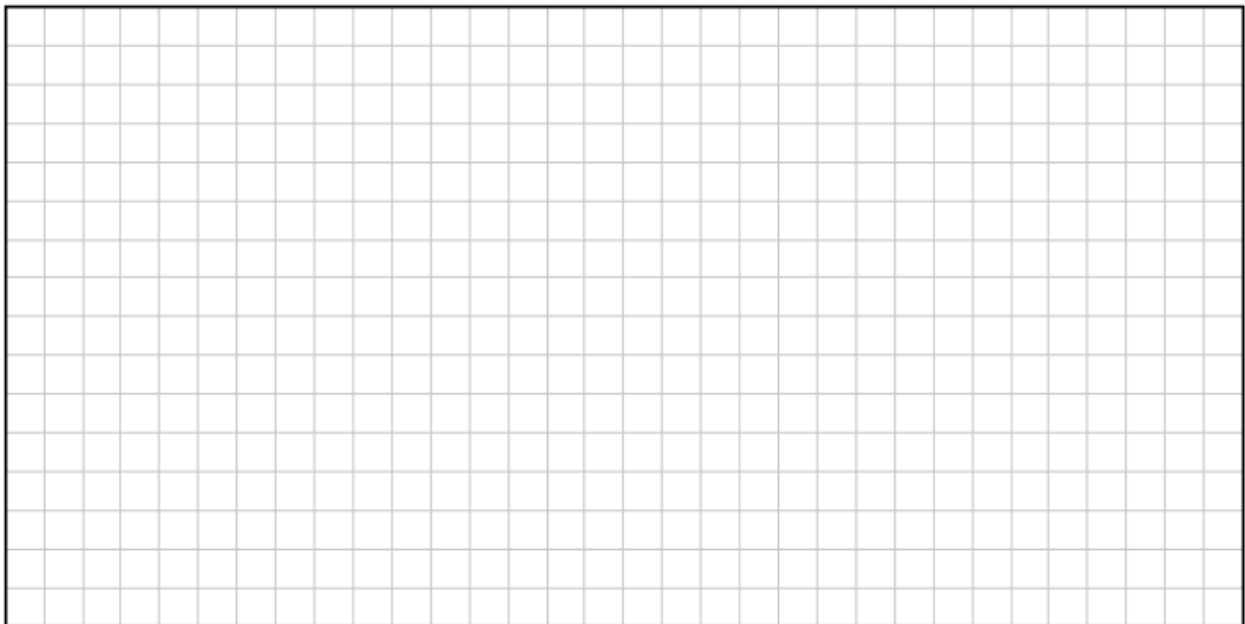
(i) The distance swimmer A swims. Leave your answer correct to one decimal place.

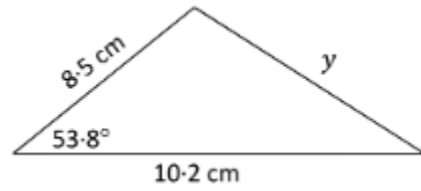
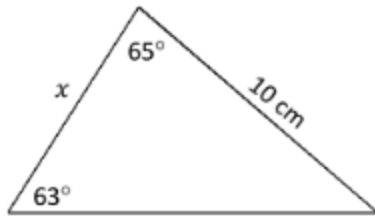


(ii) Find the measure of angle θ . leave your answer correct to one decimal place.

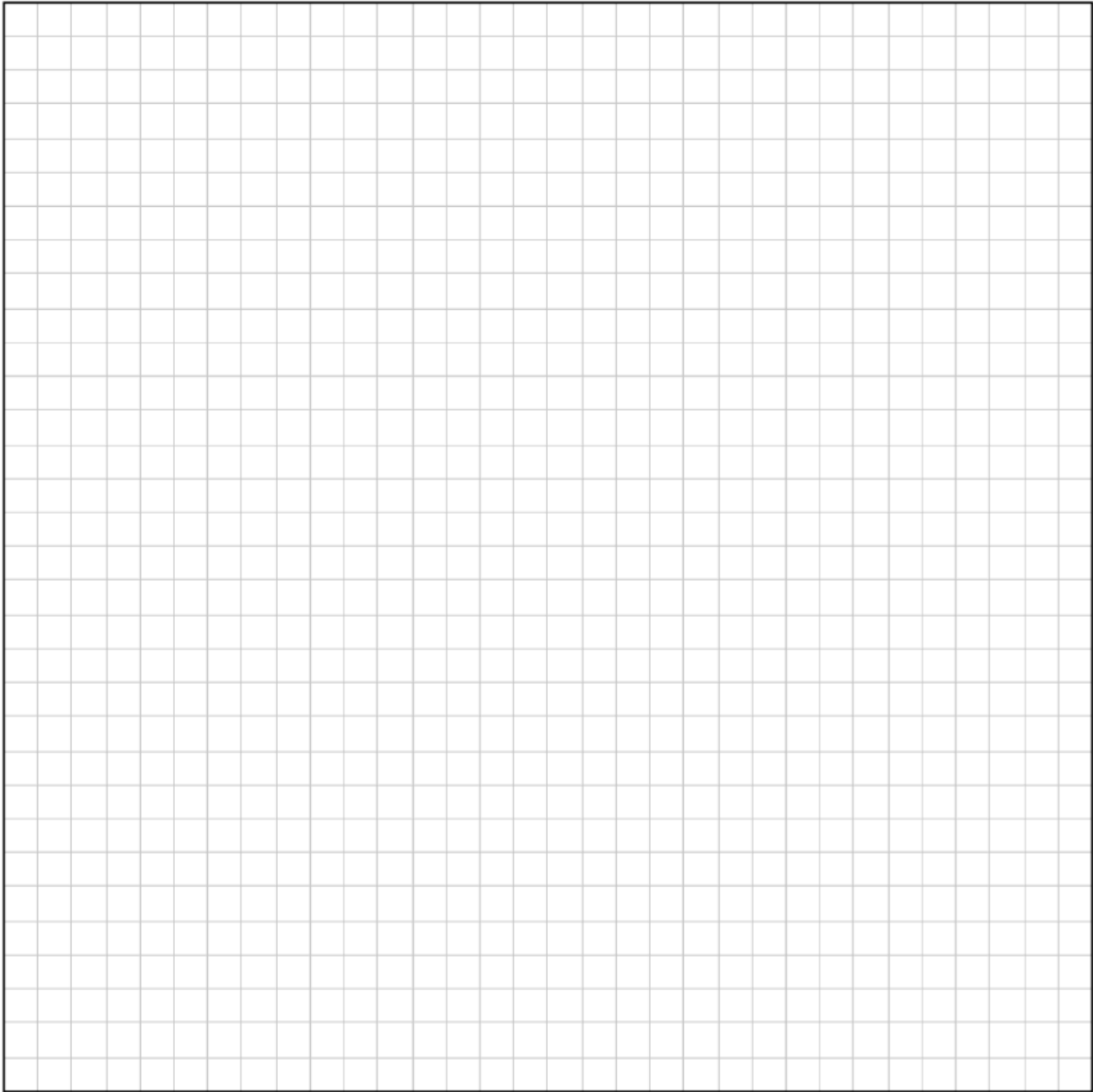


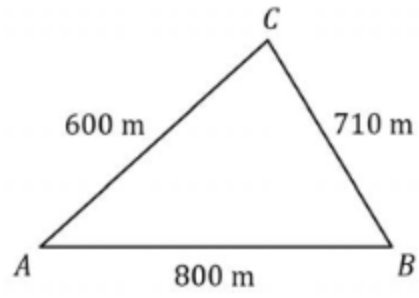
(iii) Find the distance the swimmers are apart when they reach the opposite side of the lake. Leave your answer correct to the nearest metre.



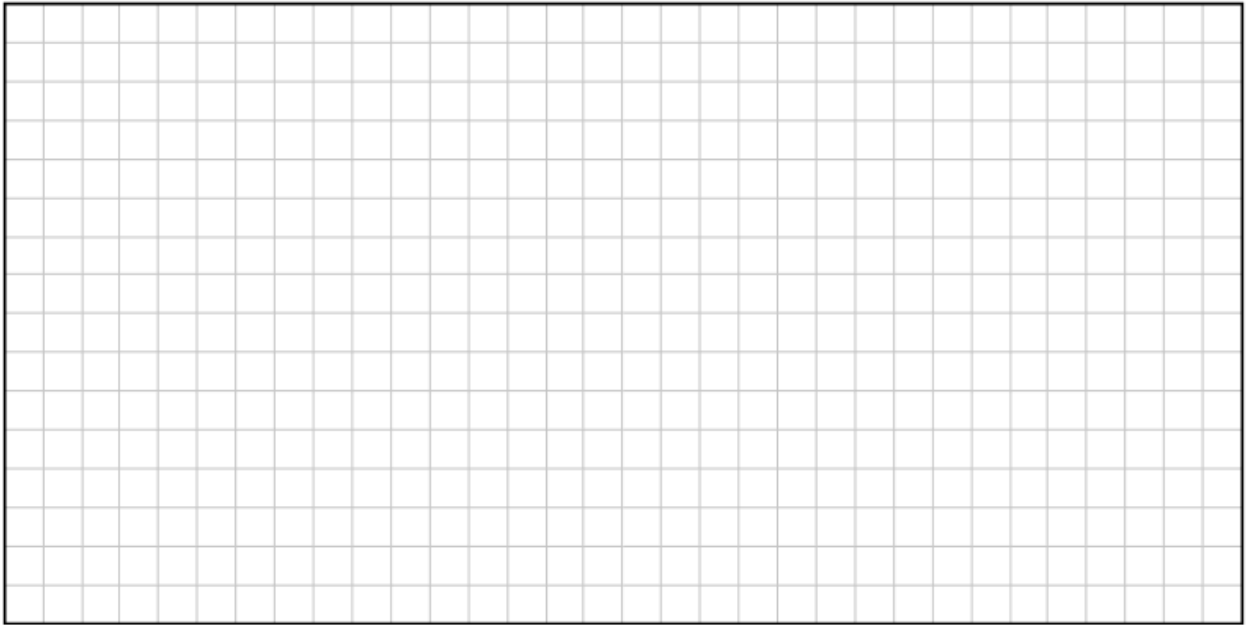


John says x has the same value as y . Is John correct? Justify your answer.



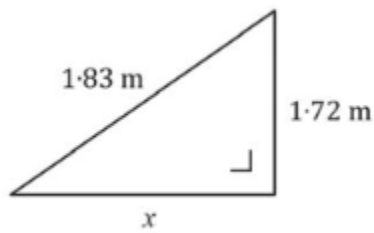
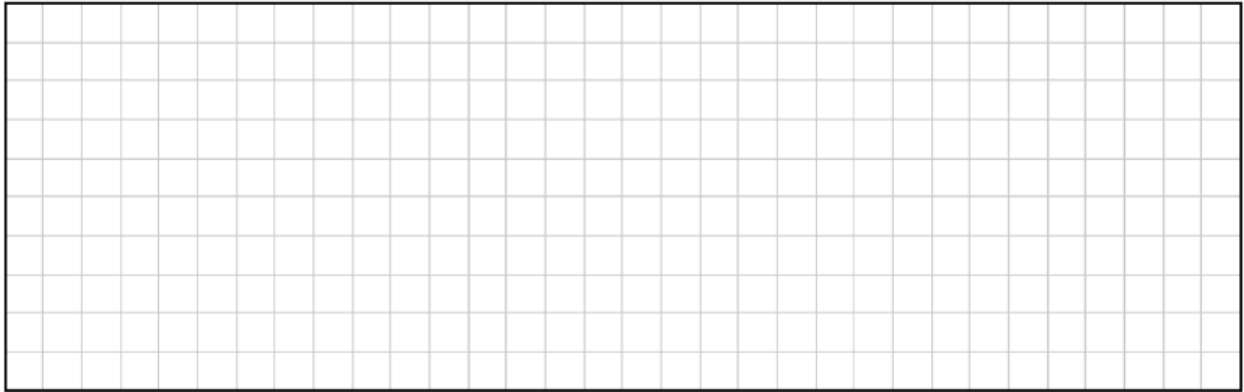
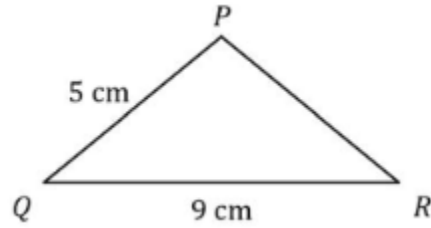


Find the measure of the angle at vertex C .

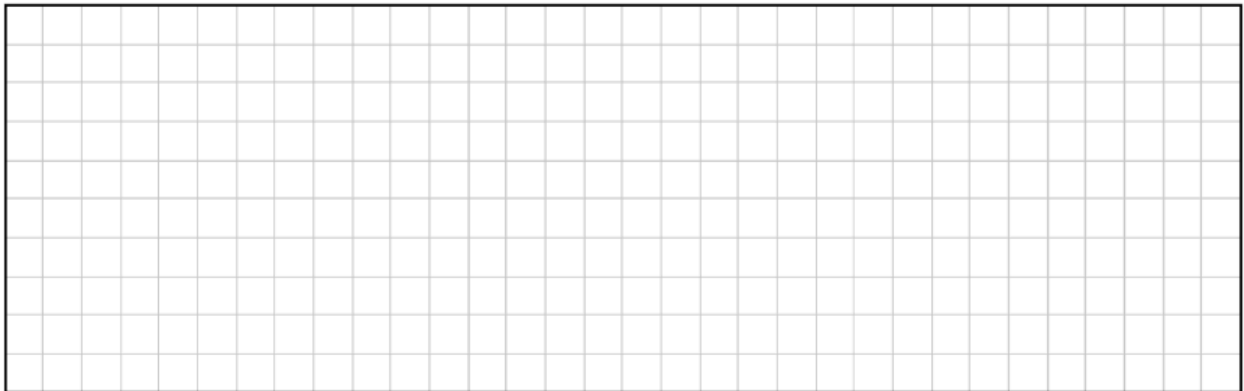


Area of a triangle

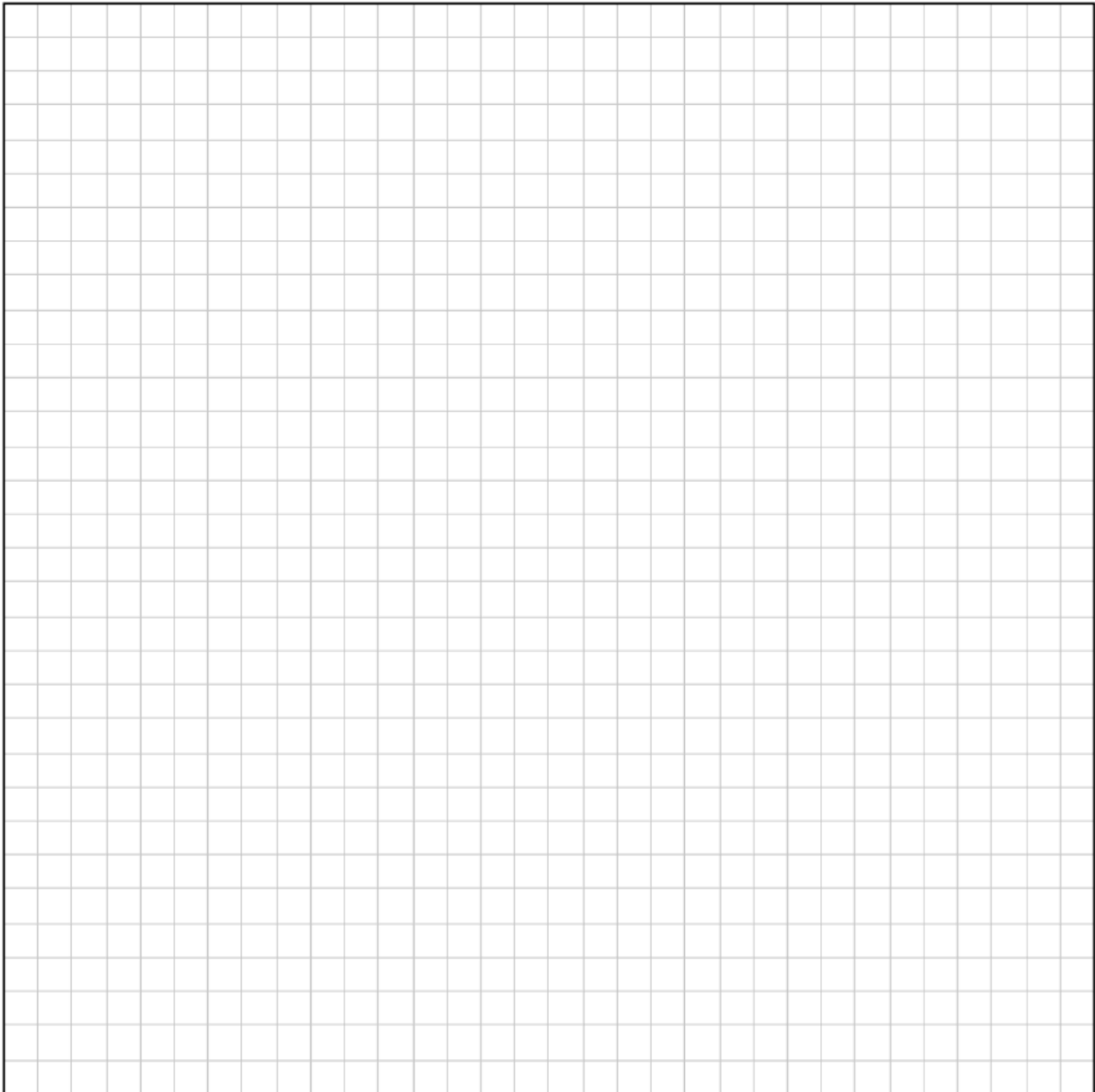
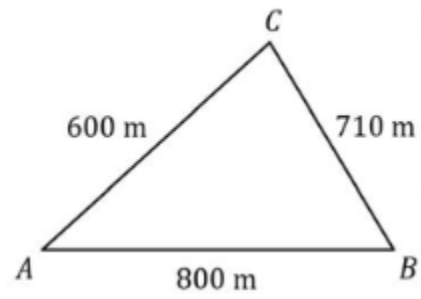
The area of triangle PQR is 13.71 cm^2 . Find $|\angle PQR|$.



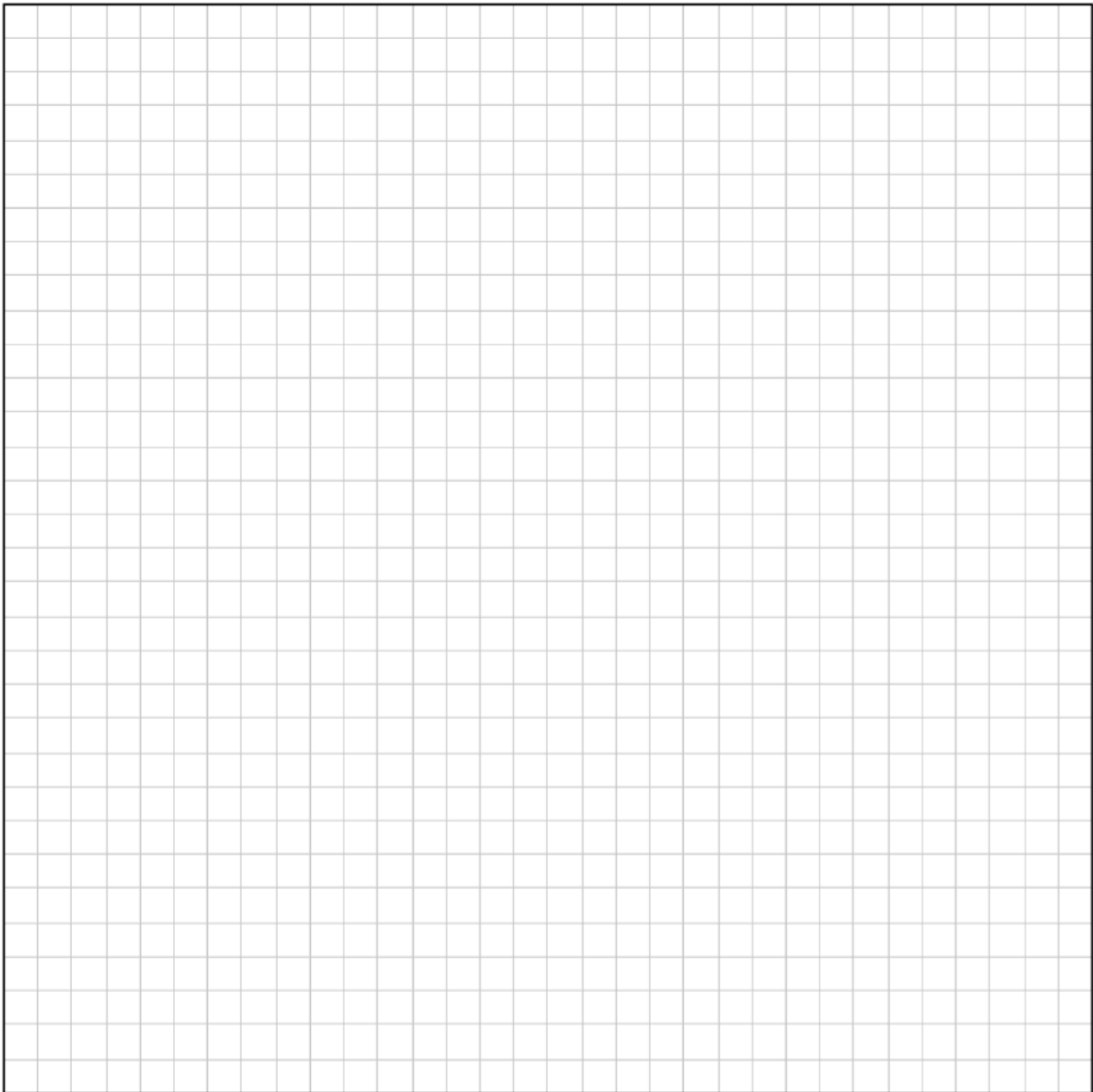
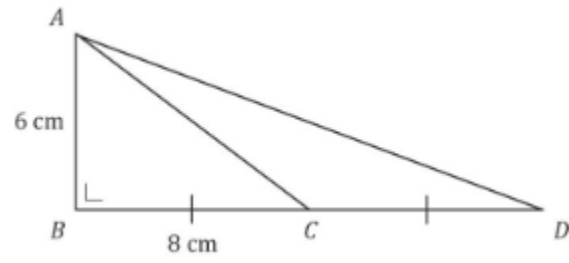
Find the value of x , and hence find the area of the above triangle.



Work out the area of $\triangle ABC$.

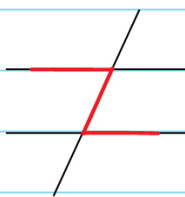


Prove that the area of $\triangle ACB$ is equal to the area of $\triangle ACD$



Chapter 6

GEOMETRY AND AREA + VOLUME



Triangles

Scale

Geometry theorems

Area and volume

Triangles

Equilateral → All 3 sides and angles the same

Scalene → All 3 sides and angles different

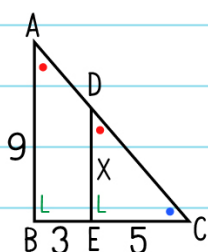
Isosceles → 2 sides and angles the same

Circumcircle / Circumcentre :

The point in a triangle where the lines that cut each side in half at right angles meet. It's the center of the circle that goes through all three corners of the triangle.

Similar Triangles

Triangles with all 3 angles the same



$$\begin{aligned} |\angle ABC| &= |\angle DEC| \\ |\angle ACB| &= |\angle DCE| \\ |\angle BAC| &= |\angle EDC| \end{aligned}$$

$$\frac{9}{X} = \frac{8}{5}$$

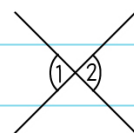
continue

Congruent Triangles

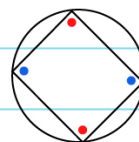
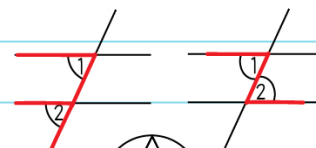
→ Identical

Proofs: SSS, SAS, ASA, RHS

Geometry theorems



$|\angle 1| = |\angle 2|$ for all examples



Cyclic Quadrilateral
= opposite angles add to 180°

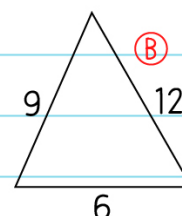
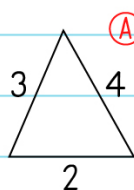
$X = 60^\circ$
 $120/X$

Straight lines add to 180°

Scale

Triangle B is triangle A, scaled by a factor of 3

e.g.





● Area and volume

- 2-d shapes
- 3-d shapes

↳ cones
cylinders
spheres
cubes/ cuboids

- Trapezoidal rule

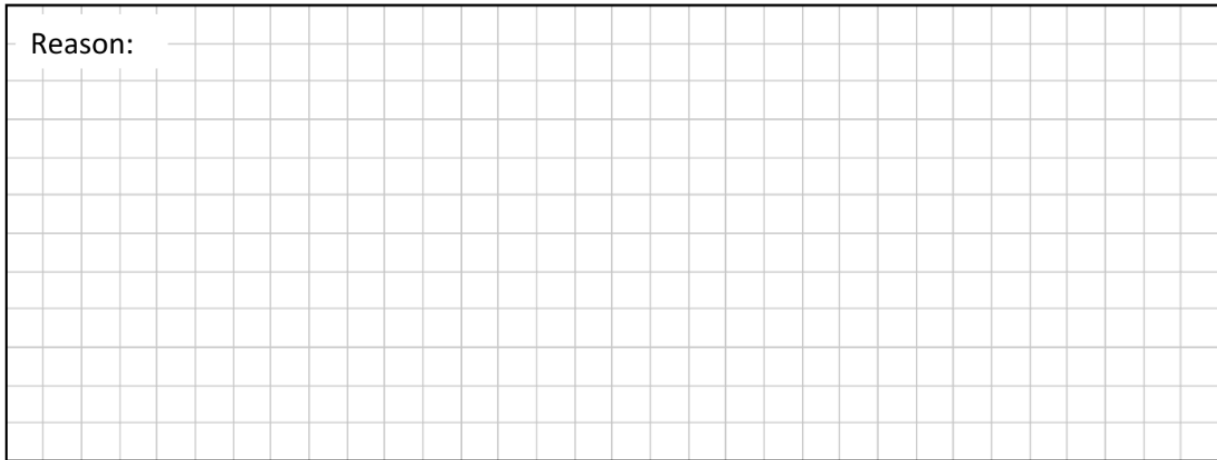
Similar and congruent triangles

Statement: If two triangles are similar, they must be congruent.

true

false

Reason:

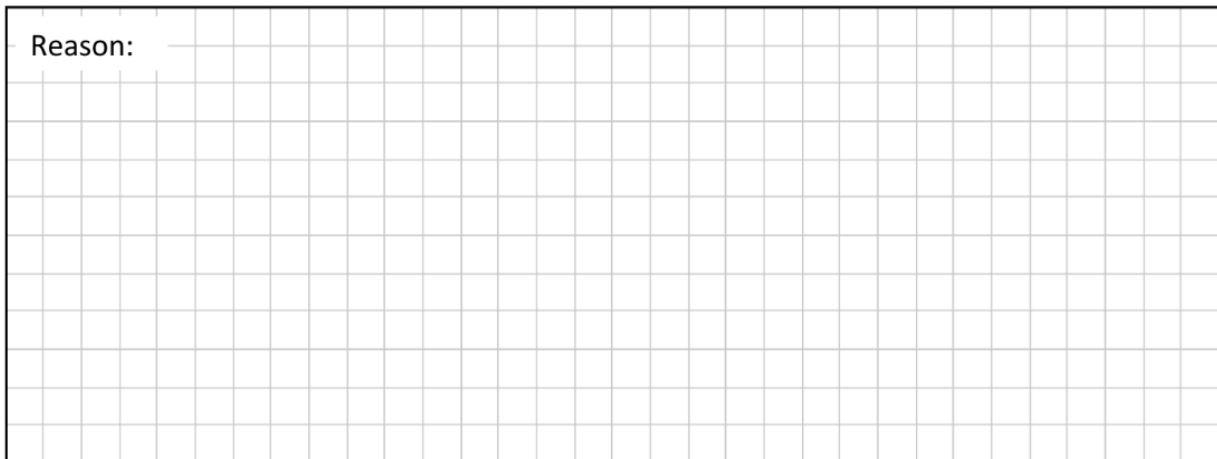


Statement: If two triangles are congruent, they must be similar.

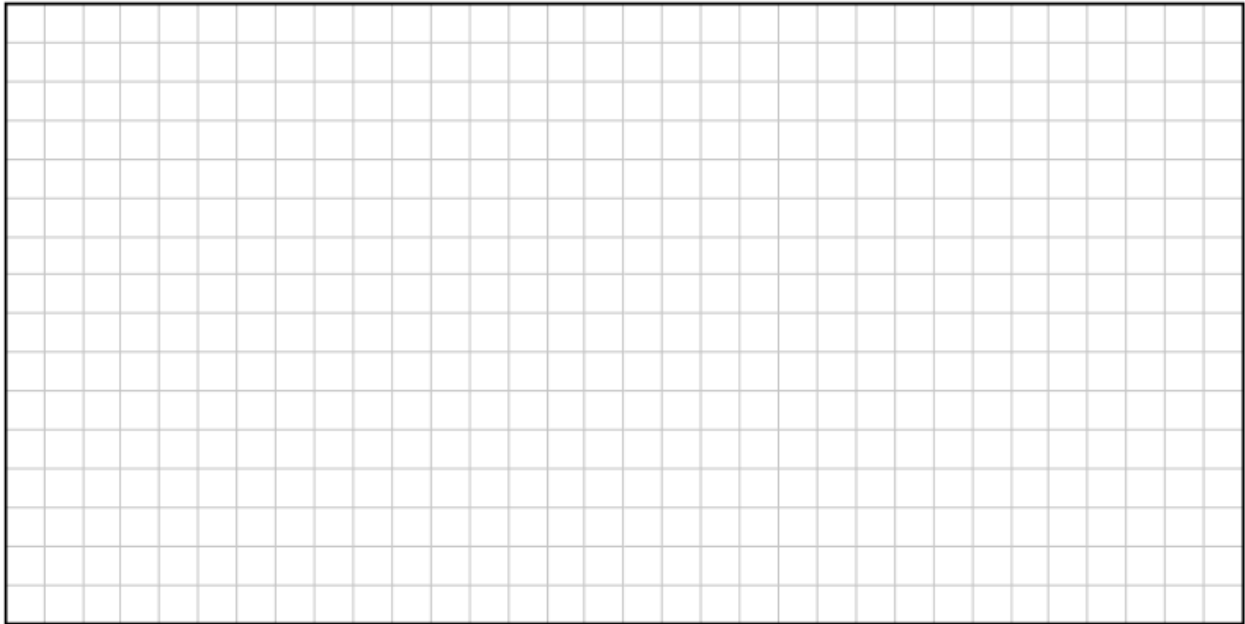
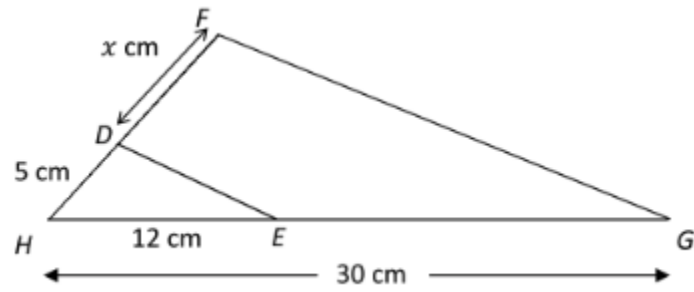
true

false

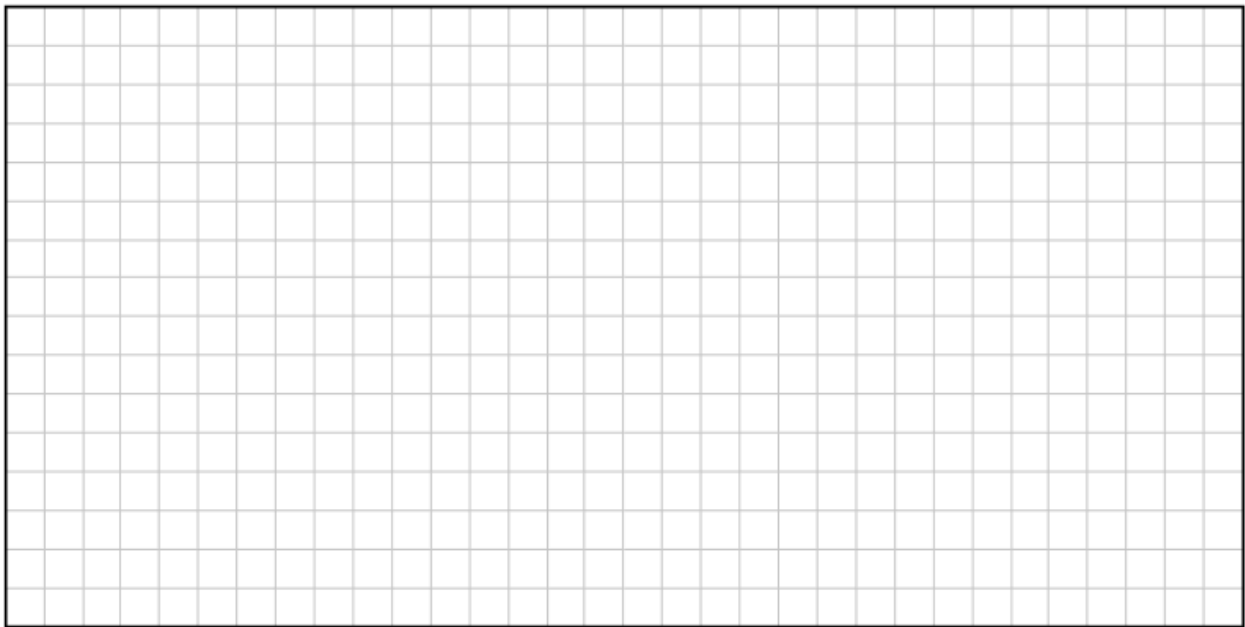
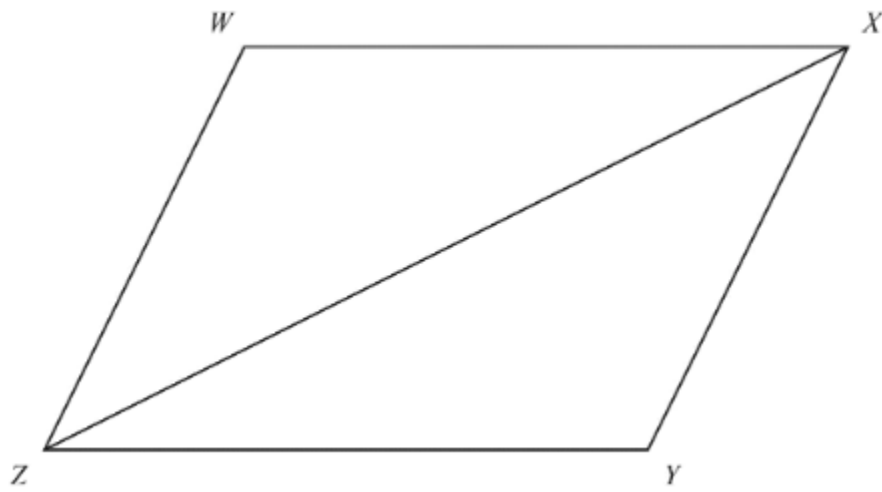
Reason:



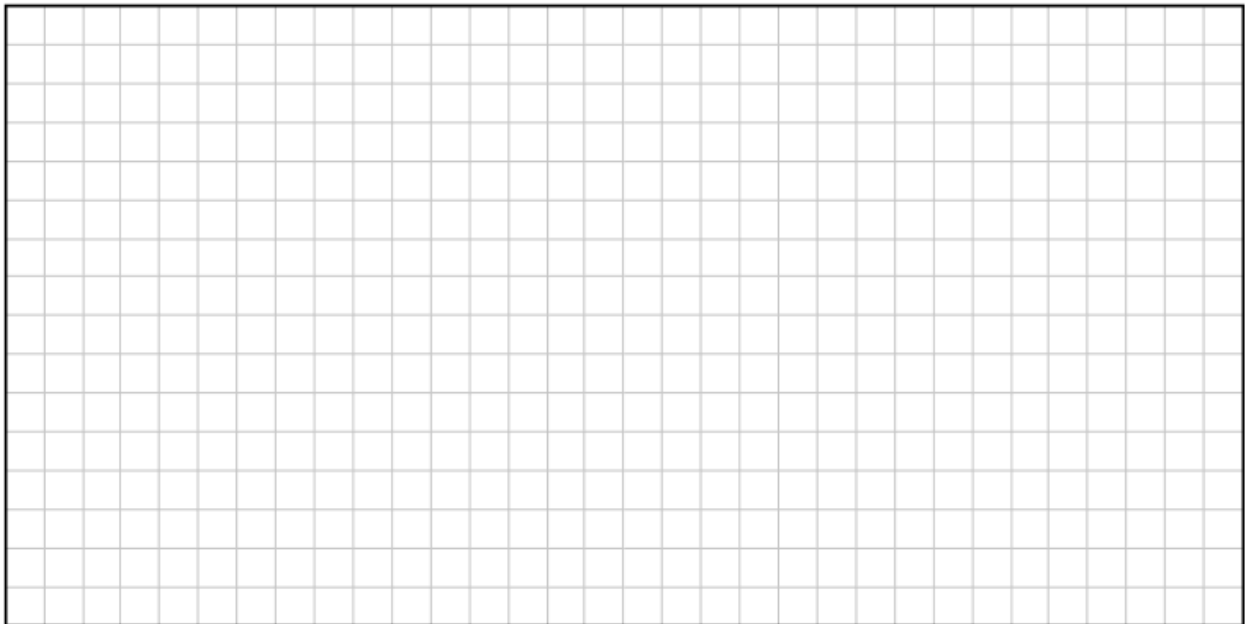
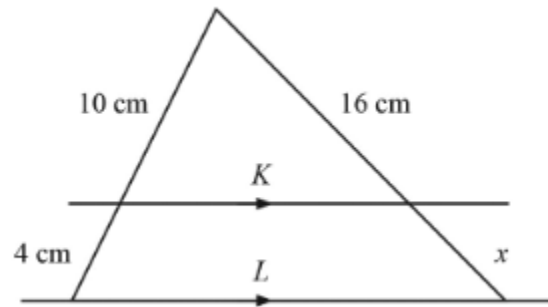
In the diagram below, DE is parallel to FG . Find the value of x .



$WXYZ$ forms a parallelogram as shown. Prove that the diagonal $[XZ]$ divides the parallelogram into two congruent triangles.

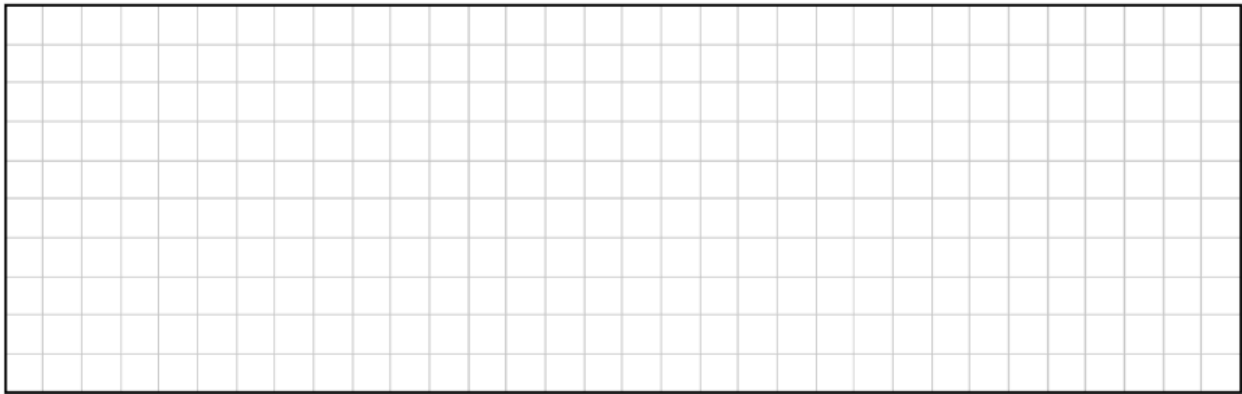
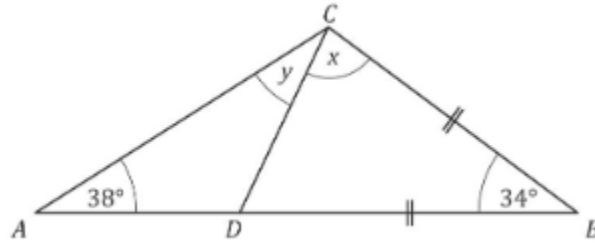


In the diagram $K \parallel L$. Find the value of x .



Geometry theorems

ABC is a triangle. Calculate the value of x and the value of y and tick the correct box that describes triangle DCB

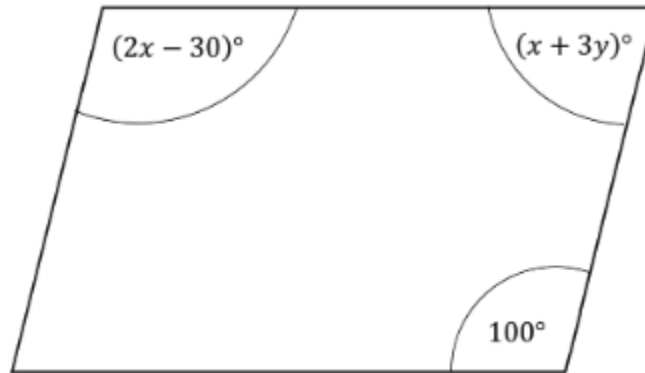


Isosceles

Scalene

Equilateral

The diagram below is a parallelogram. Find the value of x and the value of y



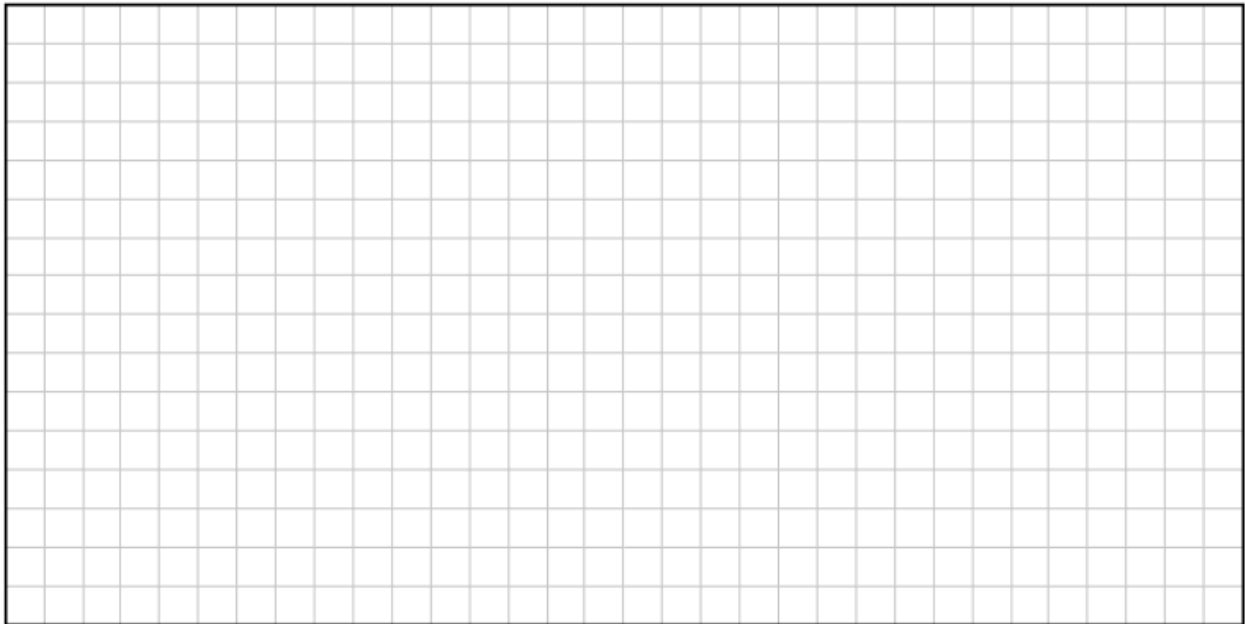
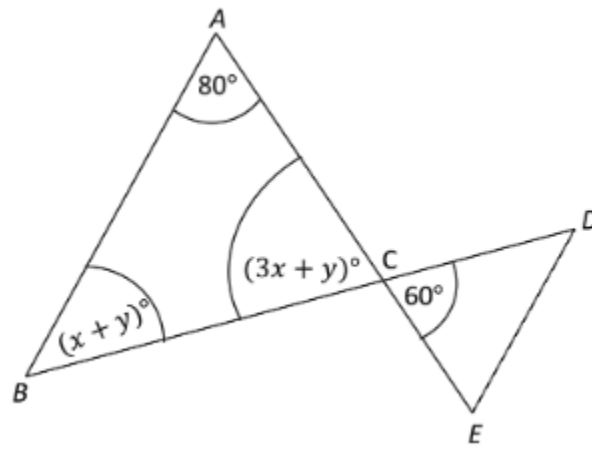
A large grid area for working out the solution to the problem.

Statement: Every square is a parallelogram → true false

Reason:

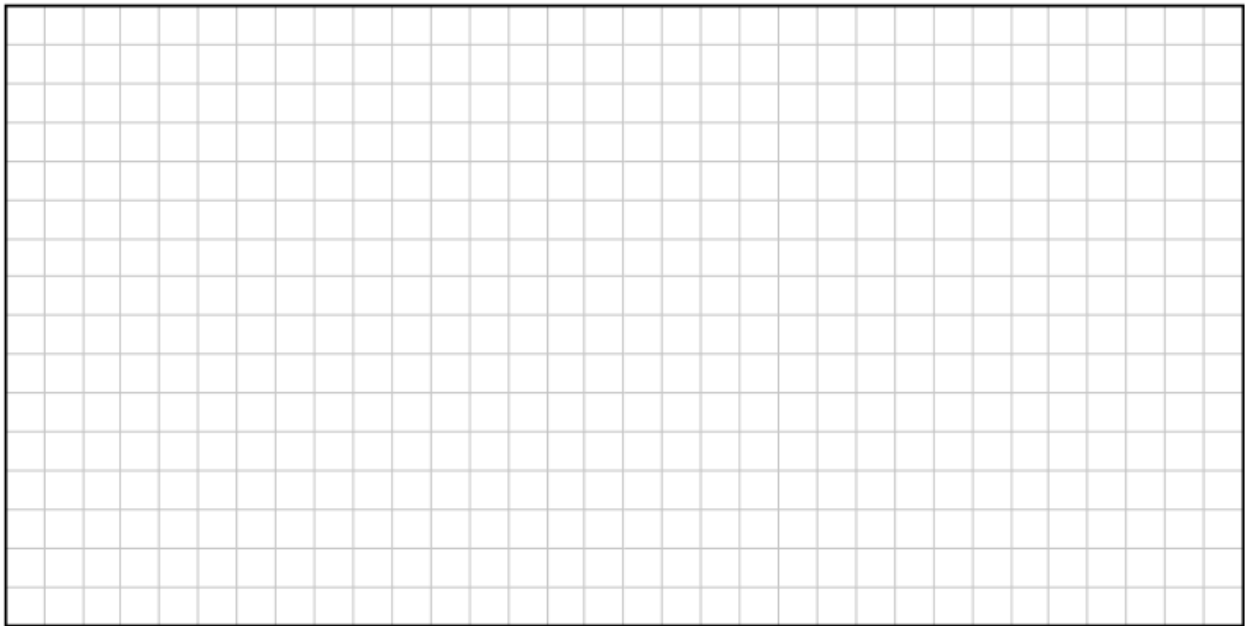
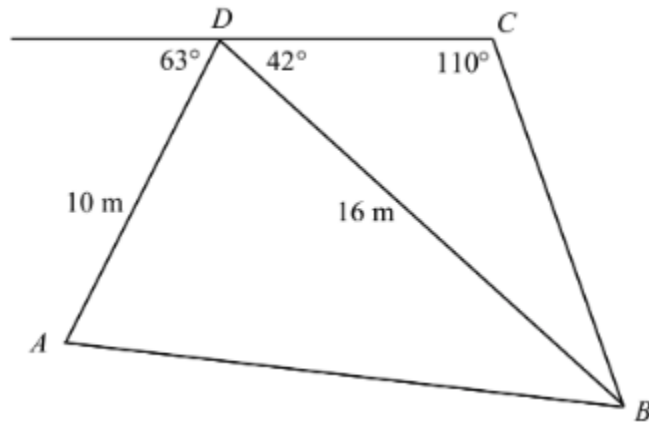
A grid area for providing a reason for the statement.

Find the value of x and the value of y

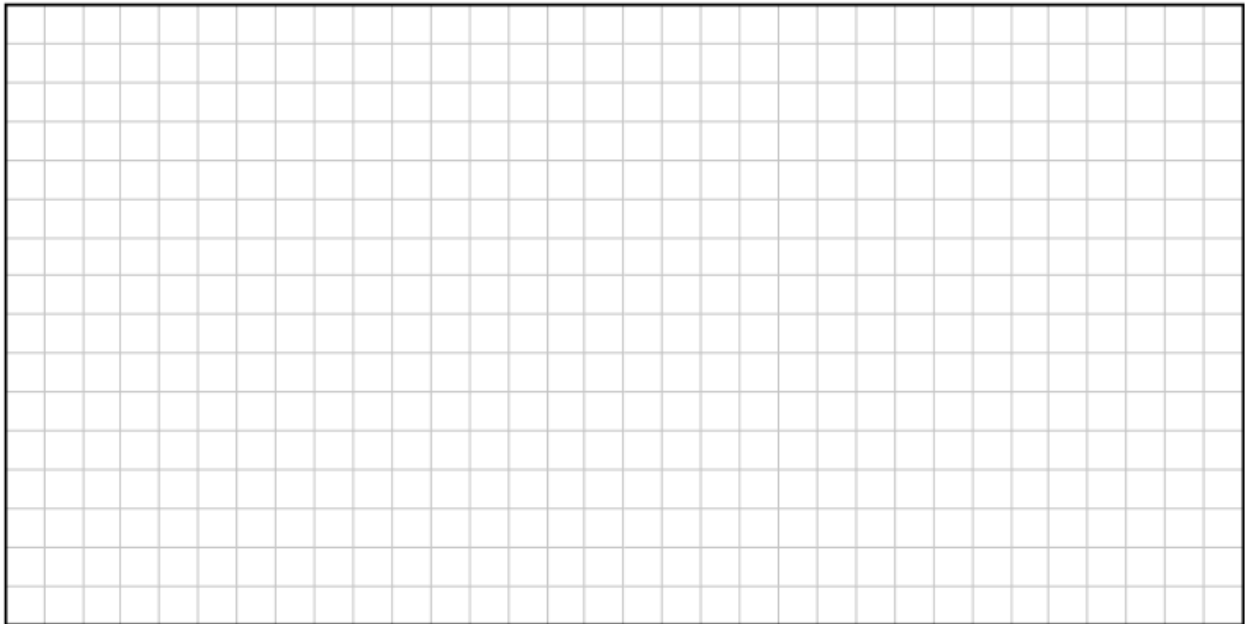
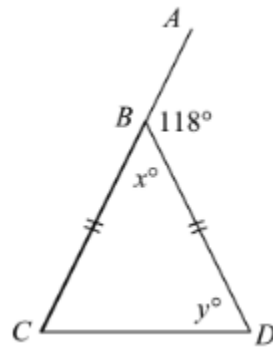


The diagram below shows the triangles BCD and ABD , with some measurements given.

Find (i) $|BC|$, (ii) $|AB|$ and (iii) the area of triangle BCD , all correct to two decimal places.

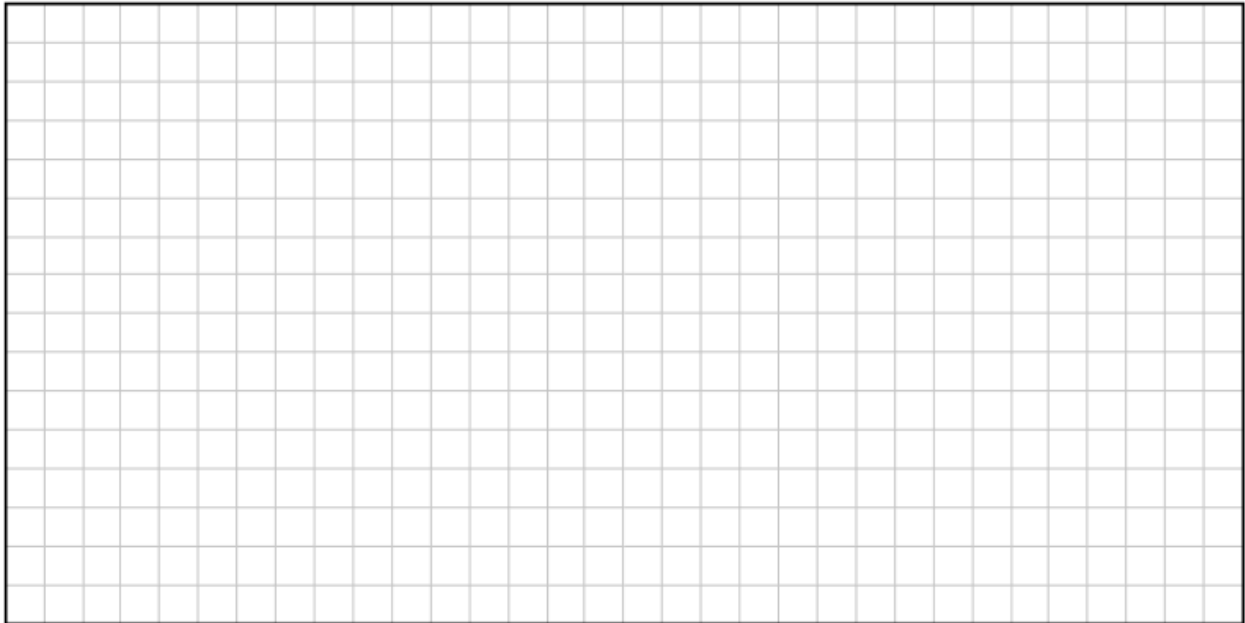
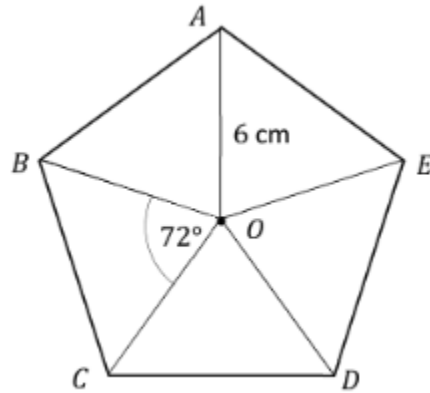


Calculate the value of x and the value of y



A toy company makes spinners in the shape of a regular pentagon, as shown in the diagram. Each of the five sides of the pentagon are the same length. The pentagon is divided into 5 congruent triangles.

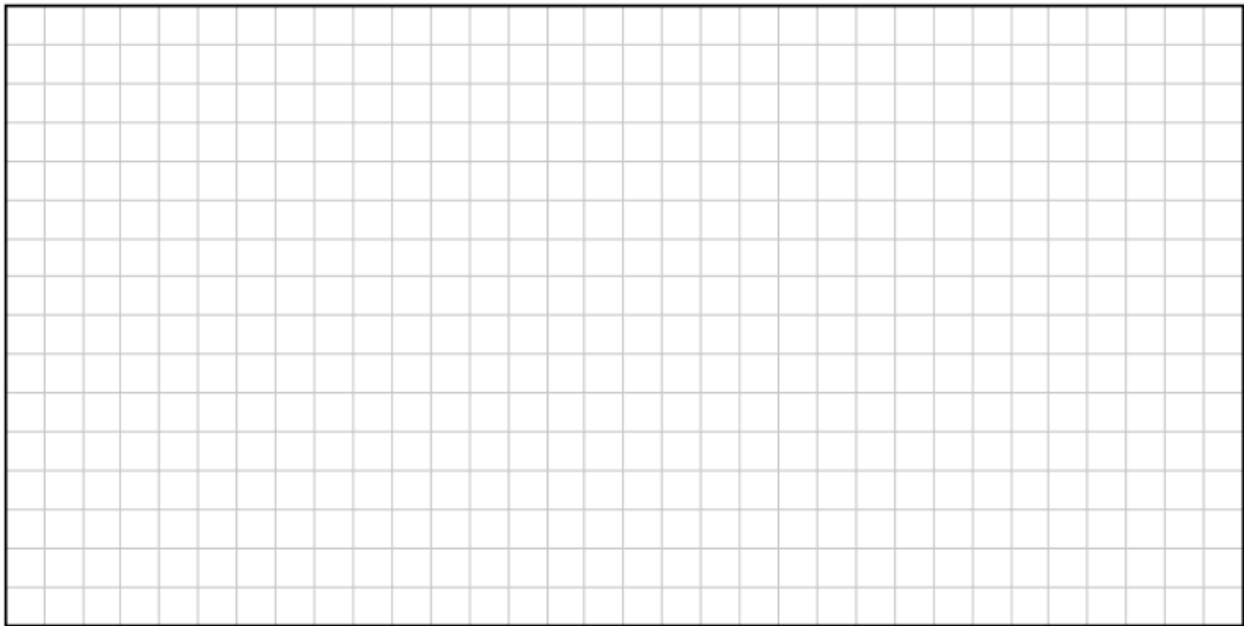
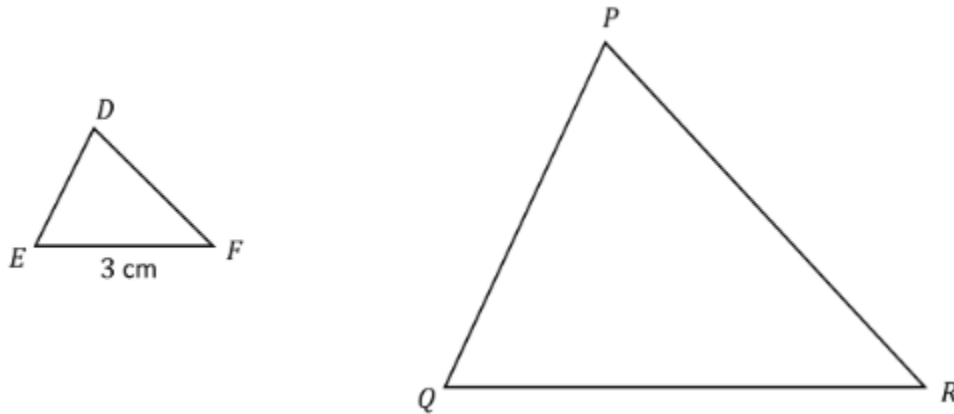
Work out the size of angle CBA .



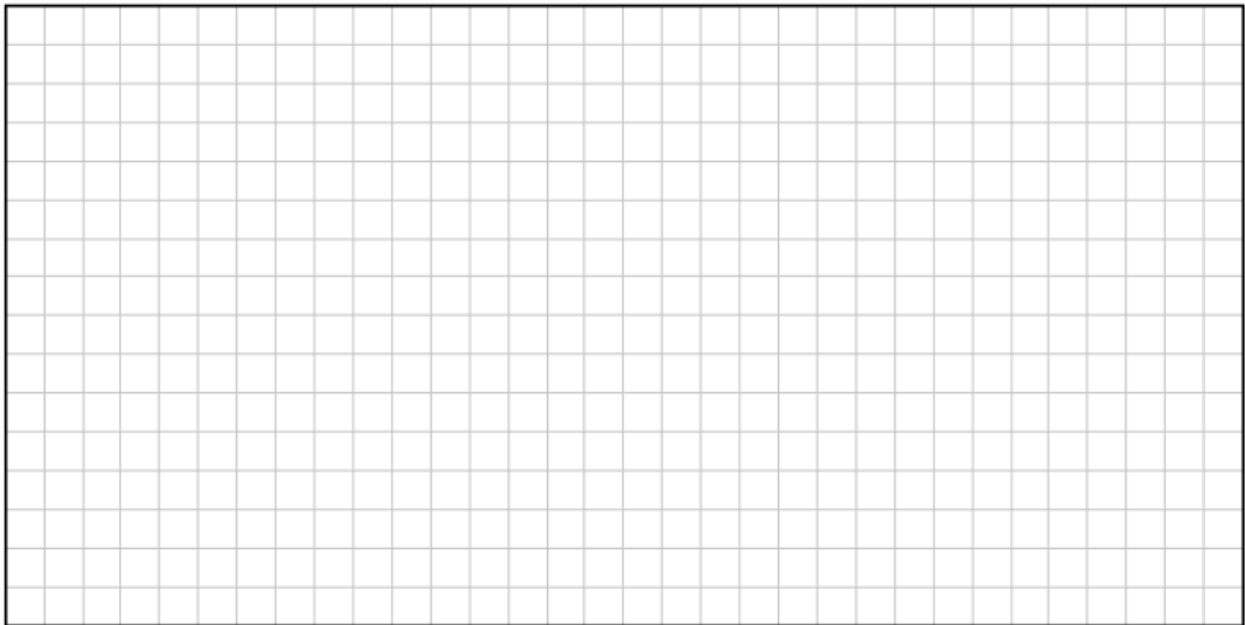
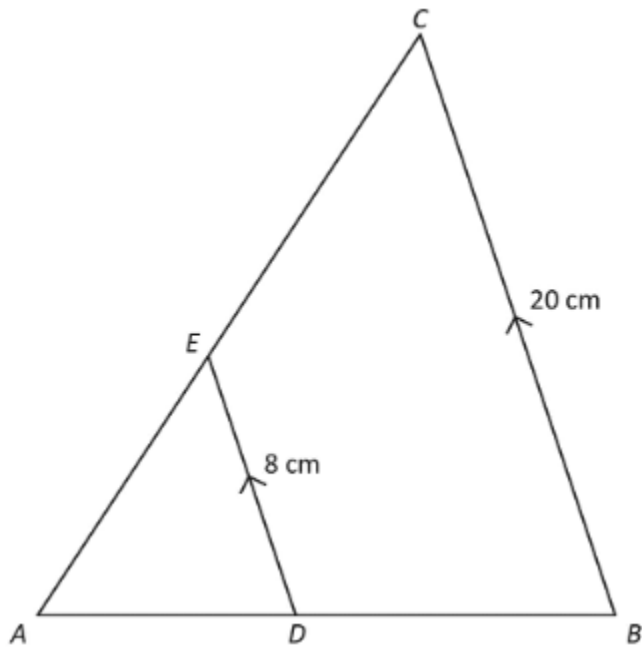
Scale

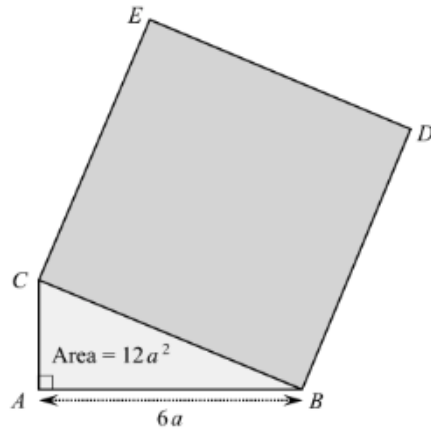
The triangle PQR is the image of the triangle DEF under an enlargement. The scale factor is 2.5.

Find the area of DEF if the area of PQR is 18.75 cm^2 .

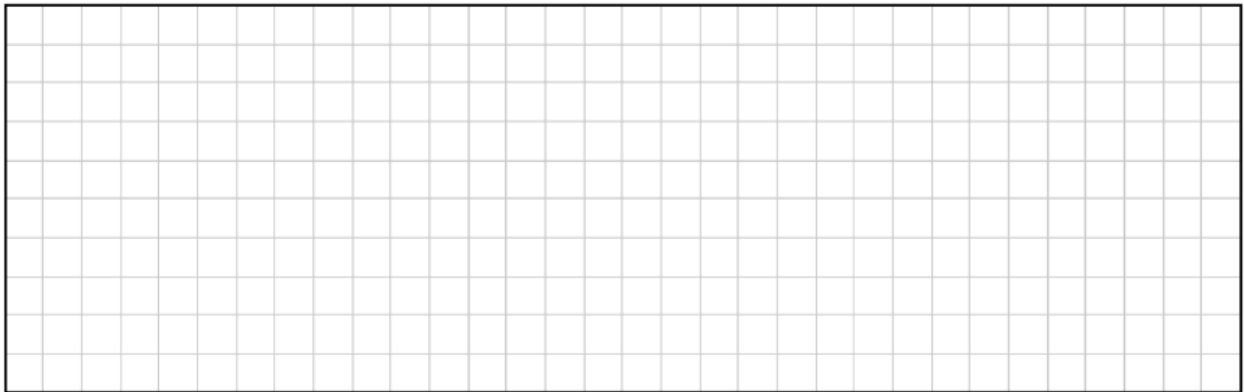


The triangle ABC is the image of triangle ADE under an enlargement with centre A .
Find the value of the scale factor, and hence find $|EC|$ given that $|AC|$ is 18.75 cm.



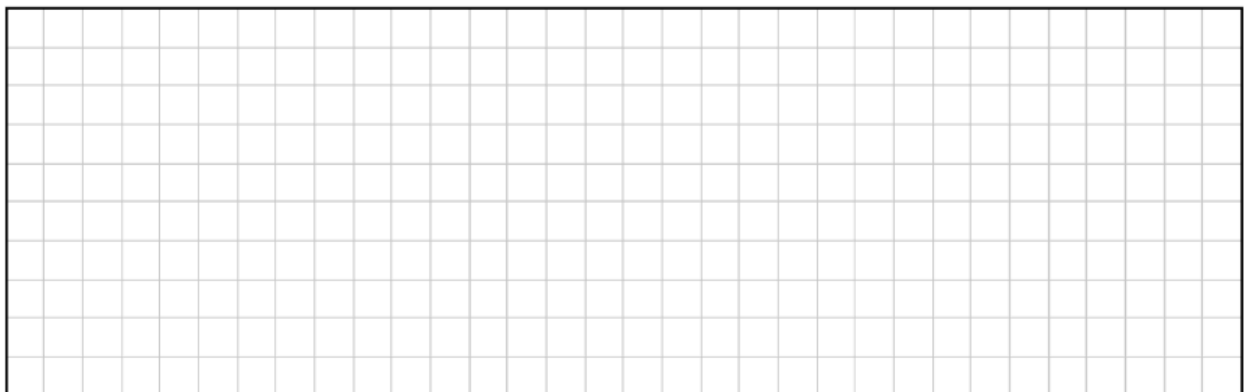
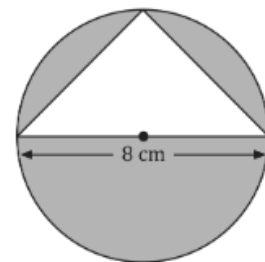


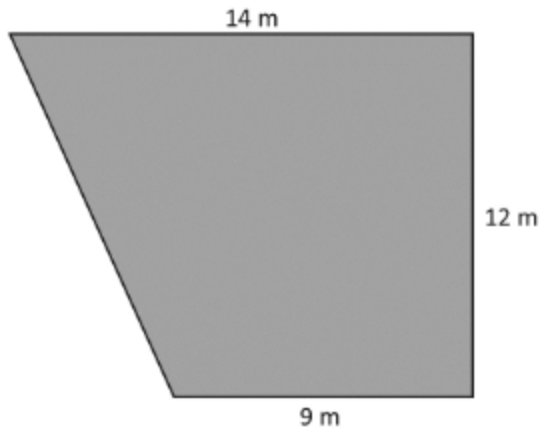
Find, in terms of a , the area of square $CEBD$.



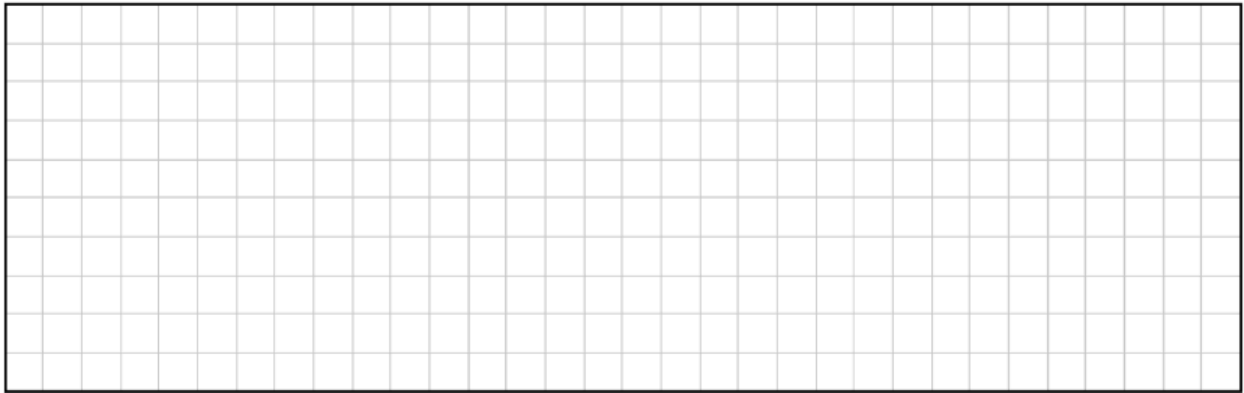
An isosceles triangle is inscribed inside a circle.

Find the area of the shaded region, giving your answer correct to two decimal places.

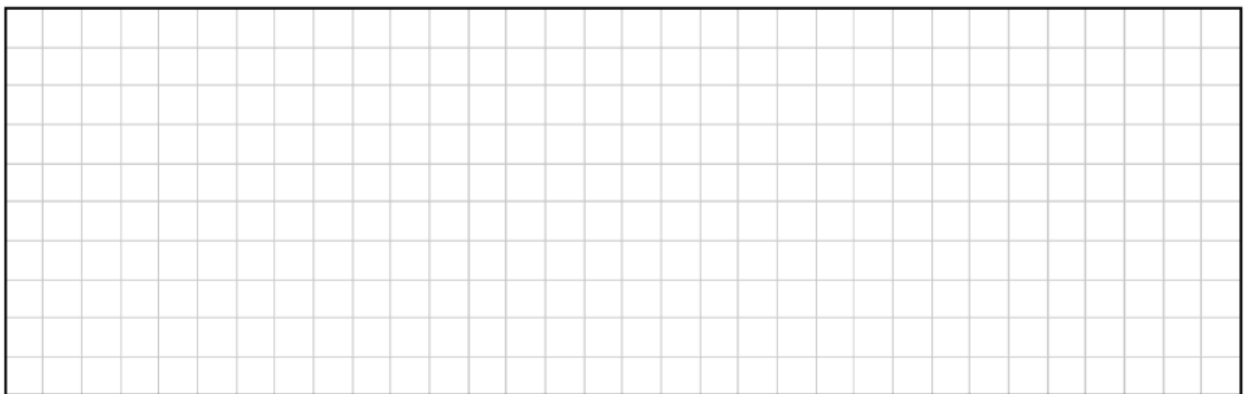


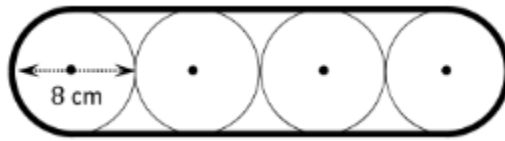


Calculate the perimeter and area of the above shape.

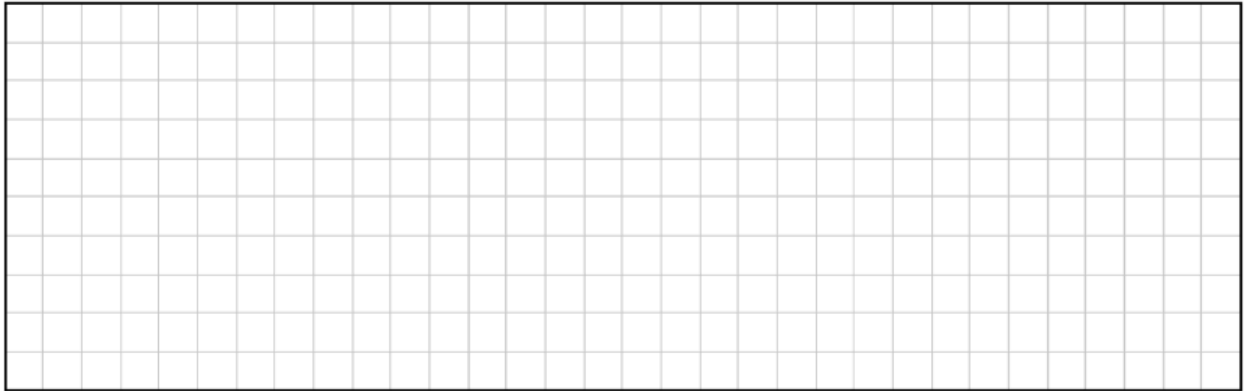


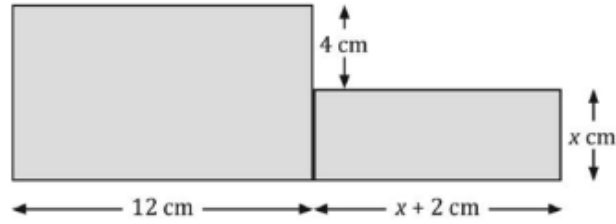
The area of a semicircle is $6\pi \text{ cm}^2$. Find the radius of this semi circle.



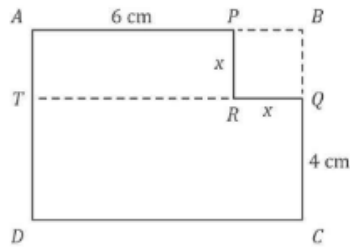
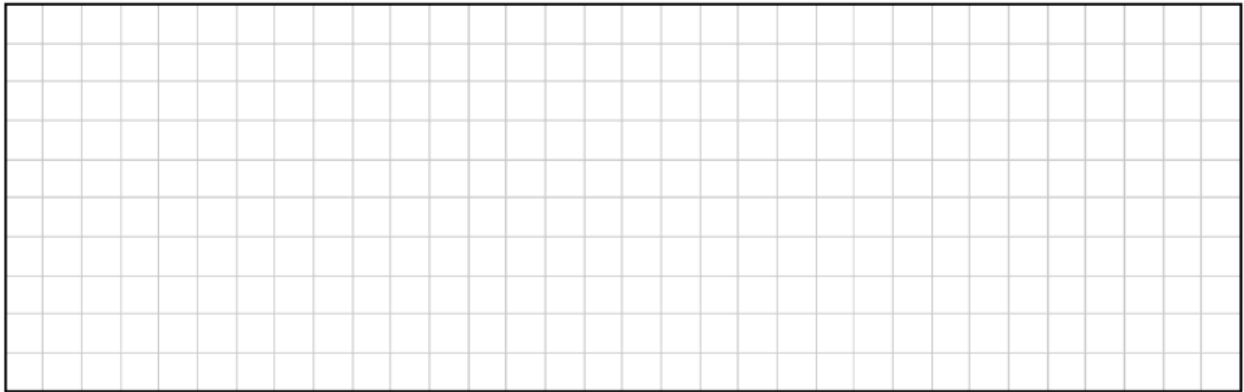


The rubber track for a toy digger goes around four circular wheels as shown. Calculate the length of the rubber track. Give your answer correct to one decimal place.

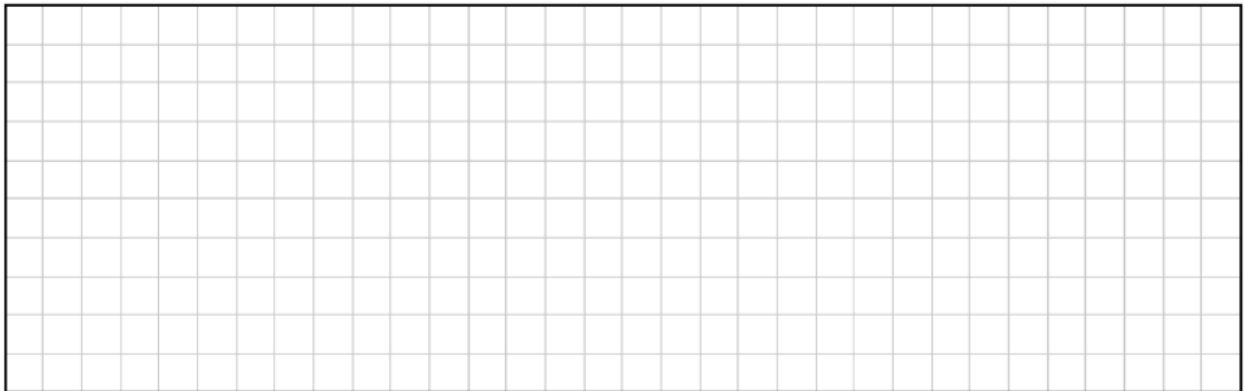


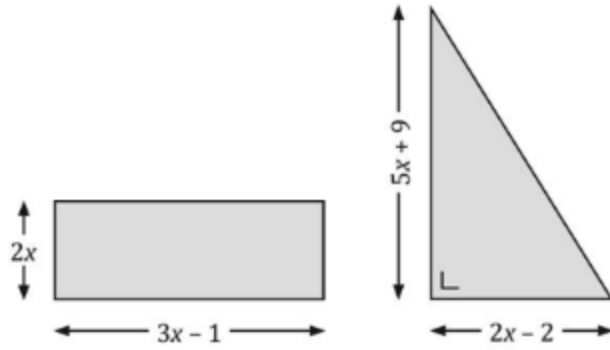


The diagram shows a composite shape formed by joining two rectangles. The area of the larger rectangle is 4 times the area of the smaller rectangle. Calculate the dimensions of the smaller rectangle.



The area of rectangle $TQCD$ is twice the area of rectangle $APRT$. Solve for x .





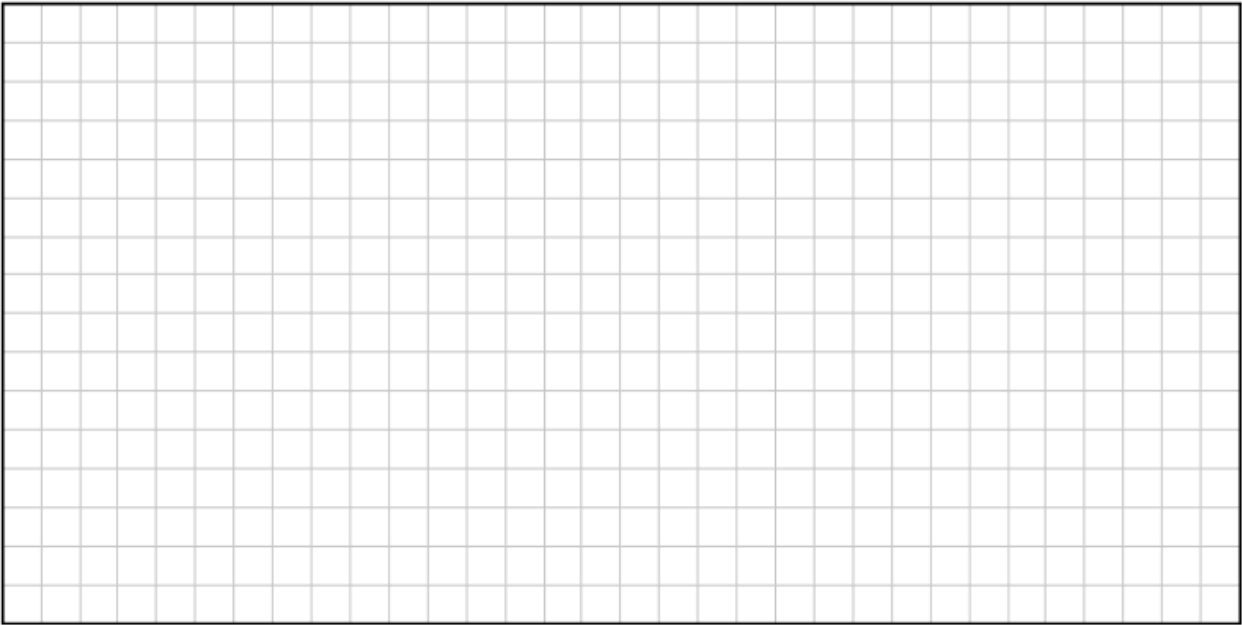
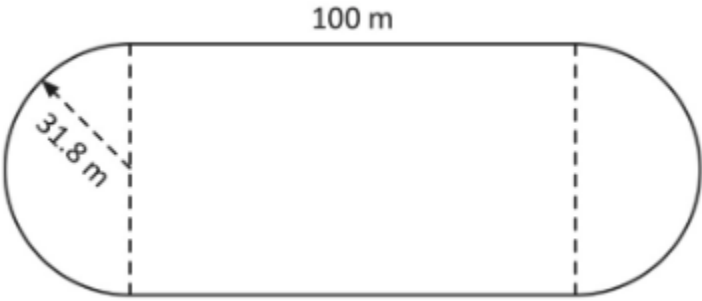
Find the value of x such that the area of the rectangle is the exact same as the area of the triangle.



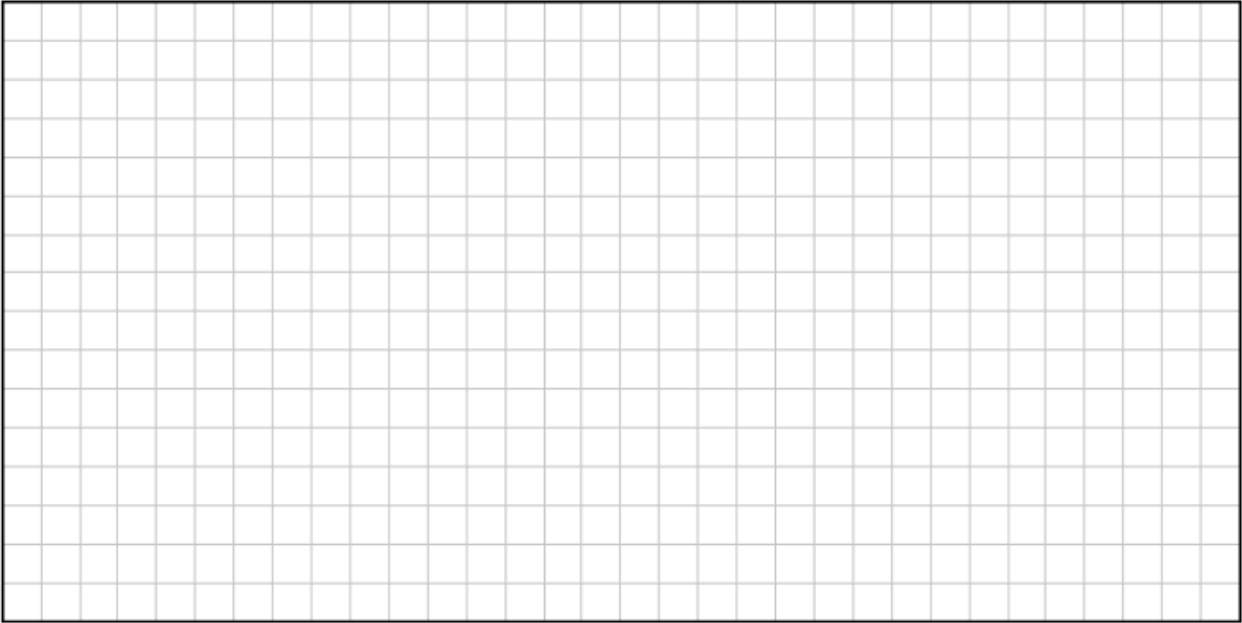
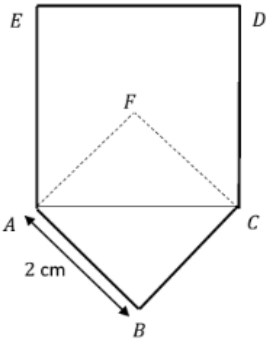
The width of a vegetable patch is x . Its length is 5 metres longer than its width. If the area of the patch is 36 m, find the dimensions of the garden.



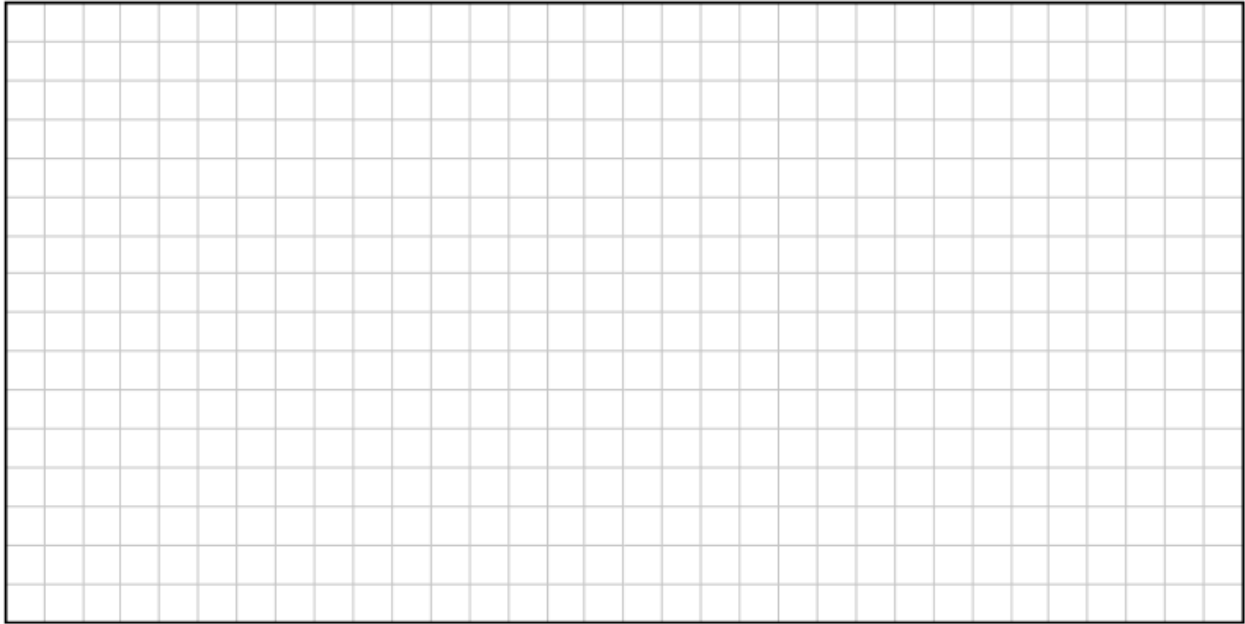
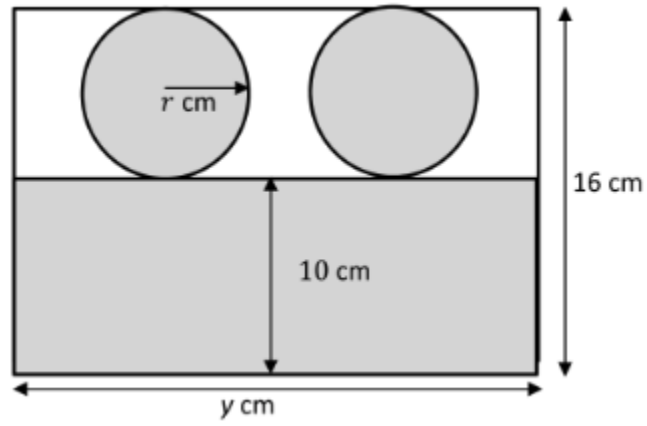
The below shape represents a running track. How many laps of this track will a runner have to complete in order to finish a 10 000 metre race?



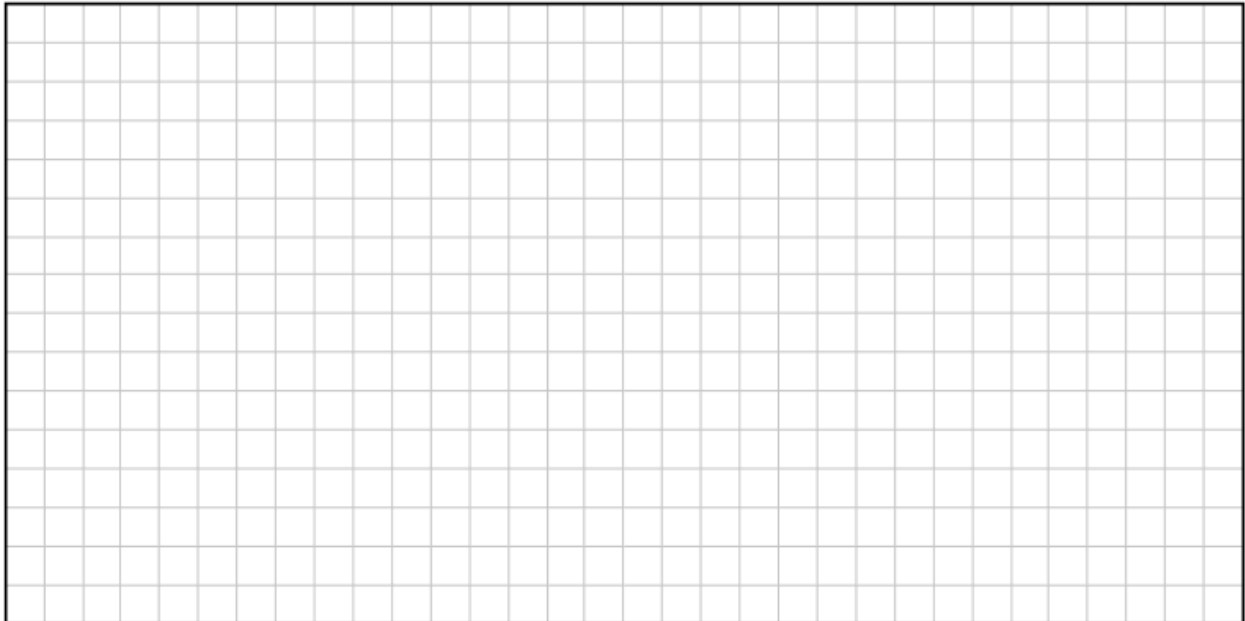
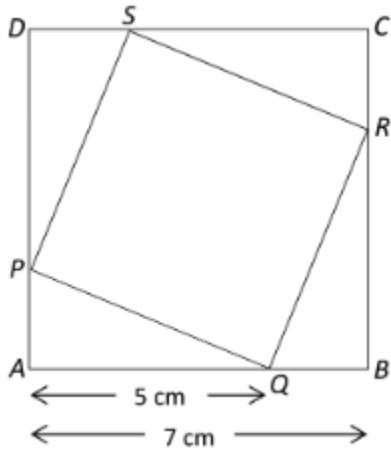
The figure below shows two squares $EDAC$ and $ABCF$. Find the perimeter and the area of the full figure.



The shape below represents the open net of a cylinder. Find the area of this open net.



The square $ABCD$ has sides of length 7 cm. $PQRS$ has vertices that lie on the perimeter of $ABCD$ as shown. Find the area of $PQRS$



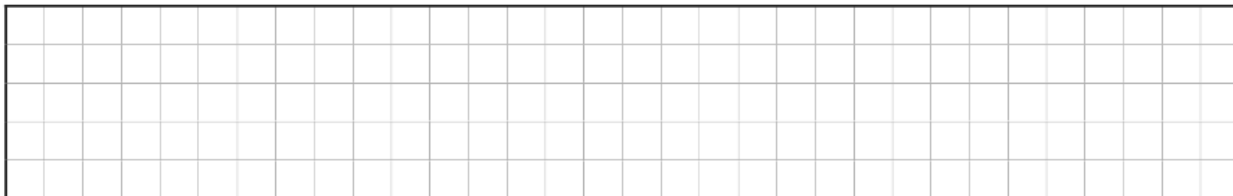
9 inch pizza = 2 slices of BIG PIZZA.

A 9 inch pizza is in the shape of a circle, with a diameter of 9 inches.

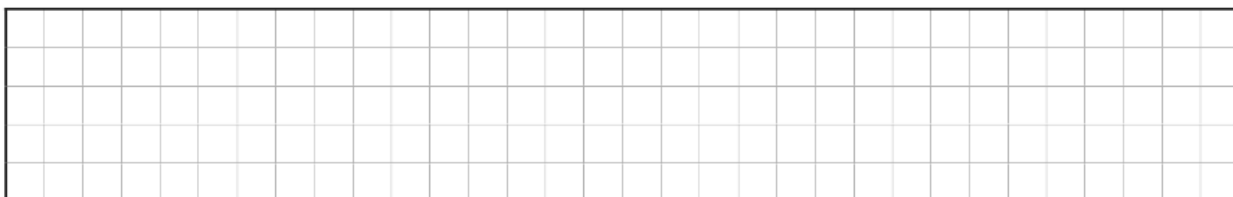
Each BIG PIZZA is in the shape of a bigger circle, and is divided into 6 slices of equal area. Work out the radius of a BIG PIZZA.



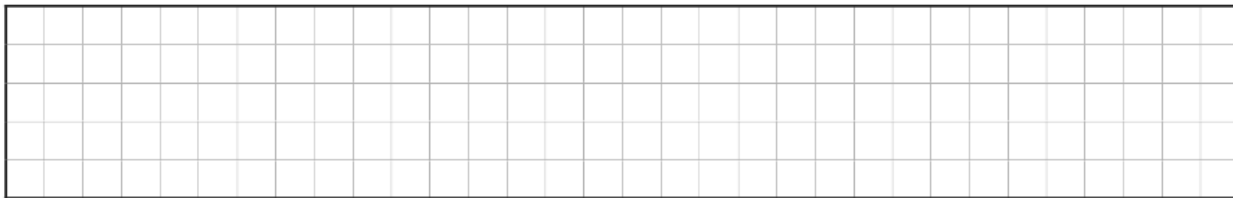
Find the volume of a cylinder with a diameter of 18 mm and a height of 5 mm. Give your answer correct to the nearest mm^3 .



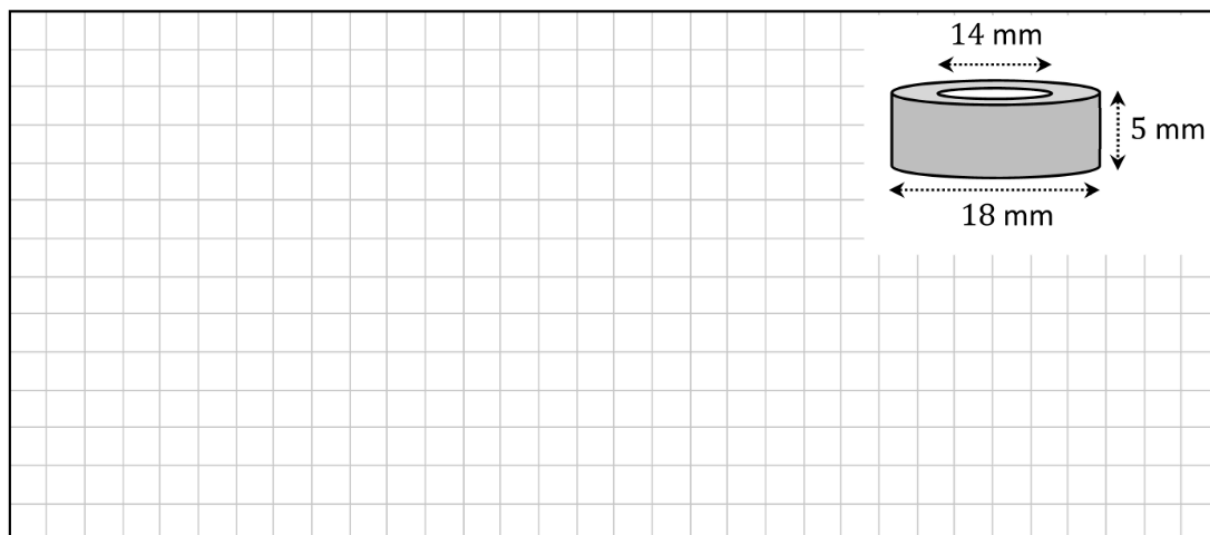
Find the volume of a cone with radius of 3 cm and a height of 5 cm. Give your answer correct to the nearest mm^3 .



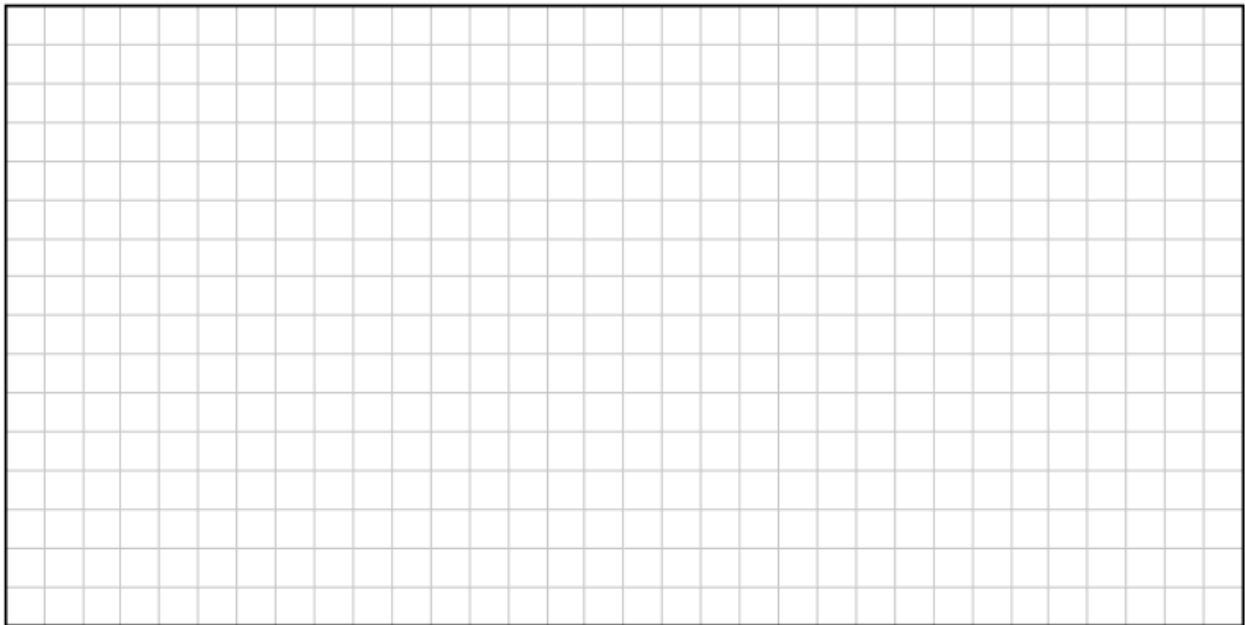
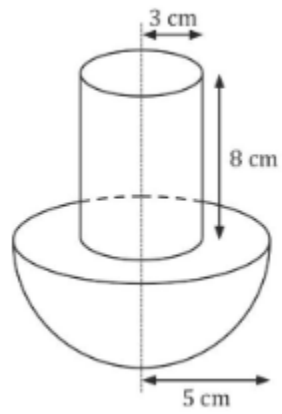
Find the radius of a cylinder with height 10 cm and volume 250 cm^3 . Give your answer correct to the nearest cm^3 .



A metal ring can be made by removing a cylinder from another cylinder as seen below. Work out the volume of this ring.

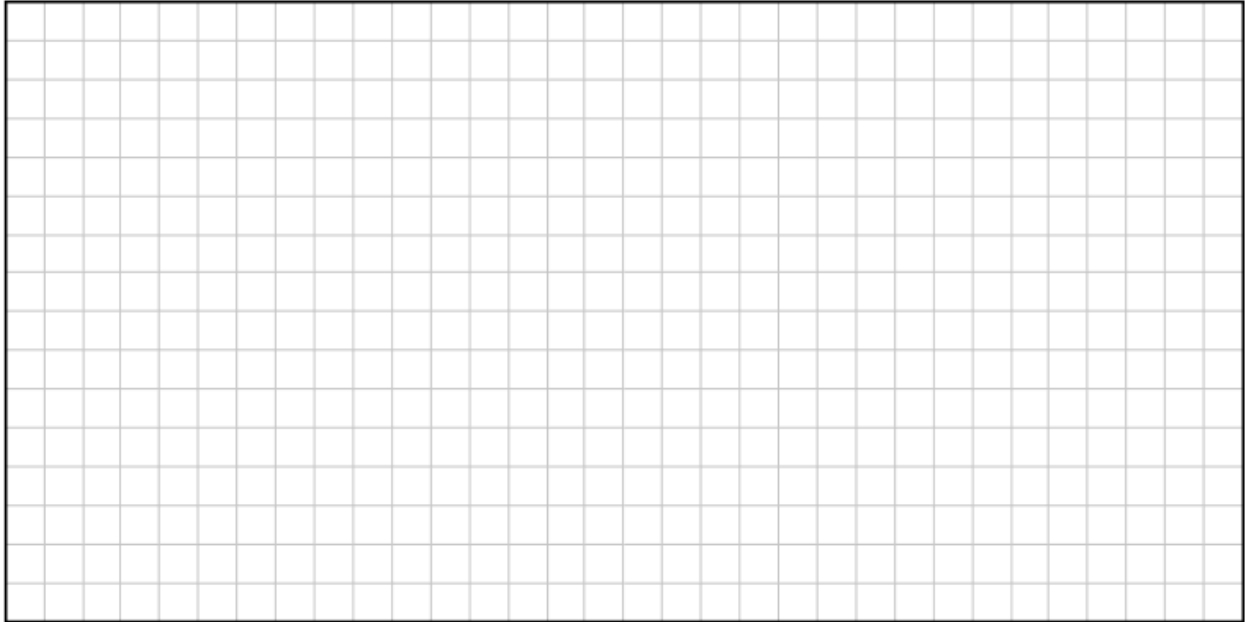


The diagram shows a toy which is made from a cylinder and a hemisphere. They share the same centre line. Find the total volume of this toy.

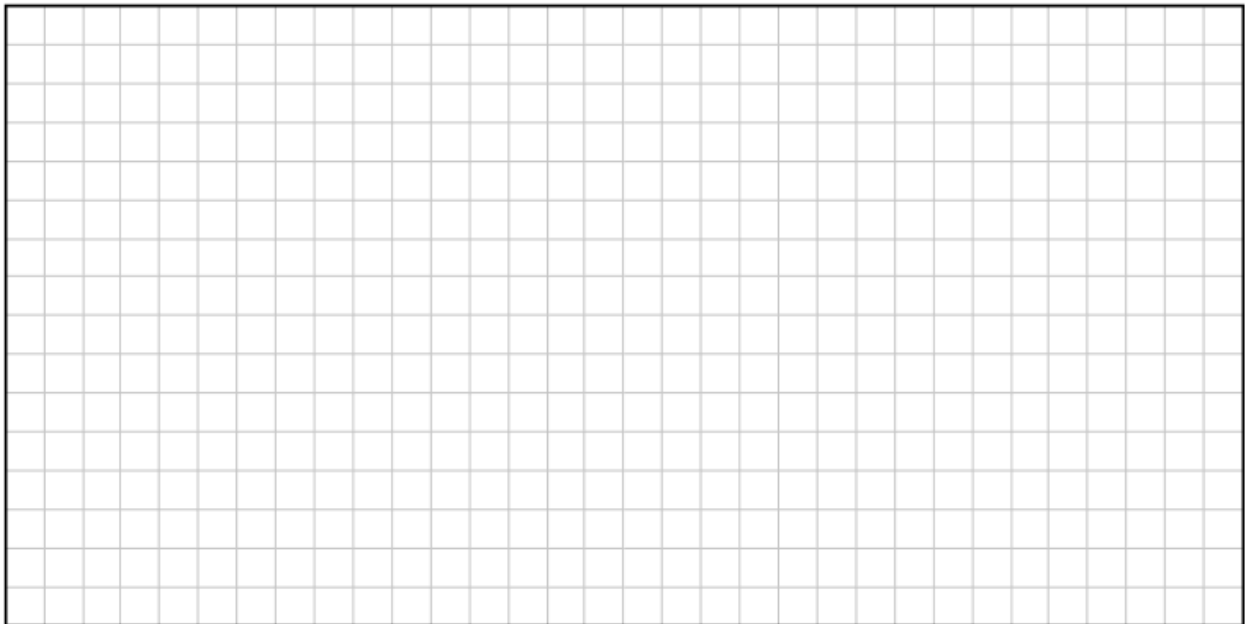
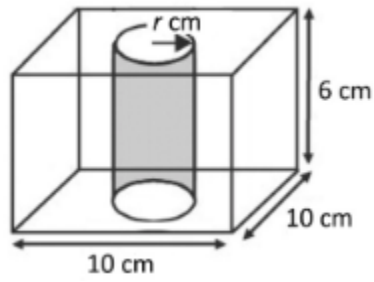


A family buys a paddling pool for their back garden, in the shape of a cylinder. It has an internal diameter of 2.4 m and an internal depth of 0.9 m.

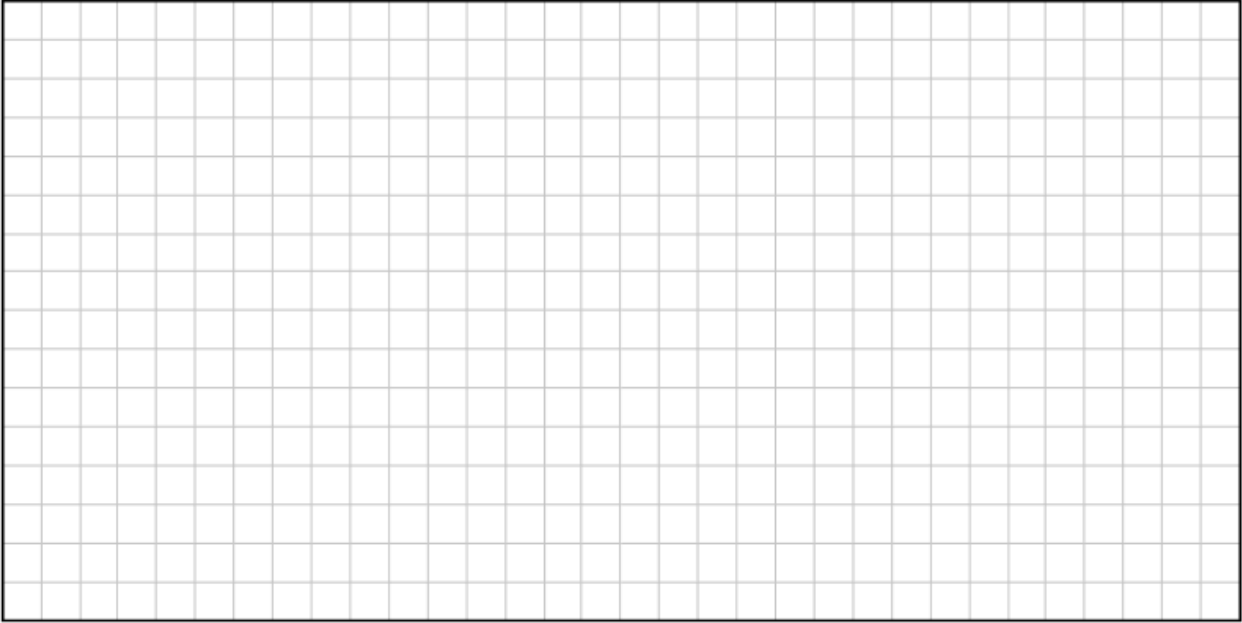
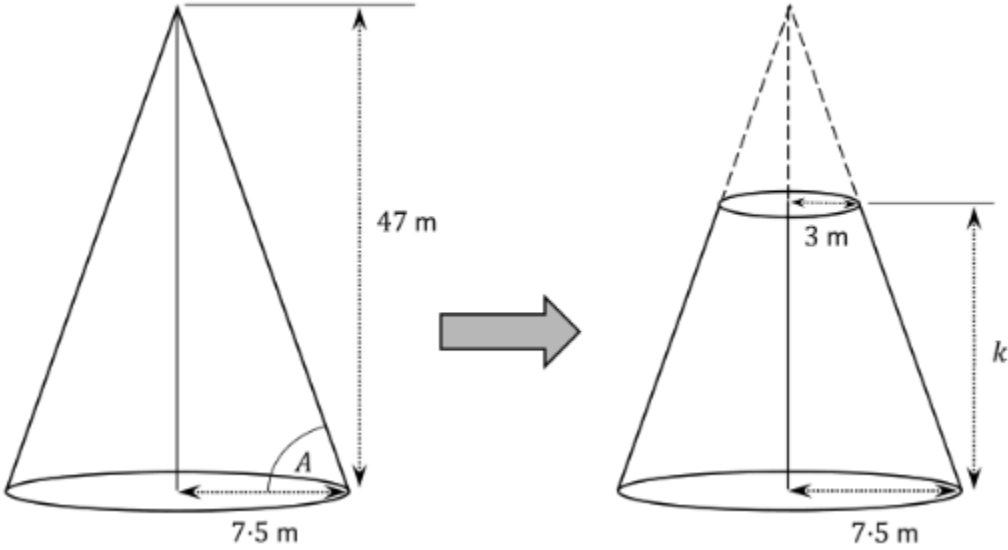
The instructional manual says that the pool should only be filled to 0.75 m. 3 pool seats in the shape of cubes are dropped into the pool. They each have side lengths of 0.5 m. Has the water in the pool overflowed?



A cylindrical hole of height 6 cm and radius r is drilled all the way through the below block. After the hole is drilled, 61.5% of the original block remains. Find the value of r .



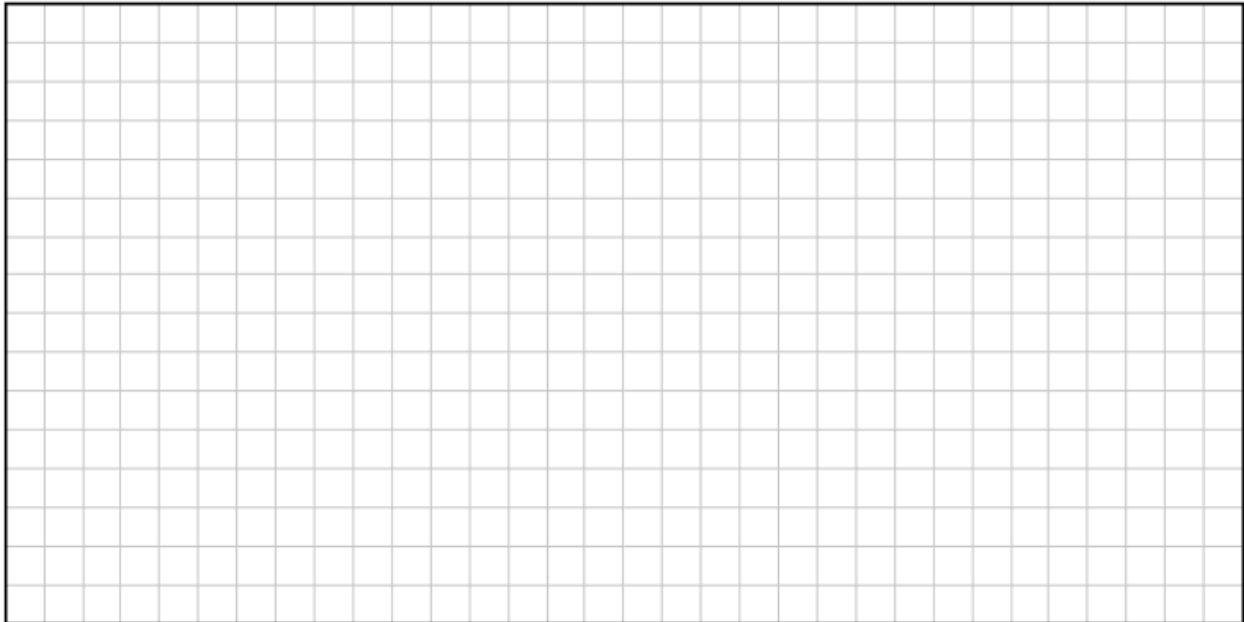
Part of a right-cone is removed as seen below. Find the angle marked A and the distance marked k .

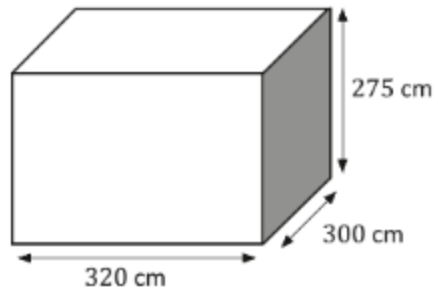


A class group is investigating the effect a change in the radius has on the surface area and volume of a sphere. They decided to see how the surface area and the volume will change if the radius is doubled. Some students think the surface area will double in size, and the volume will triple in size. Investigate if they are correct. Show your workings and state your conclusion.

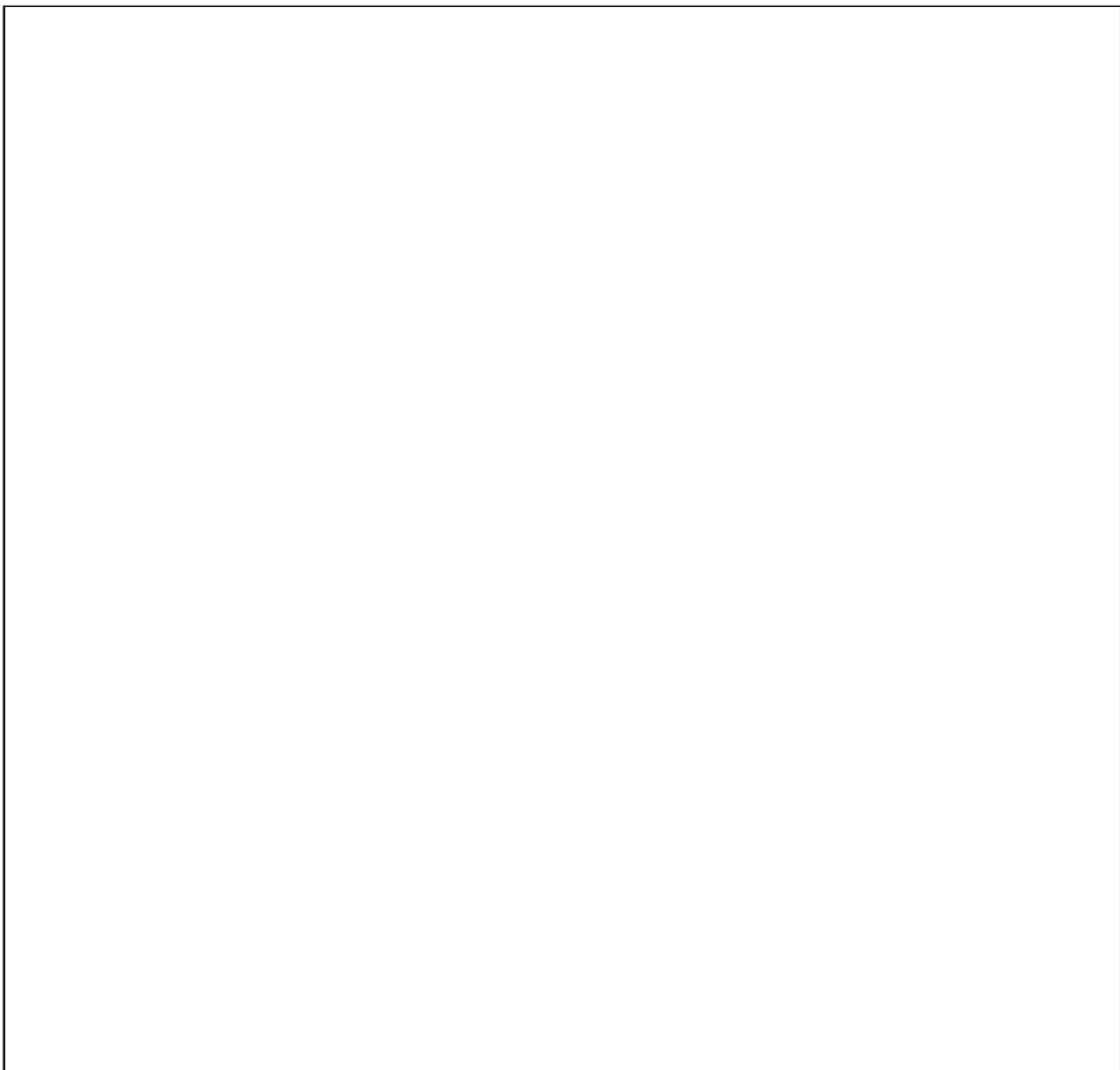


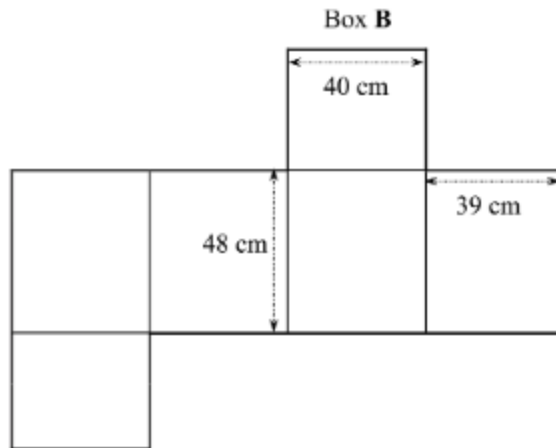
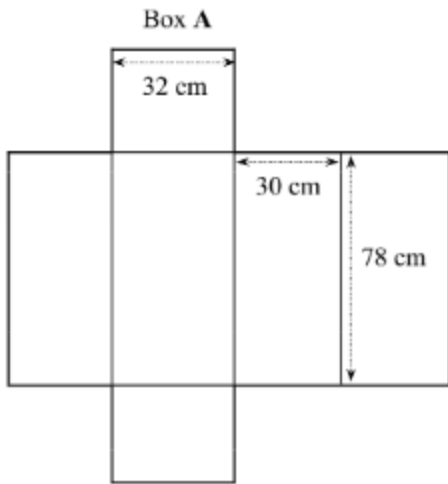
A cylindrical jug has a height of 24 cm, and a diameter of 20 cm. The jug is filled to 65% of its capacity. 50 ice cubes, each of side 2 cm, are added to the jug. Calculate the rise in the level of water in the jug. Give your answer correct to one decimal place.



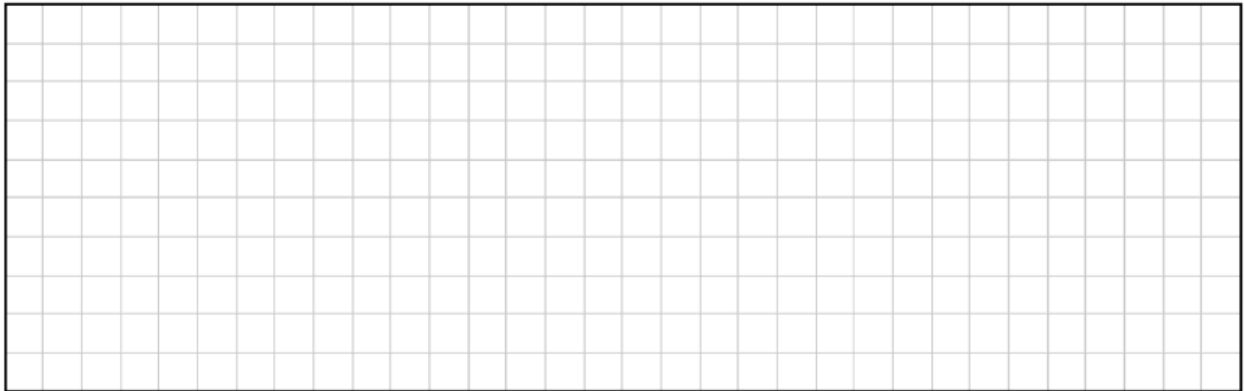


Draw a net of the above cuboid, clearly showing all measurements.



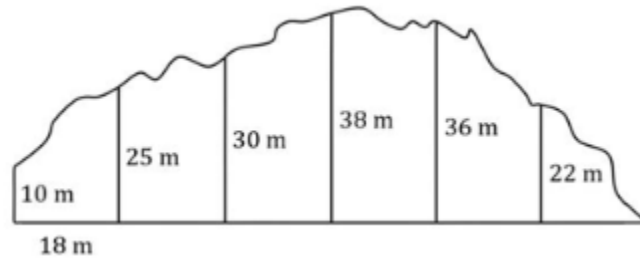


Show that Box A and box B have the same volume.



Trapezoidal rule

An oil spill next to a road is measured as follows.



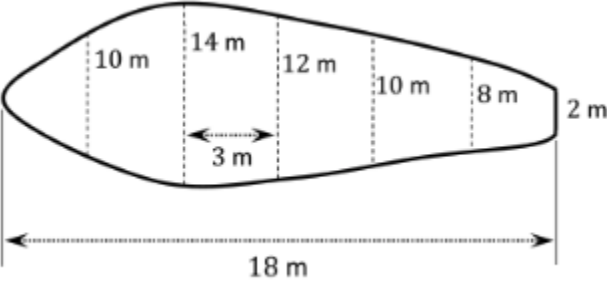
(i) Estimate the area of the spill.

A large grid for estimating the area of the spill.

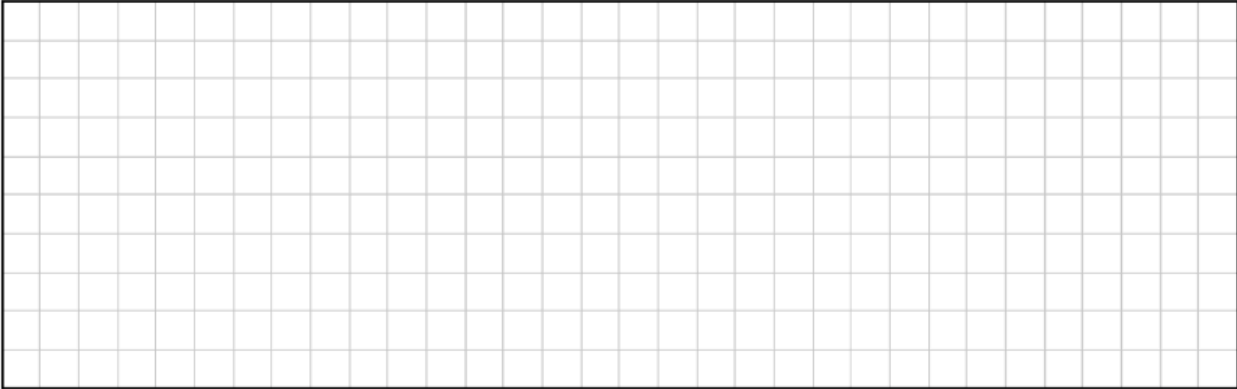
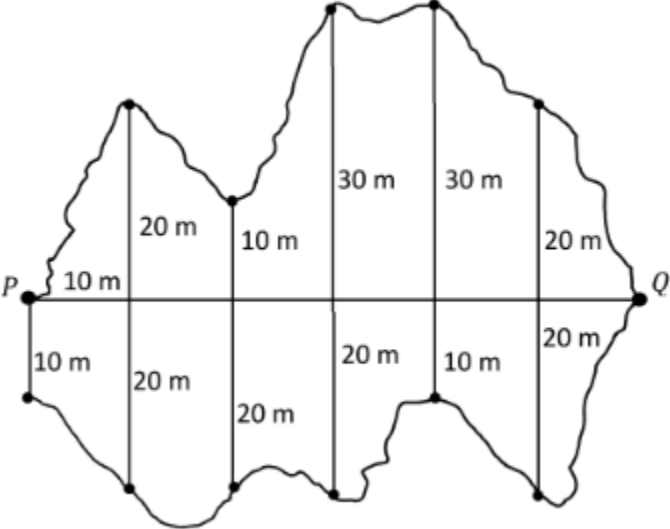
(ii) The actual area of the spill is 2980 m^2 . Calculate the percentage error of your estimate from above. Give your answer to two significant figures.

A large grid for calculating the percentage error.

The width of a pond is measured at intervals of three metres starting at the right hand edge of the pond. These measurements are given on the diagram below. Estimate the area of the pond correct to the nearest metre.



The diagram below shows the plan of a lake. Estimate the surface area of the Lake.



Chapter 7

FINANCIAL MATHS

- Payslips and USC
- Exchange rates
- Different percentages
- % increase and decrease
- Compound interest

● Payslips and USC

Gross → before tax

Net → after tax

Tax payable → total tax to be paid

Tax credits → Money we "get back" on tax

USC → Universal Social Charge

● Exchange Rates

Converting between currency, units etc

Example:

$$€1 = €1.15$$

$$€10 = €11.50$$

● Compound Interest

$$F = P(1+i)^t$$

Final value ← F
Principle ← P
Interest rate ← i
time ← t
-What we put in

[2% is 0.02]

● % Increase / decrease

A mas bar costs €1.10 before VAT@23% is added. How much does it cost after VAT is added?

$$€1.10 \times 1.23 = €1.35$$

A galaxy bar costs €1.50 after VAT @ 23% has been added. How much did it cost before VAT?

$$\frac{€1.50}{1.23} = €1.22$$

● Different percentages

$$\% \text{ profit} = \frac{\text{profit}}{\text{selling price}} \times 100$$

$$\% \text{ discount} = \frac{\text{discount}}{\text{original cost}} \times 100$$

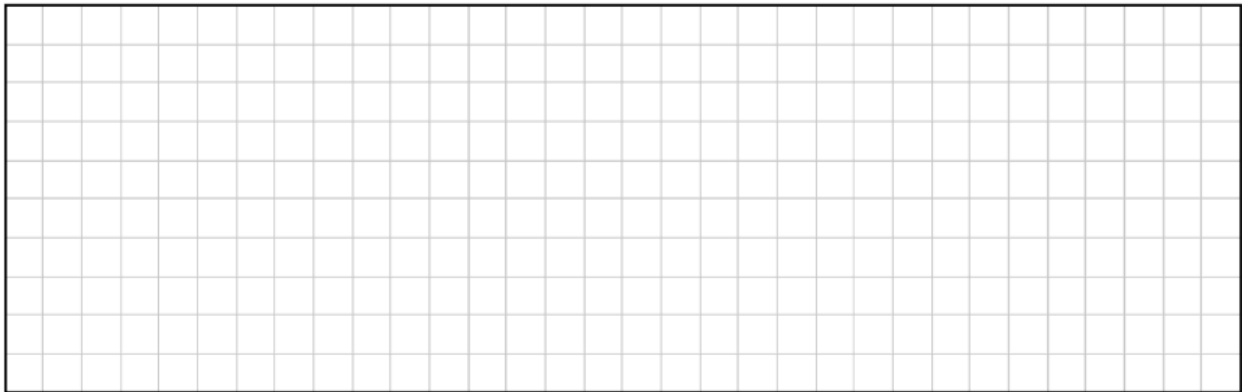
$$\% \text{ increase} = \frac{\text{increase}}{\text{original cost}} \times 100$$

Rachel earns a gross salary of €43,500. Rachel has a tax credit of €3200. The standard rate cutoff point is €37,000. The standard rate of tax is 20%, while the higher rate is 42%. Calculate Rachel's net salary for that year.



The standard rate of income tax is 20% and the higher rate is 41%. The standard rate cut off point is €36,500. Ashling has a gross income of €47,500 and total tax credits of €1,830.

(i) Calculate Ashling's net income.



(ii) The following year, Aishling's gross income increased. Her new net tax amounts to €15015. Her tax credits and tax rates remain the same. Calculate her new gross income.



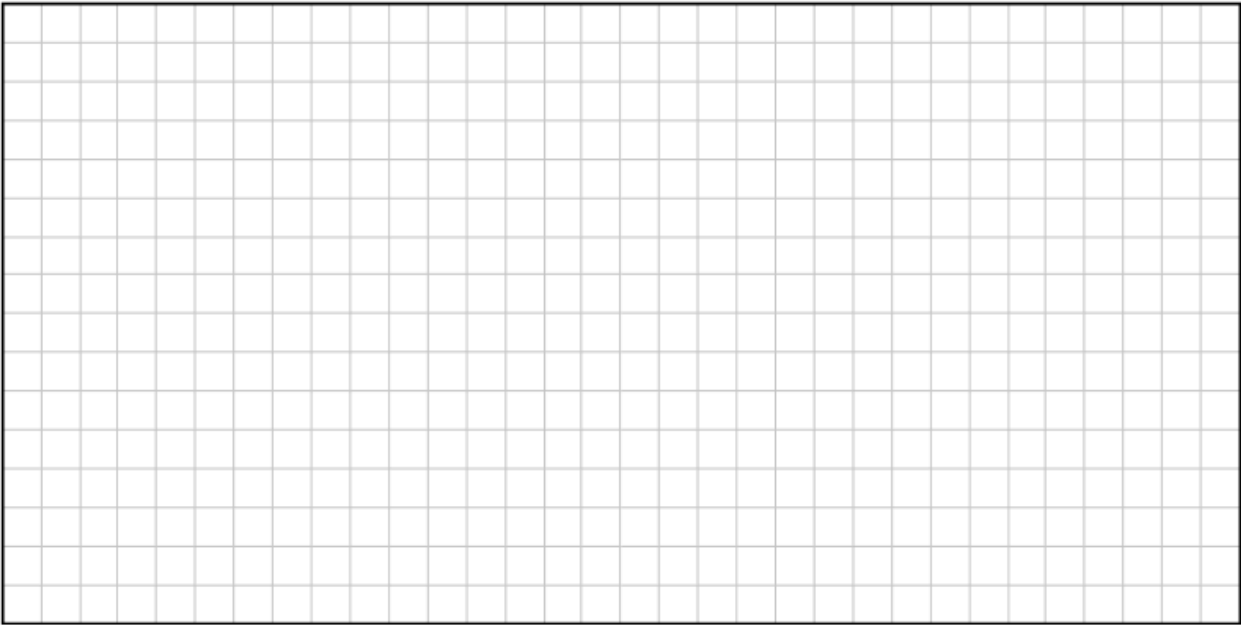
Ryan has a gross annual income of €72,000. He pays income tax on his gross income at a rate of 20% on the first €42,600, and 40% on the balance.

He must also pay USC on all of his gross income. The income band rates for USC are as follows:

Income band	Rate
Up to €12 012	1.5%
Next €5 564	3.5%
Above €17 576	7%

Ryan must also pay PRSI at 4% of the gross income.

Given that his tax credits are €3,800, calculate his net income for the year.

A large grid for calculations, consisting of 20 columns and 25 rows of small squares.

Joan pays €8825 in income tax. Her standard rate cut off point of tax is €44,000 and she has a tax credit of €1775. If the standard rate of tax is 20% and the higher rate is 40%, calculate her gross income.

Michelle must pay USC on all of her gross income. The income bands and rates of USC that Michelle must pay are given in the table below.

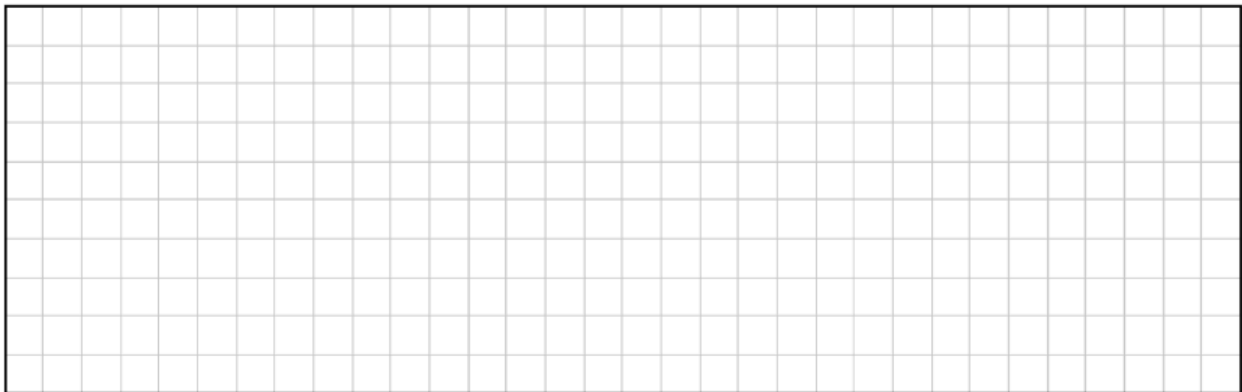
Michelle's gross income is x . Michelle pays a total of 5% of her gross income in USC. Find the value of x .

Income Band	Rate of USC
Up to €12 012	1.5%
Next €5564	3.5%
Above €17 576	7%

The Universal Social Charge was introduced in 2011. Here are the current rates:

Income	Rate
First €12,012	0.5%
Next €6,760	2.5%
Next €51,272	5%
Balance	8%

Karen earns €80,000. Calculate Karen's USC bill.

A large grid for calculations, consisting of 20 columns and 20 rows, intended for the student to work out the USC bill for Karen.

Peter has a new income of €55 000 per annum. He pays USC at the following rates. Calculate his USC payment for the year.

Rate of USC	Charged on income from
2%	€0 to €10,036
4%	€10,036 to €16,016
7%	Above €16,016

A large grid for calculations, consisting of 20 columns and 20 rows, intended for the student to show their work in calculating the USC payment.

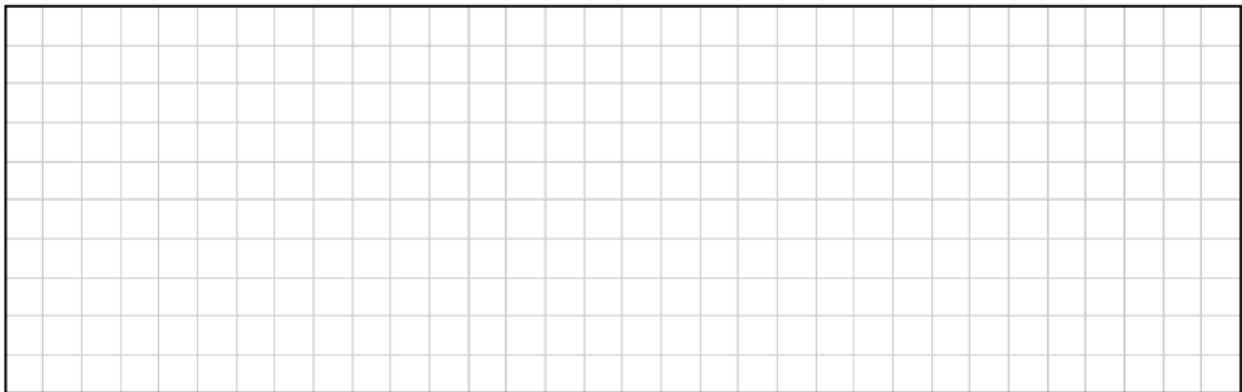
Exchange rates

Jane buys a laptop online for \$699, plus a shipping cost of \$30. The exchange rate is

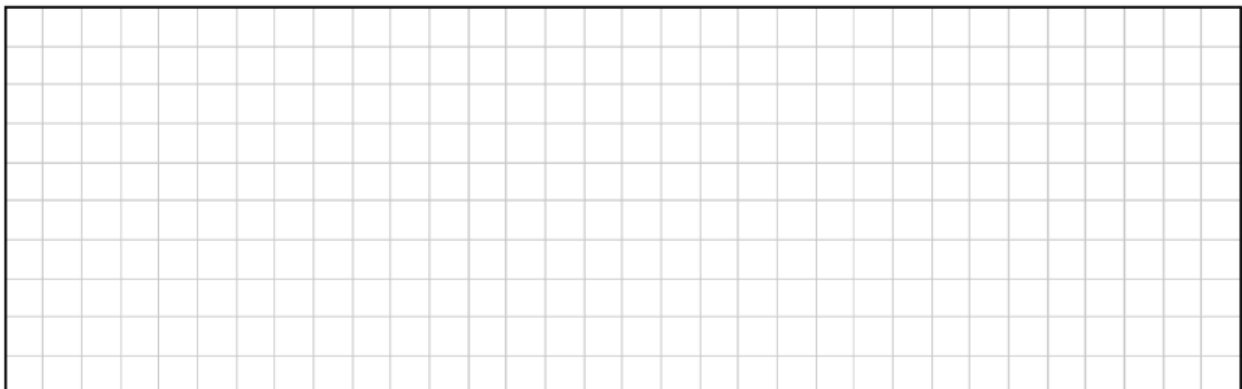
$\$1 = \text{€}0.90$. Work out in euro, the total cost to Jane of buying the laptop online.



Faith is going to spend her savings on a trip to New York. She has €3800 saved with Bank A. They offer her the following exchange rate: $\text{€}1 = \$1.06$ with no exchange fee. A rival bank offers $\text{€}1 = \$1.08$ with a 2% exchange charge. Which bank should she choose?



A monument is valued at \$6,500,000 Egyptian dollars. Find the value of the monument in euros, given that $\text{€}1 = \$19$.



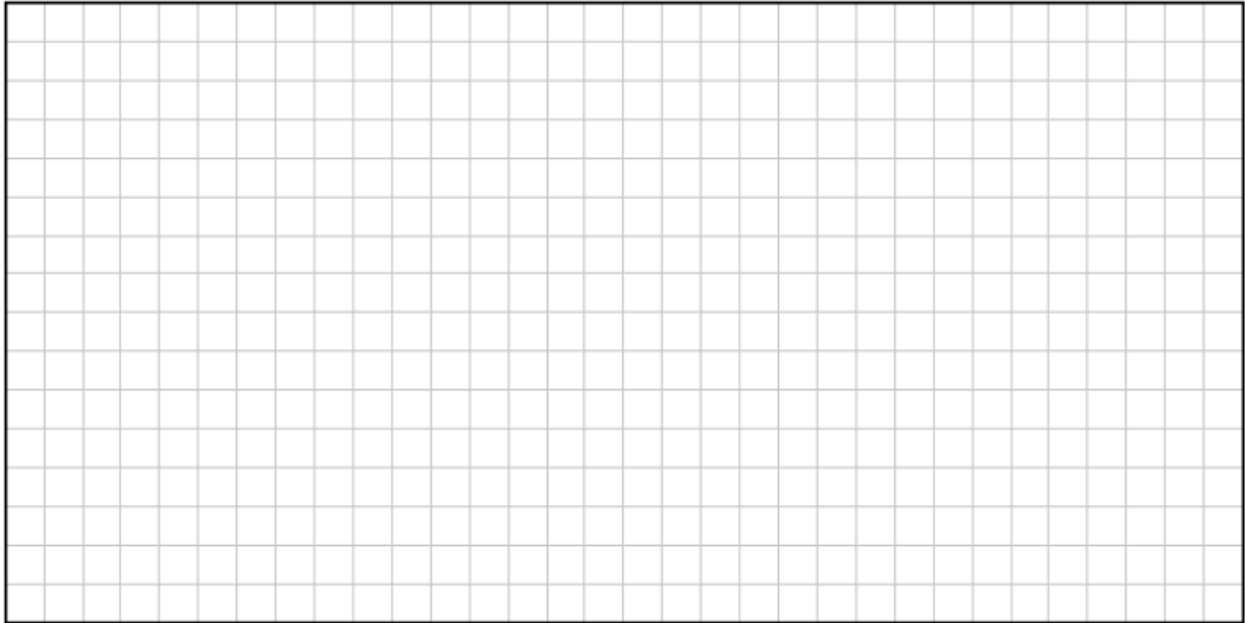
Jane bought an 800 square foot apartment in Florida for 270,000 USD. Mei bought a $100m^2$ apartment in Beijing for 1,250,000 CNY.

Given that:

$$1 \text{ CNY} = 0.15 \text{ USD}$$

$$1 \text{ square foot} = 0.093m^2$$

Which apartment is better value?



A shop buys juice-boxes from the UK for £380. The exchange rate is €1 = £0.7241. Find the price of the juice-boxes in euro, correct to the nearest cent.



Demot has €5000. He would like to invest it for two years. A special savings account is offering a rate of 3% for the first year, and a higher rate for the second year, if the money is retained in the account. Tax of 41% will be deducted each year from the interest earned (DIRT).

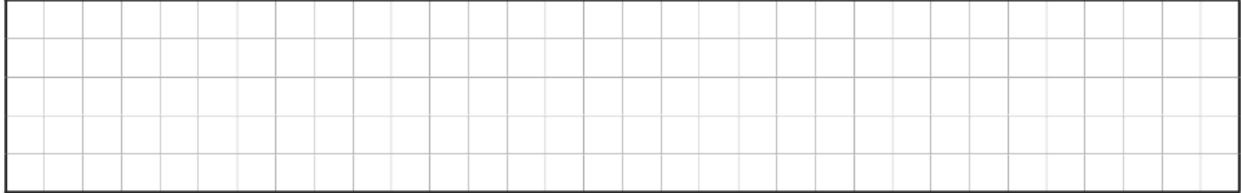
(i) How much will the investment be worth at the end of the first year, after tax is deducted.

(ii) Dermot calculates that, after tax has been deducted, the investment will be worth €5223.60 at the end of the second year. Calculate the rate of interest for the second year. Give your answer as a percentage, correct to one decimal place.

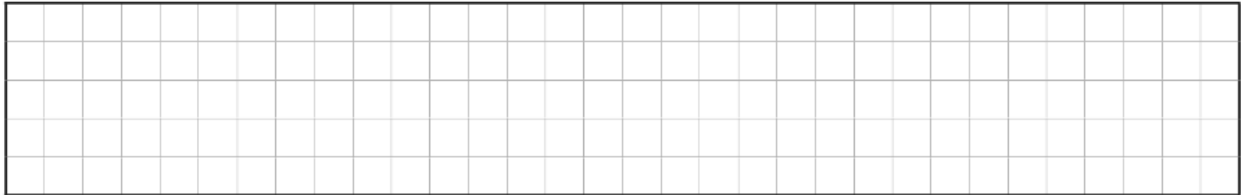
A small business purchases a photocopier for €5 500. Its purchase value decreases by 20% each year for 4 years. Find the value of the photocopier at the end of the 4 years.

Different percentages

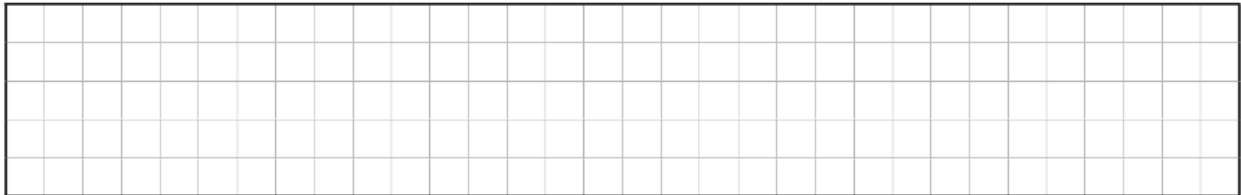
Rory buys a new football. It usually costs €20, but he is charged €17. What percentage discount does he receive?



A house is valued at €460,000. One year later, the value of the house has gone up to €472,000. Work out the percentage increase in the value of this house over the year.



The monthly price of an online TV subscription service increased from €12.50 to €13.75 after one year. Express this increase as a percentage of the original price.



Shane borrows €25,000 at 4% compound interest per annum for five years. Calculate as a percentage of the principle, the total interest Shane pays on the loan, correct one decimal place.

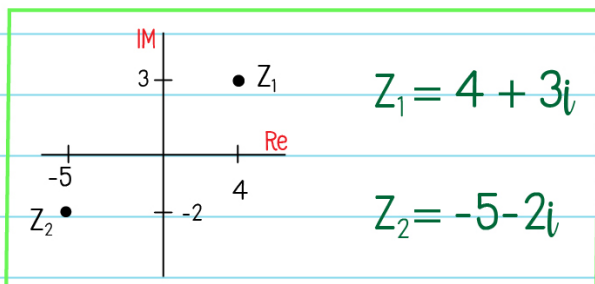


Chapter 8

COMPLEX NUMBERS

Examples:

$i = \sqrt{-1}$	i^{57}	i^{67}
$i^2 = -1$	$= (i^4)^{14} \times i^1$	$= (i^4)^{16} \times i^3$
$i^3 = -i$	$= 1 \times i$	$= 1 \times -i$
$i^4 = 1$	$= i$	$= -i$



i) $Z_1 + Z_2$

$$4 + 3i + (-5 - 2i) = -1 + i$$

ii) $Z_1 - Z_2$

$$4 + 3i - (-5 - 2i) = 9 + 5i$$

iii) $Z_1 Z_2$

$$(4 + 3i)(-5 - 2i) = -20 - 8i - 15i - 6i^2 = -14 - 23i$$

(-1)

iv) Z_1 and Z_2

$$Z_1 = 4 + 3i \quad Z_2 = -5 - 2i$$

$$\bar{Z}_1 = 4 - 3i \quad \bar{Z}_2 = -5 + 2i$$

v) $\frac{Z_1}{Z_2}$

$$\frac{4 + 3i}{-5 - 2i} \times \frac{-5 + 2i}{-5 + 2i}$$

Fraction? Multiply top and bottom by conjugate of bottom

vi) $|Z_1|$

$$3^2 + 4^2 = |Z_1|^2$$

$$|Z_1| = 5$$

vii) Complex Quadratics

$2 + i$ is a solution of $z^2 - 4z + 5 = 0$
 because $(2 + i)^2 - 4(2 + i) + 5 = 0$
 $(2 + i)(2 + i) - 4(2 + i) + 5 = 0$
 $4 + 4i + i^2 - 8 - 4i + 5 = 0$
 $0 = 0$

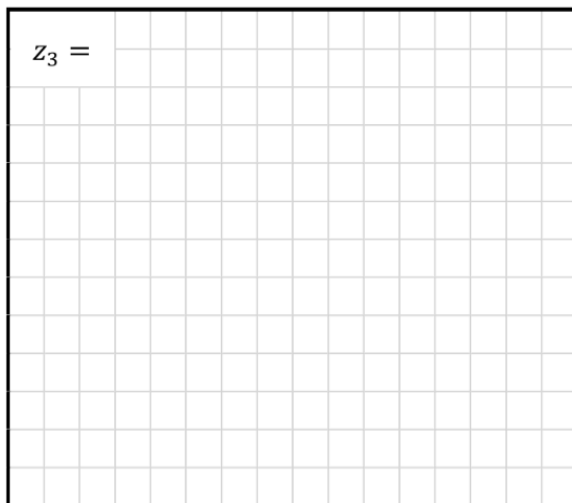
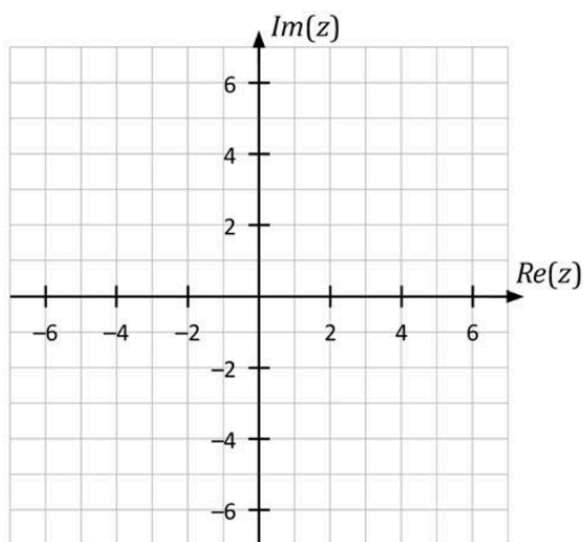
Test whether $3 - 2i$ is a solution

$z_1 = -1 + 3i$ and $z_2 = 2 - 4i$, where $i^2 = -1$.

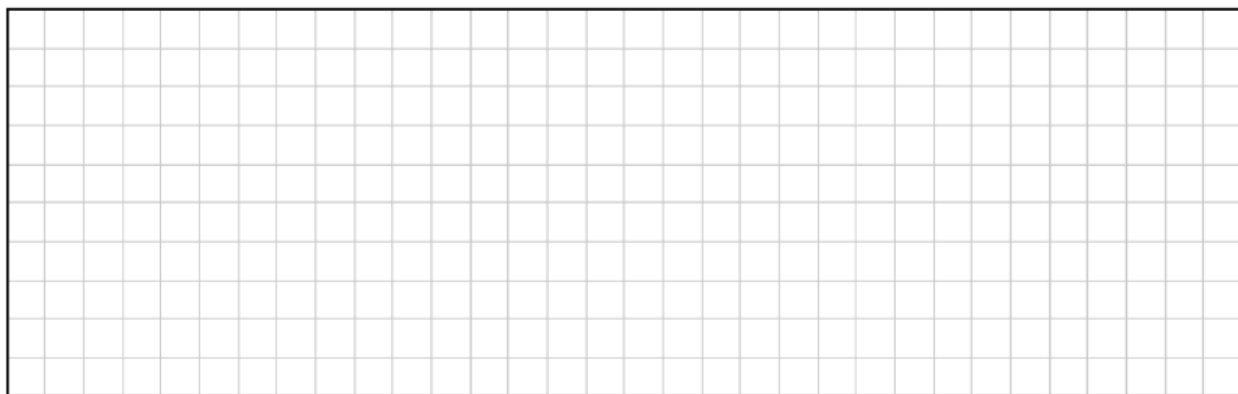
$$z_3 = \overline{z_1} + \overline{z_2}$$

(i) Find the value of z_3 and plot it on the given Argand diagram

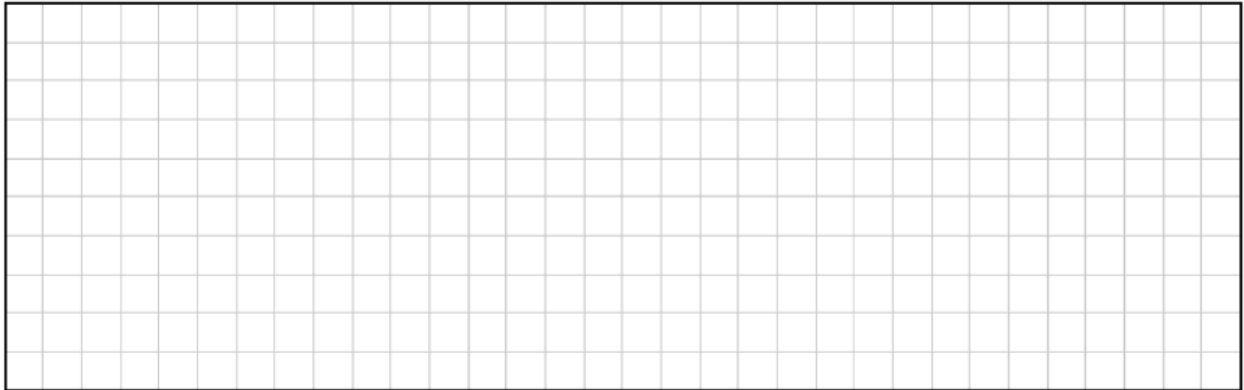
$z_3 =$

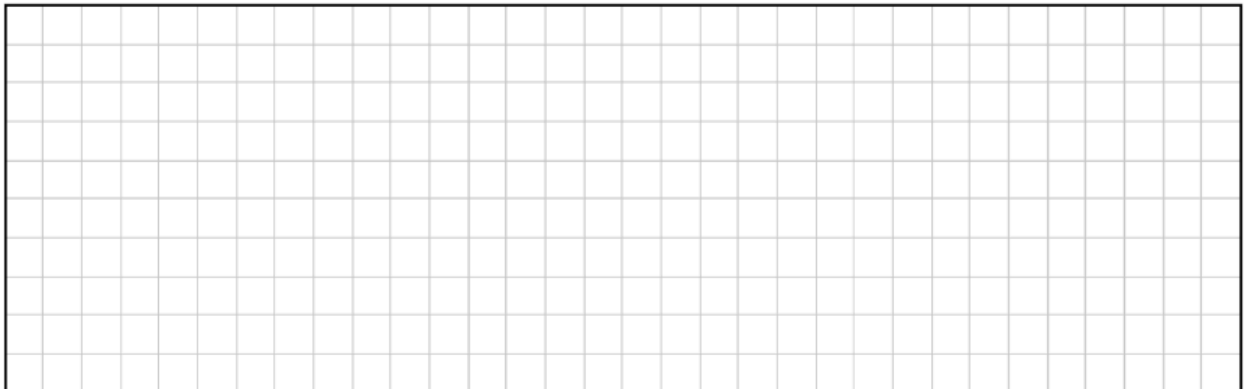
(ii) Prove that $|z_1| > |z_2|$



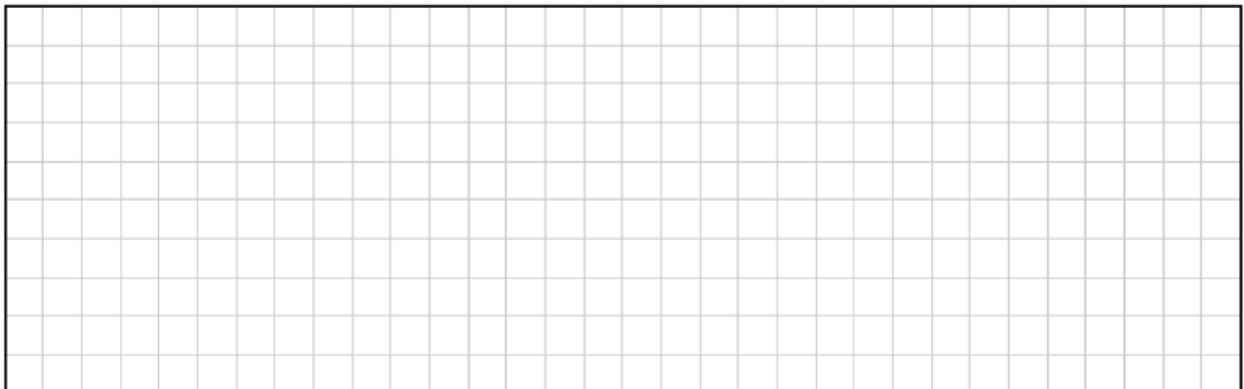
(iii) Find $\frac{z_3}{z_1}$ in the form $a + bi$.



(iv) Find $z_1 - z_2$ in the form $a + bi$.

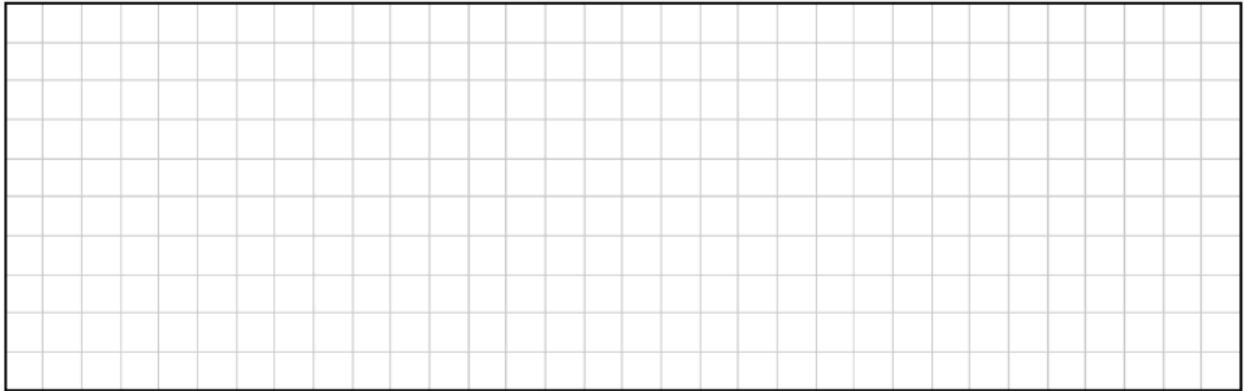


(v) Find $\overline{z_1 z_2}$ in the form $a + bi$.

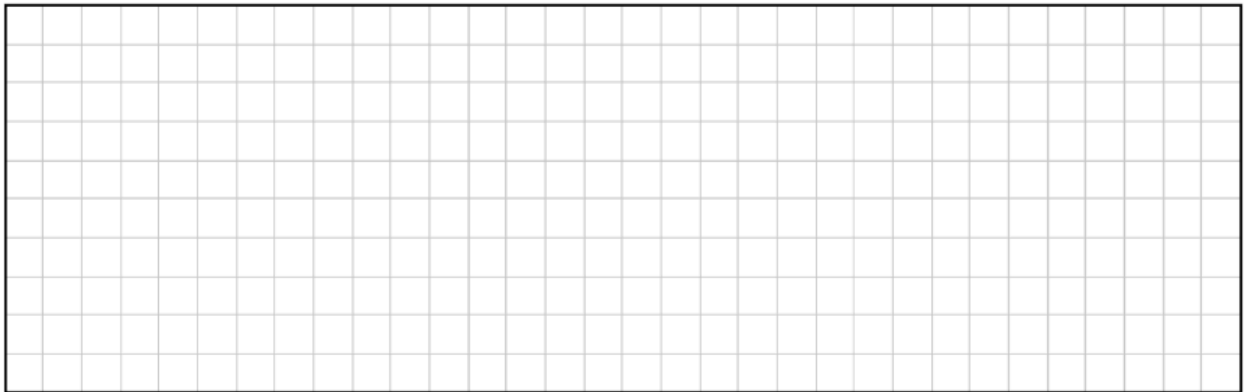


Simplify $4(3 + 2i) + i(5 - i)$.

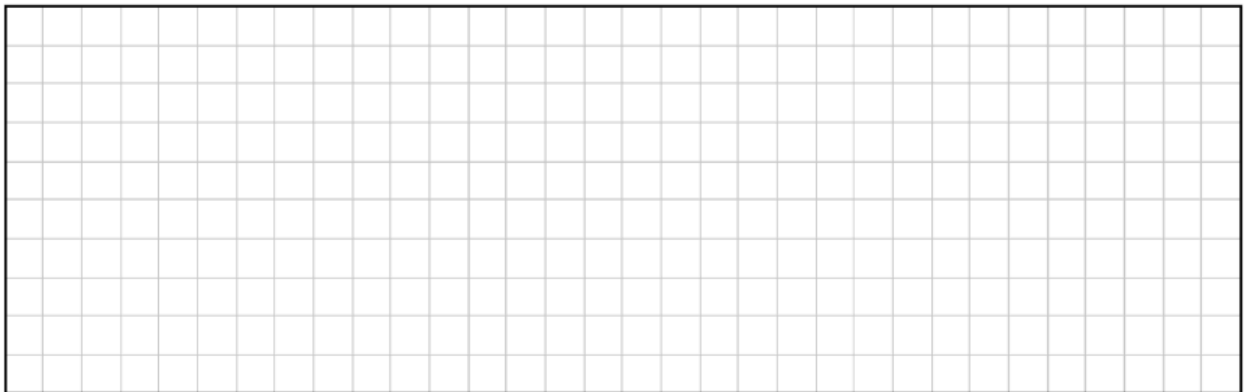
Leave your answer in the form $a + bi$.



If $z = 3 + 3i$, express $\frac{z}{z} + i$ in the form $a + bi$



$z_2 = 5 + i$, $z_3 = 6 - 2i$, $z_4 = 4z_2 - 5z_3$. Work out the value of $|z_4|$.



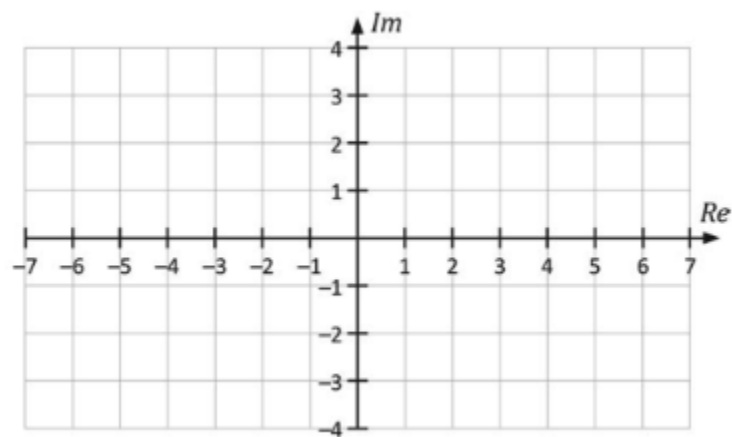
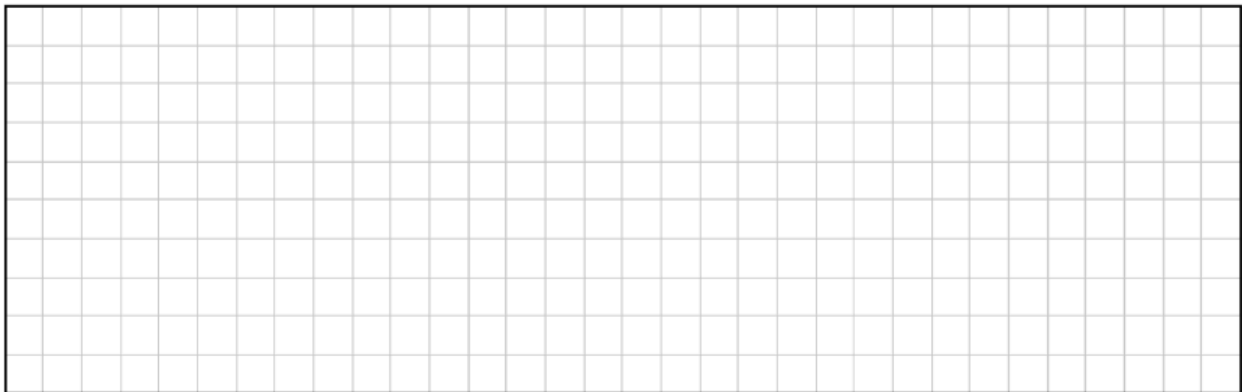
$z_2 = 5 + i$, $z_3 = 6 - 2i$. Find $|z_2 - z_3|$



$w = 3 - 2i$.

(i) Show that $\frac{13}{w} = \bar{w}$

(ii) Plot w , \bar{w} and $(w + \bar{w})$ on the Argand Diagram below.



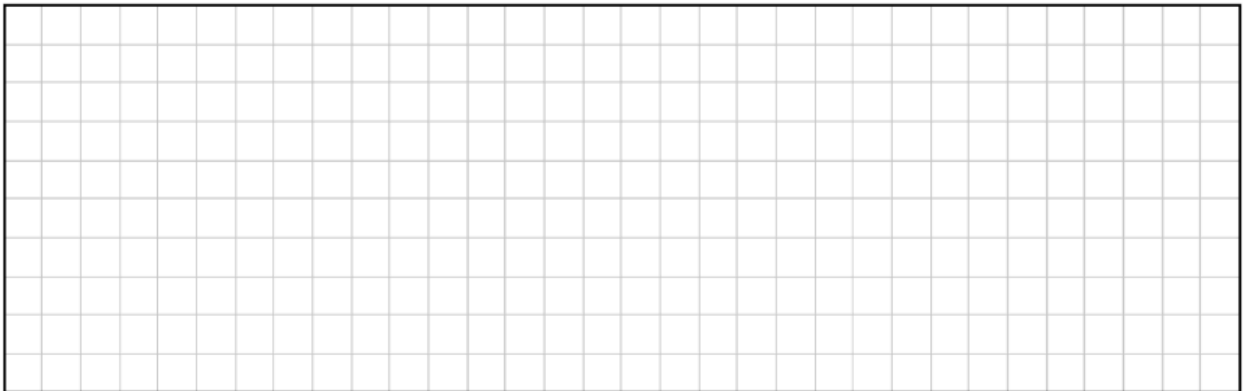
Solve for z :

$$2z - 6(4 - 6i) = (-1 + i)(4 - 2i)$$

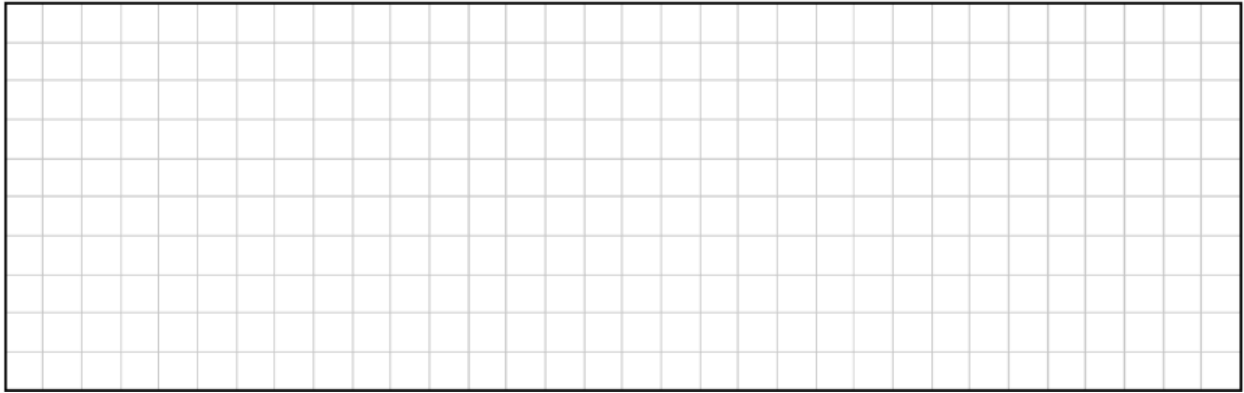


$$z_1 = 3 - 4i.$$

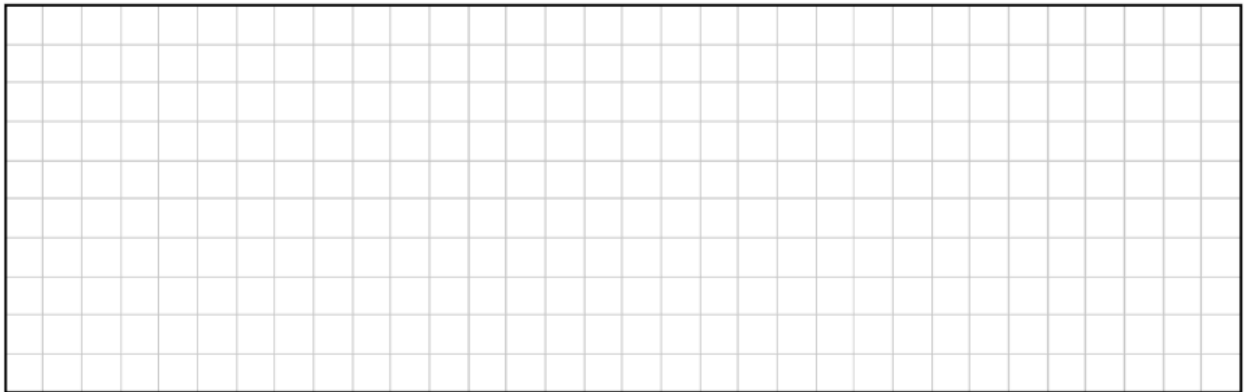
If $z_1 \times z_4 = 29 + 3i$, write z_4 in the form $a + bi$.



Show that $z = 2 + i$ is a solution of the equation $z^2 - 4z + 5 = 0$.



Verify that $(2 + 3i)$ is a root of the quadratic $z^2 - 4z + 13 = 0$.



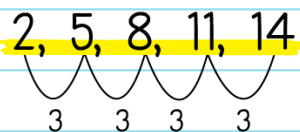
Chapter 9

$$T_n = an^2 + bn + c$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

SEQUENCES AND SERIES

• Linear / Arithmetic



1st difference is the same

$$T_n = a + (n-1)d$$

a = 1st term

d = common difference

n = term number

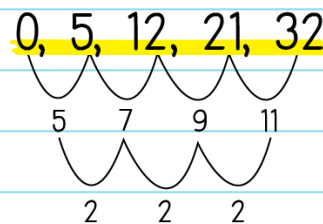
Example: Find the 10th term of the above pattern

$$\begin{aligned} T_{10} &= 2 + (10-1)(3) \\ &= 29 \end{aligned}$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

↖
Use when asked about Total

• Quadratic



2nd difference is the same

$$T_n = an^2 + bn + c$$

$$a = \frac{1}{2} \times (\text{2nd difference})$$

Example: $a = \frac{1}{2} (2) = 1$

$$1(2)^2 + b(2) + c = 5 \quad [\text{2nd term is 5}]$$

$$\rightarrow 2b + c = 1 \quad \textcircled{1}$$

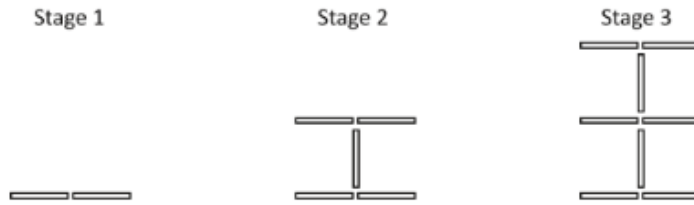
$$1(3)^2 + b(3) + c = 12 \quad [\text{3rd term is 12}]$$

$$\rightarrow 3b + c = 3 \quad \textcircled{2}$$

$$2b + c = 1 \quad \textcircled{1} \quad \text{Solve simultaneously}$$

$$3b + c = 3 \quad \textcircled{2}$$

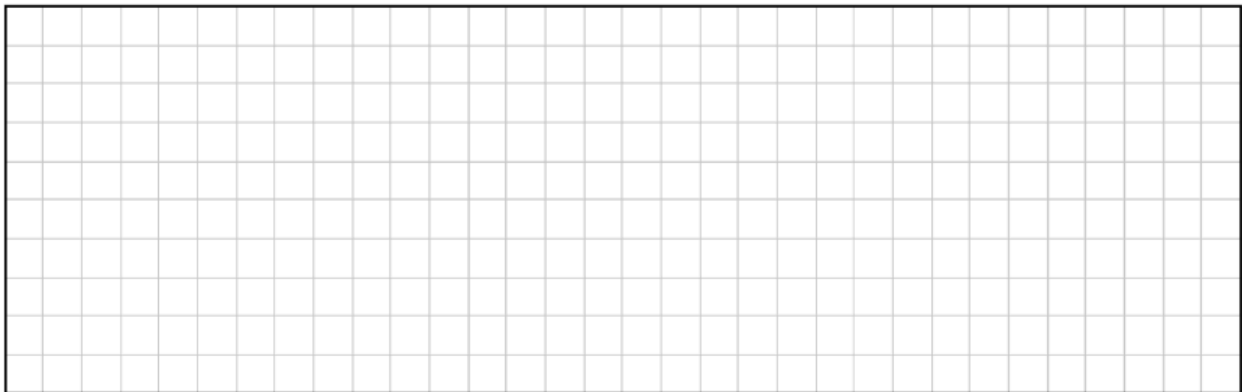
$$T_n = 1n^2 + 2n - 3$$



(i) Draw stage 4 and stage 5 of the pattern:

<p>Stage 4:</p>	<p>Stage 5:</p>
-----------------	-----------------

(ii) Find a general formula for the number of matchsticks needed for the n^{th} stage of the pattern. Use this formula to determine how many matchsticks there will be on the 21st stage. Determine which stage will have 101 matchsticks.




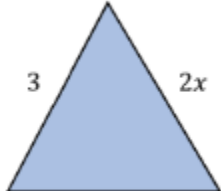
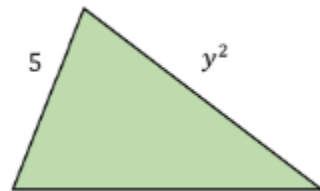
In a particular linear sequence, the 2nd term is 22, and the 5th term is 73. Fill in the boxes below to show the rest of the terms in the sequence.

	22			73	
--	----	--	--	----	--

The three triangles A, B, C are shown below. Their perimeters are in linear sequence.

The common difference is 16cm, and the length of the missing side in triangle A is 8cm.

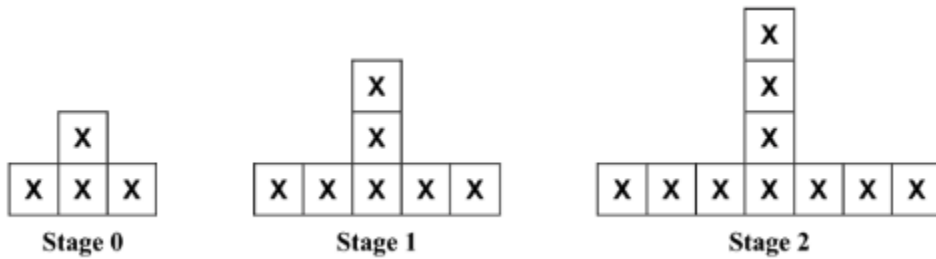
Find the value of x and the value of y .

Triangle A	Triangle B	Triangle C
		

--

In a particular linear sequence, the second term is 40, and the sixth term is 116. Use this to figure out the first term, and the common difference.

From there, work out what value the 30th term is.



There are exactly 130 X's in stage k of the above sequence. Find the value of k .

The first three terms of an arithmetic sequence are $-5, p, 3$.

(i) Find the value of p .

(ii) Find T_{12} .

(iii) Which term in the sequence has a value of 219.

The first 4 terms of an arithmetic sequence are 3, 7, 11, 15.

(i) Find the sum of the first 24 terms of the sequence.

(ii) Which term number gives this sequence a total of 9870 when all the terms are added together.

Maya makes a pattern using sticks. She sees that pattern 1 has 4 sticks, pattern 2 has 7 sticks and pattern 3 has 10 sticks.

(i) Pattern j uses 67 sticks. Find the value of j .

(ii) Maya continues the same pattern. How many sticks are used in total to make 10 sequences.

Gemma is training for a marathon. Her training includes a run every Saturday, starting with a run of five kilometres on the first Saturday. Each Saturday she increases the length of her run from that of the previous Saturday by 2 kilometres.

(i) Find an expression in terms of n for the distance Gemma runs on the n^{th} Saturday of her training

(ii) To be ready for the marathon Gemma needs to be able to run 41 kilometres. How many Saturdays of training in this way will it take for Gemma to be able to run 41 kilometres?.

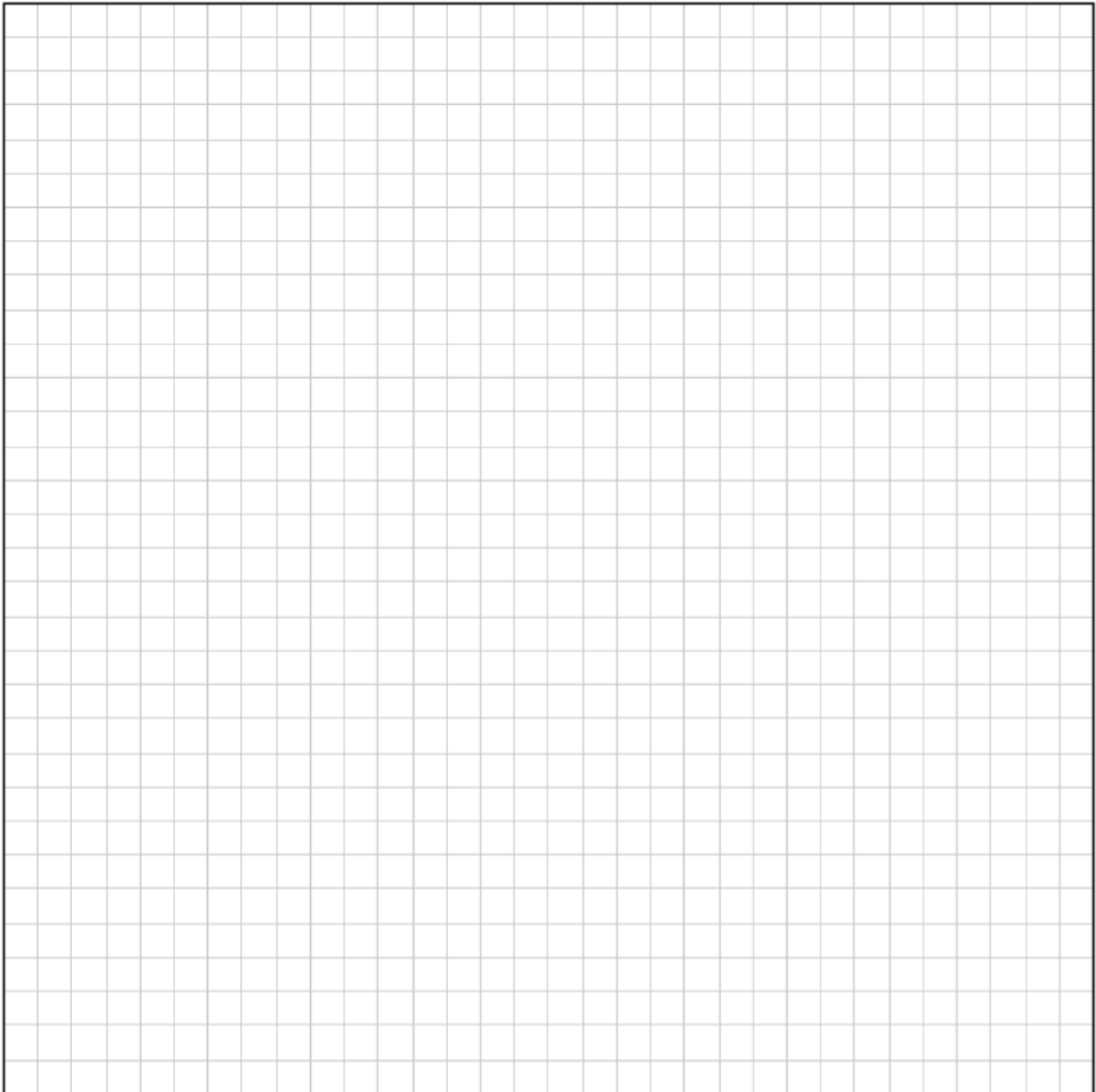
(iii) Calculate the total combined distance Gemma has run on all of her Saturdays during the first 12 weeks of training.

Find T_n in terms of n for the following pattern, and hence find T_{10} :

2, 9, 16, 23, 30

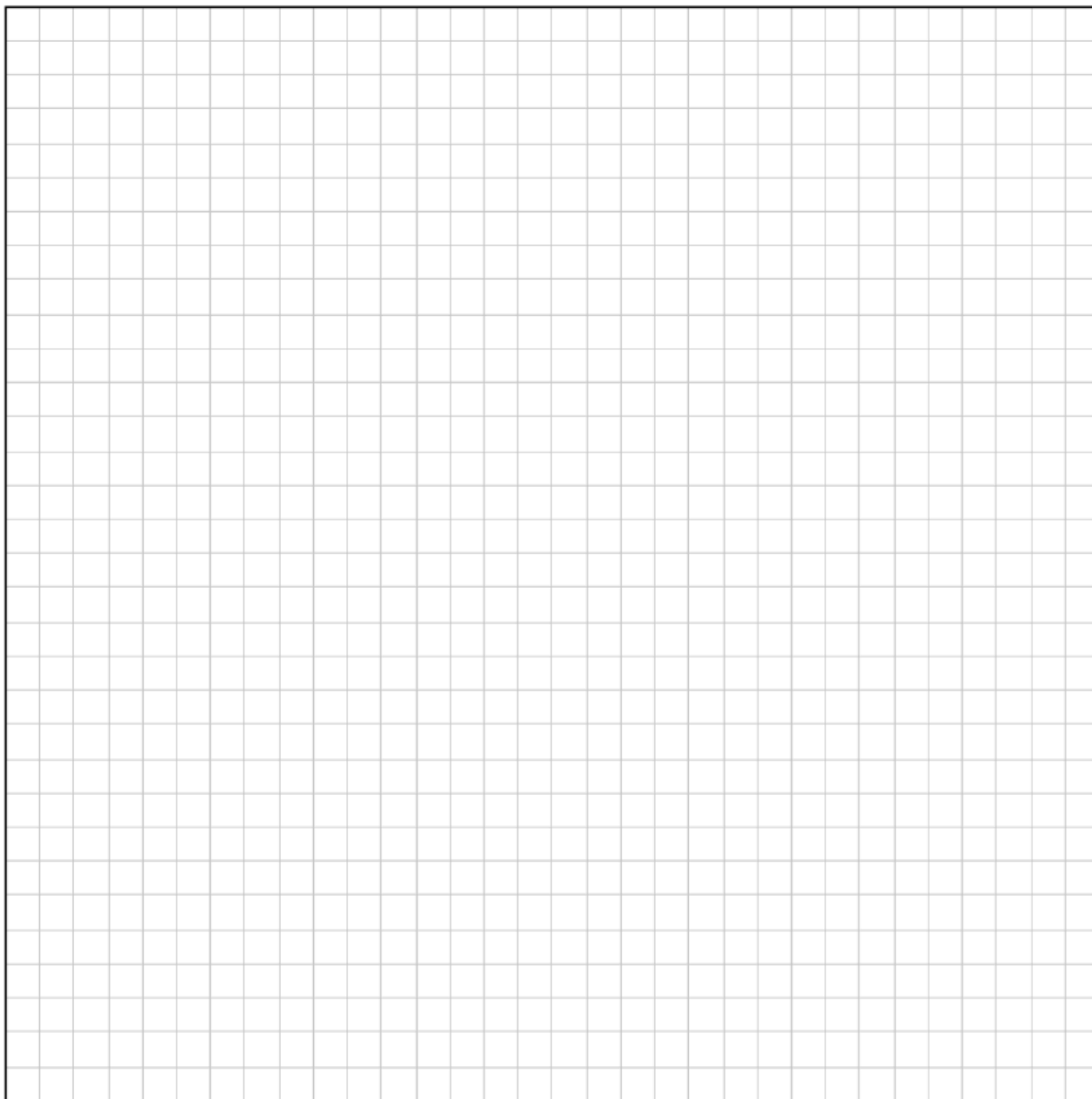
Prove that the following sequence is quadratic, and hence find the function in the form $ax^2 + bx + c$, where $a, b, c \in \mathbb{Z}$.

0, 5, 12, 21, 32...



Time (mins)	0	1	2	3	4	5	6	7	8	9	10
Temp (°C)	10	14.5	20	26.5	34	42.5	52	62.5	74	86.5	100

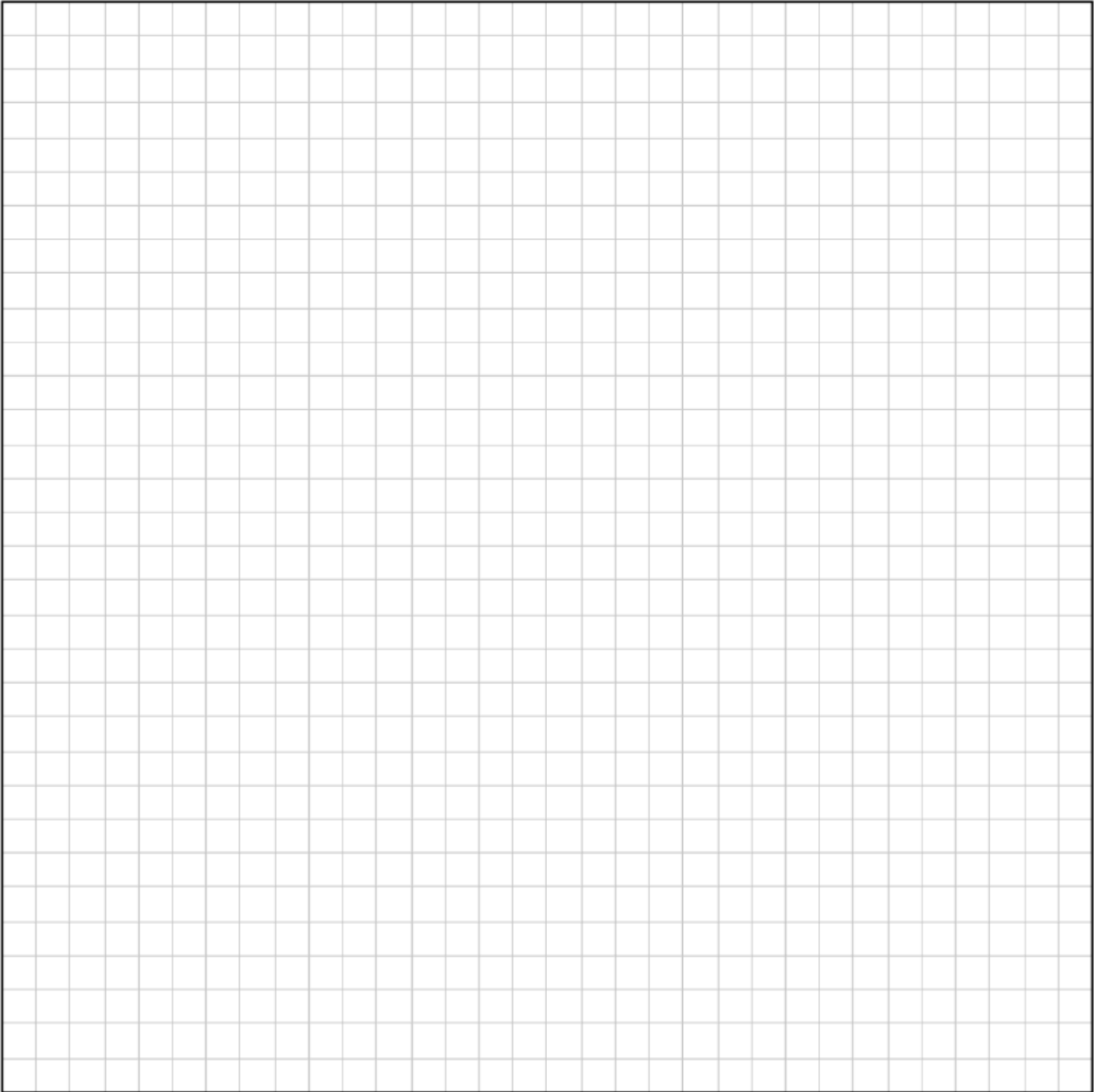
The temperature of water in a pot with the lid off creates a quadratic pattern. Find the function for the temperature of the water in the form $ax^2 + bx + c$, where $a, b, c \in R$.



Cans are stacked in rows on top of each other, to form a triangle. The total number of cans for any number of rows makes a quadratic pattern.

Total rows	1 row	2 rows	3 rows		<i>n</i> rows
Total cans	1 can	3 cans	6 cans		

How many rows would a stack of 136 cans have?.



The number of sticks in pattern n below is given by $T_n \cdot T_n$ can be written in the form $an^2 + bn + c$. Find the value of a, b and c .

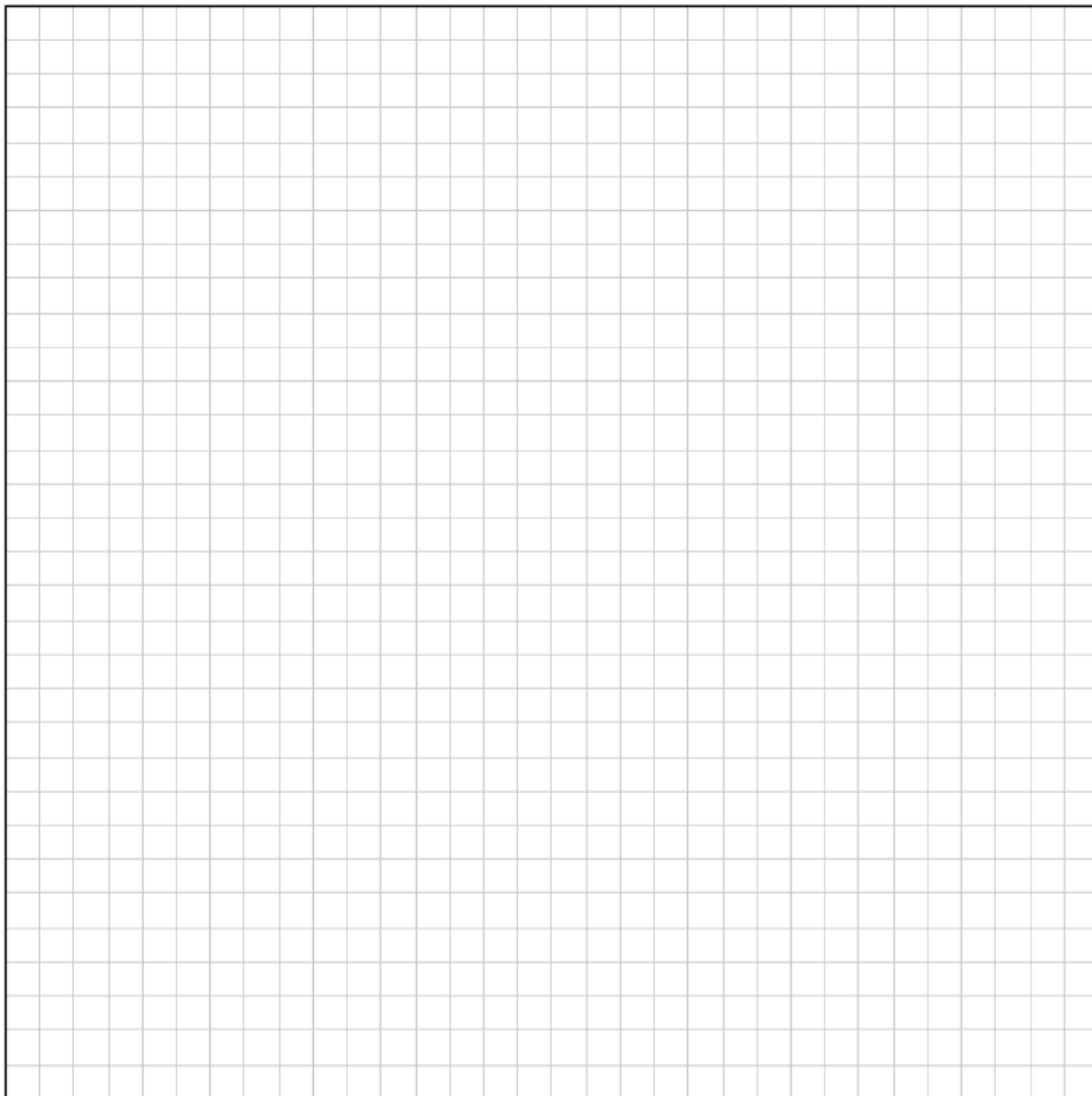
Pattern 1



Pattern 2



Pattern 3



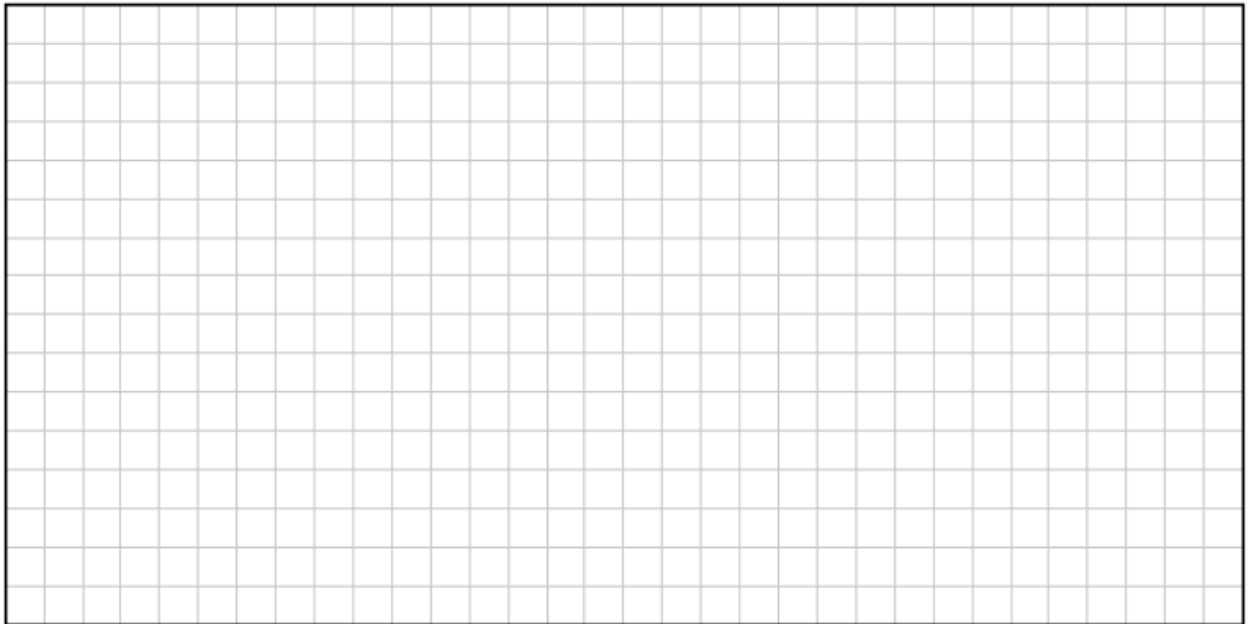
Killian is spending the summer on his uncle Donald's Farm. Donald is going to pay Killian for helping out on the farm and he gives Killian two options:

Option A: €10 per day.

Option B: €1 on day 1, €2 on day 2, €3 on day 3 etc.

The total earned using option B is given by $\frac{n^2+n}{2}$.

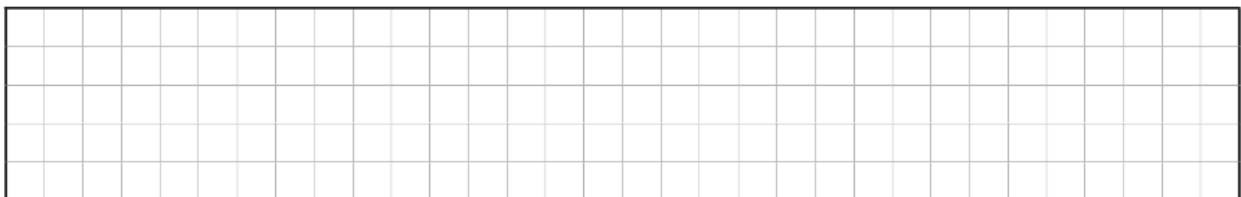
For how many days would Killian have to work such that the pay he would receive by either option is the same?



John writes down the following sequence which repeats every three terms:

3, 6, 4, 3, 6, 4

Write down the value of the 100th term in this sequence.



Chapter 10

€ PROBABILITY



- Probability theory and the fundamental principle of counting
- Permutations
- Venn diagrams
- Expected value
- Sample spaces
- Tree diagrams

All probability adds to 1

And → MULTIPLY
Or → ADD

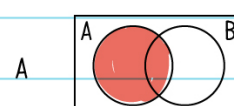
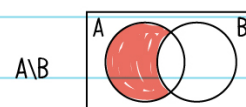
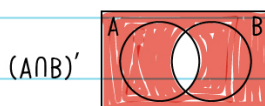
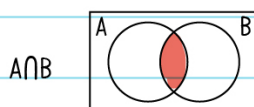
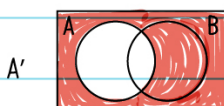
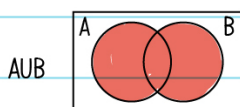
● Fundamental principle of counting

If scenerio A has m outcomes, and scenerio B has n outcomes, then the total number of outcomes is $m \times n$

● Permutations

Arranging digits, letters in which order matters

● Venn diagrams




Probability theory and the fundamental principle of counting

Michael has a bag containing 6 marbles, 3 of which are red, 1 is yellow, and 2 are blue.

(i) Michael picks a marble from the bag. What is the probability he picks a blue marble?

(ii) Michael also records all combinations of outcomes he will get if he picks a marble from the bag, and flips a coin. Work out the total number of combinations he will get.



Ruairi rolls a regular six sided die 3 times. He writes down the number on each roll, and notes whether it is even (E) or odd (O). Each outcome is equally likely. For example, EEE is just as likely as OEO. Ruairi multiplies the three numbers that he rolls and records the result. Find the probability that Ruairi will get an even number as his overall answer.



Eight cards numbered 1 to 8 are shuffled and a card is selected at random.

(i) Find the probability that a prime number is selected.

(ii) Find the probability that a factor of 4 is selected.

(iii) One card is selected at random and put to the side. Then a second card is selected from the remaining 7 cards. Find the probability that both cards are odd.

Eight cards are numbered 1 to 8 as shown below.



(i) In how many ways can these eight cards be rearranged?

(ii) How many of these arrangements start with an even number?

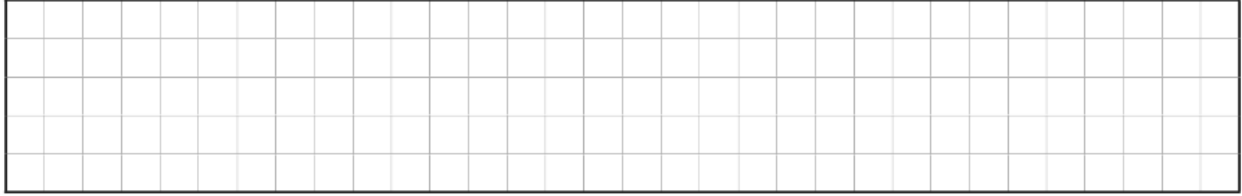
Find the number of different arrangements that can be made using all the letter of the word MACHINE if: (i) Each letter is only used once

(ii) The letter M must be first

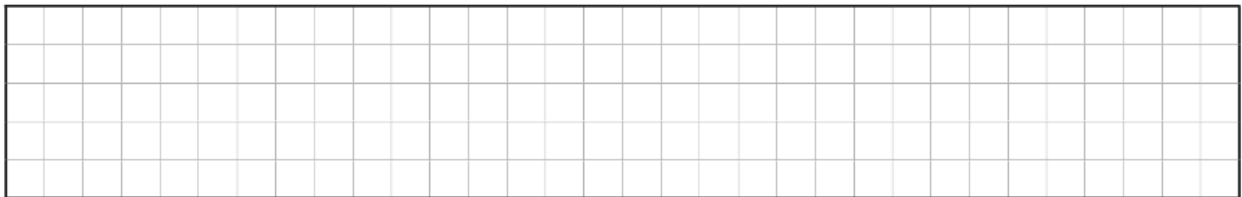
(iii) The vowels must be side by side

In how many ways can the letters of the word IRELAND be arranged if:

(i) It must begin with E



(ii) The vowels must be side by side



A student is completing an experiment that consists of drawing discs from two different bags. The first bag contains four discs numbered 1 to 4 inclusive, and the second bag contains three discs coloured red, yellow, green. A player takes one disc from each bag and notes their results.

Two-way table	R	Y	G
1			
2		(2, Y)	
3			
4			

Complete the table above, and hence answer the following questions.

What is the probability that the student draws:

(i) A disc numbered 2

(ii) A yellow or green disc

(iii) The number 3 and the colour yellow.

An experiment consists of throwing two fair dice, and recording the sum of the numbers thrown.

		Die 1					
		1	2	3	4	5	6
Die 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6			9	10
	5	6		8	9		11
	6		8	9	10	11	

(i) Complete the table above

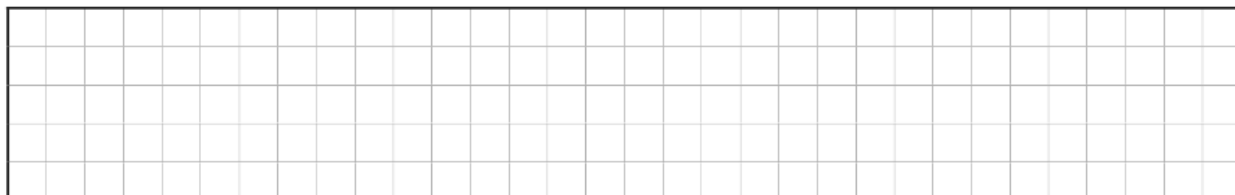
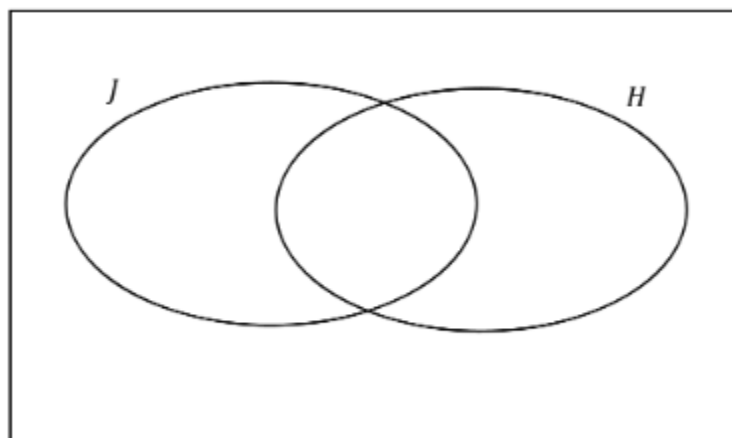
(ii) Find the probability of getting a 7 or 11 as your total.

(iii) Find the probability of getting a prime number as your total.

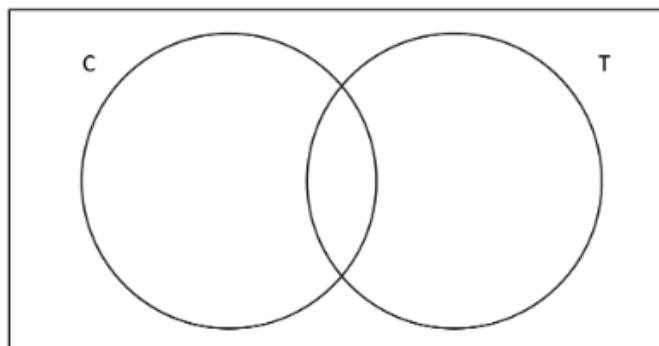
80 students in a group were asked how they spent their Summer. Some of the students got a job (J) and some went on holiday (H). Some did both.

20% of the students did neither. 25% got a job. Of those who got a job, half also went on holiday.

Complete the Venn Diagram below.



A survey has found that the probability that a student drinks coffee is 0.65. The probability that a student drinks tea is 0.7. The probability that a student drinks neither is 0.12. Fill in the Venn Diagram below.



Expected value

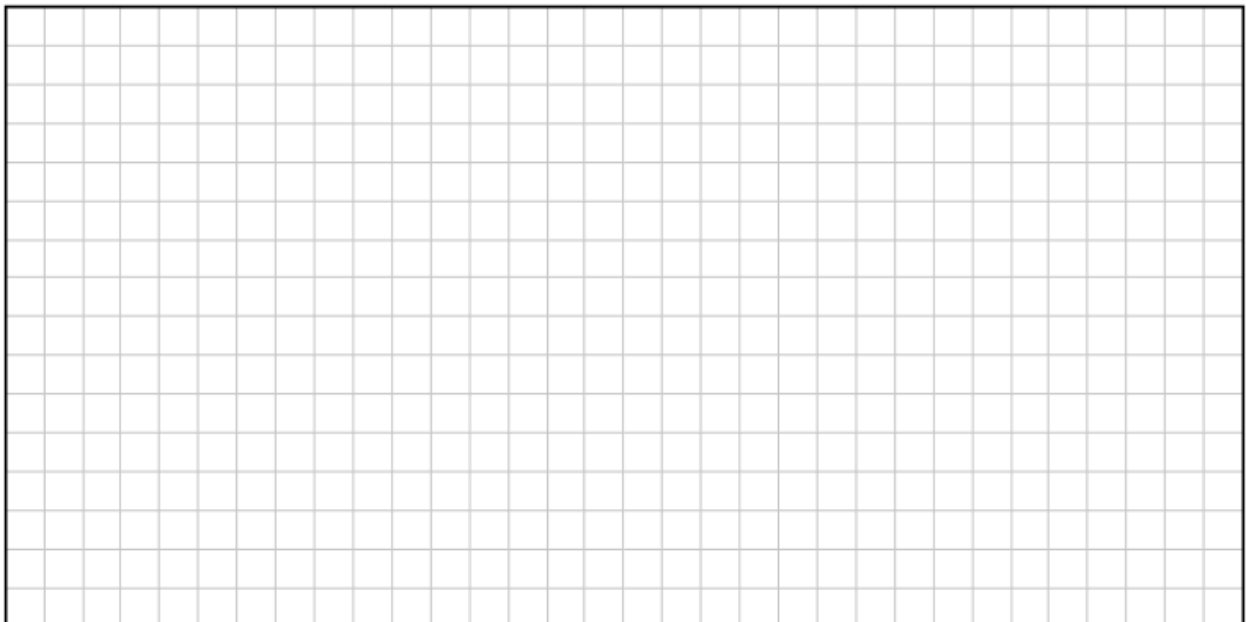
A spinner has 5 options: A , E , I , O , U .

In a game, Pawel pays €2 to spin this spinner. The table below shows the probability of each outcome, as well as the money Pawel will get, depending on the outcome.

Outcome	A	E	I	O	U
Probability	$\frac{1}{20}$	$\frac{1}{4}$	$\frac{7}{20}$	$\frac{3}{20}$	$\frac{1}{5}$
Pawel's Prize	€5	€2	€0	€1	€0.50

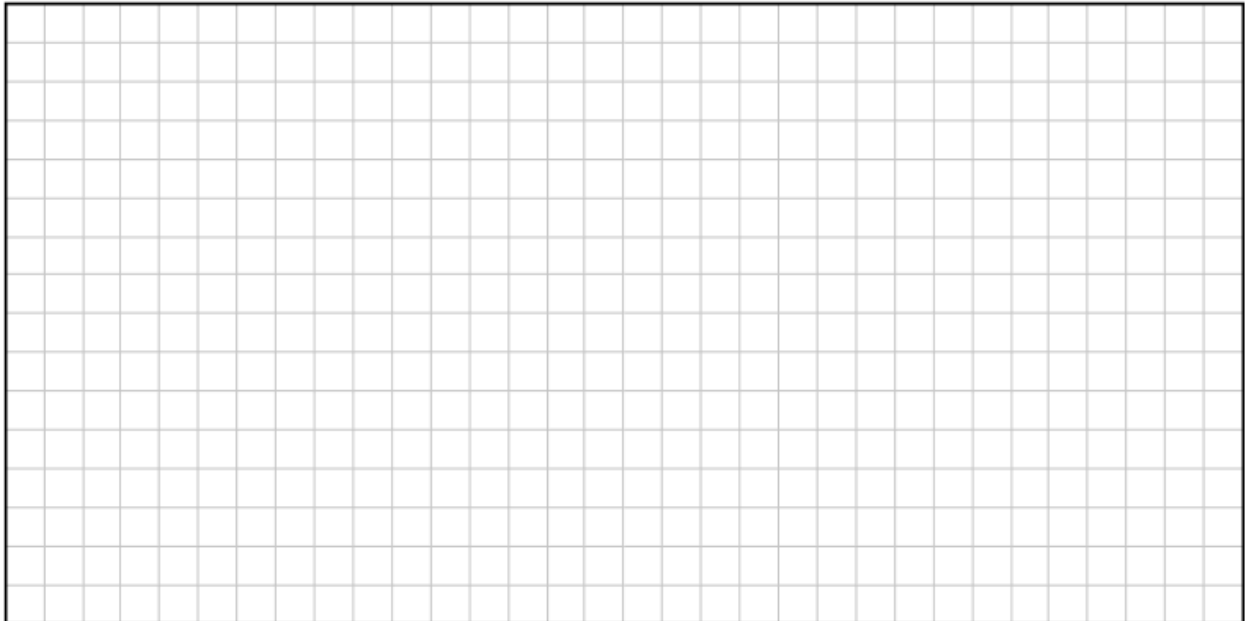
Is this game fair? Justify your answer.

Work out the expected value of the game as part of your solution.



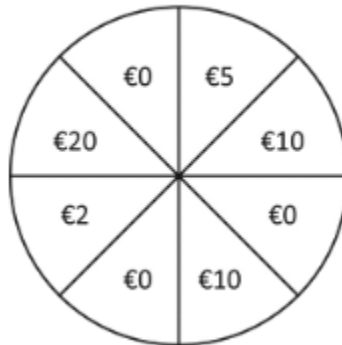
A sports club is having a raffle to raise some funds. The top prize is €5,000. There are two €1,000 prizes and three €500 prizes. The club sold 2000 tickets at €20 a ticket.

Calculate the expected value of each ticket sold, and fill in the accompanying table.



Prize (x)	€5000	€1000	€500	€0
P(x)	$\frac{1}{2000}$			

Jack has designed a game for people to play. It consists of spinning a wheel with eight equal segments. The player wins the amount shown on the wheel and the game costs 5 euro to play.



Calculate the expected value for the game, correct to the nearest cent. Is this game a good game for the club? Explain your answer fully.

A large rectangular grid for writing the answer, consisting of 20 columns and 20 rows of small squares.

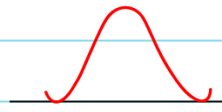
Chapter 11

STATISTICS

$$\frac{1}{\sqrt{n}}$$

$H_0:$

$H_1:$



• Definitions

• Emperical rule

• Confidence Interval and hypothesis test

• Diagrams

• Definitions

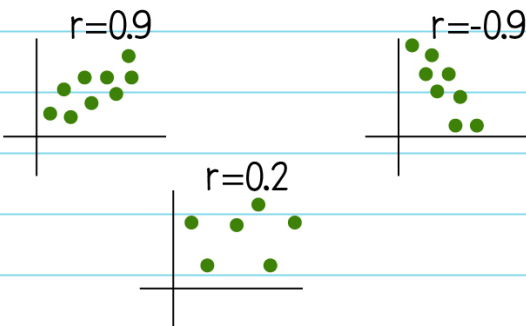
Mean → Average, Add the elements up and divide by the amount

Mode → Most frequent term in a list

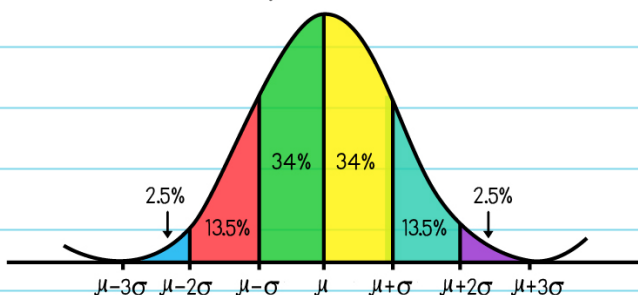
Median → Middle term of an ordered list

Range → Biggest - smallest

r → Correlation coefficient



Emperical rule



• Confidence Interval and hypothesis test

Margin of error : $\frac{1}{\sqrt{n}}$

Confidence interval :
(Proportion + MOE, Proportion - MOE)

H_0 : Original claim is true

H_1 : Original claim is not true

IF

If original value lies Inside the conf. Int,
Fail to reject H_0 .

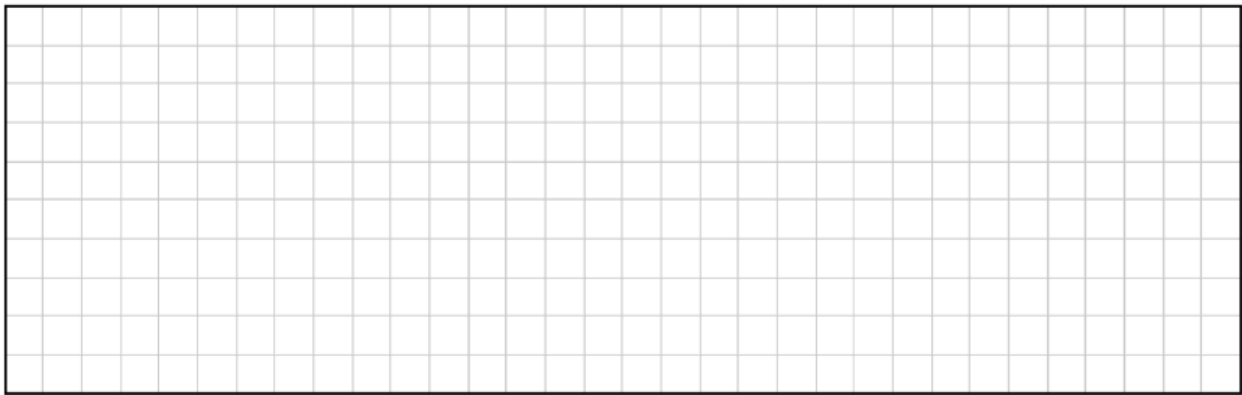
OR

If original value lies Outside the conf. Int, reject H_0 .

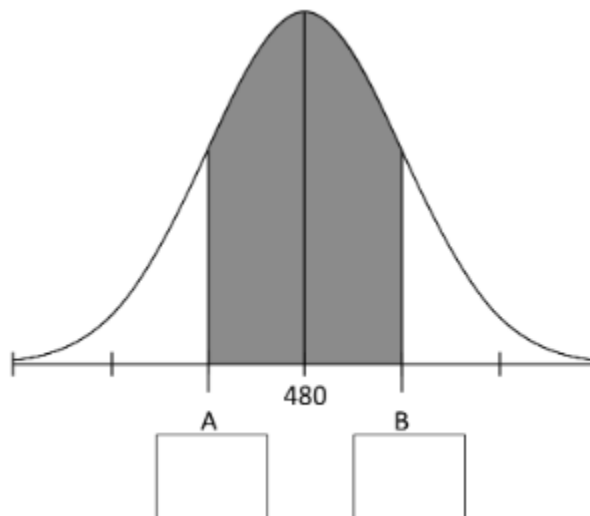
• Diagrams

- Pie charts
- Histograms
- Scatter plots
- Bar charts
- Stem and leaf

The heights of a given population are normally distributed. 95% of the population fall within the height range given below. Use this information to find the mean and the standard deviation of the heights in the population.



An aptitude test was taken by 6,500 candidates. The test scores were normally distributed. The mean score was 480 and the standard deviation was 90. On the distribution shown below, the shaded area represents all candidates who were within one standard deviation of the mean. Write the value of A and the value of B into the boxes below.



Confidence intervals and hypothesis tests

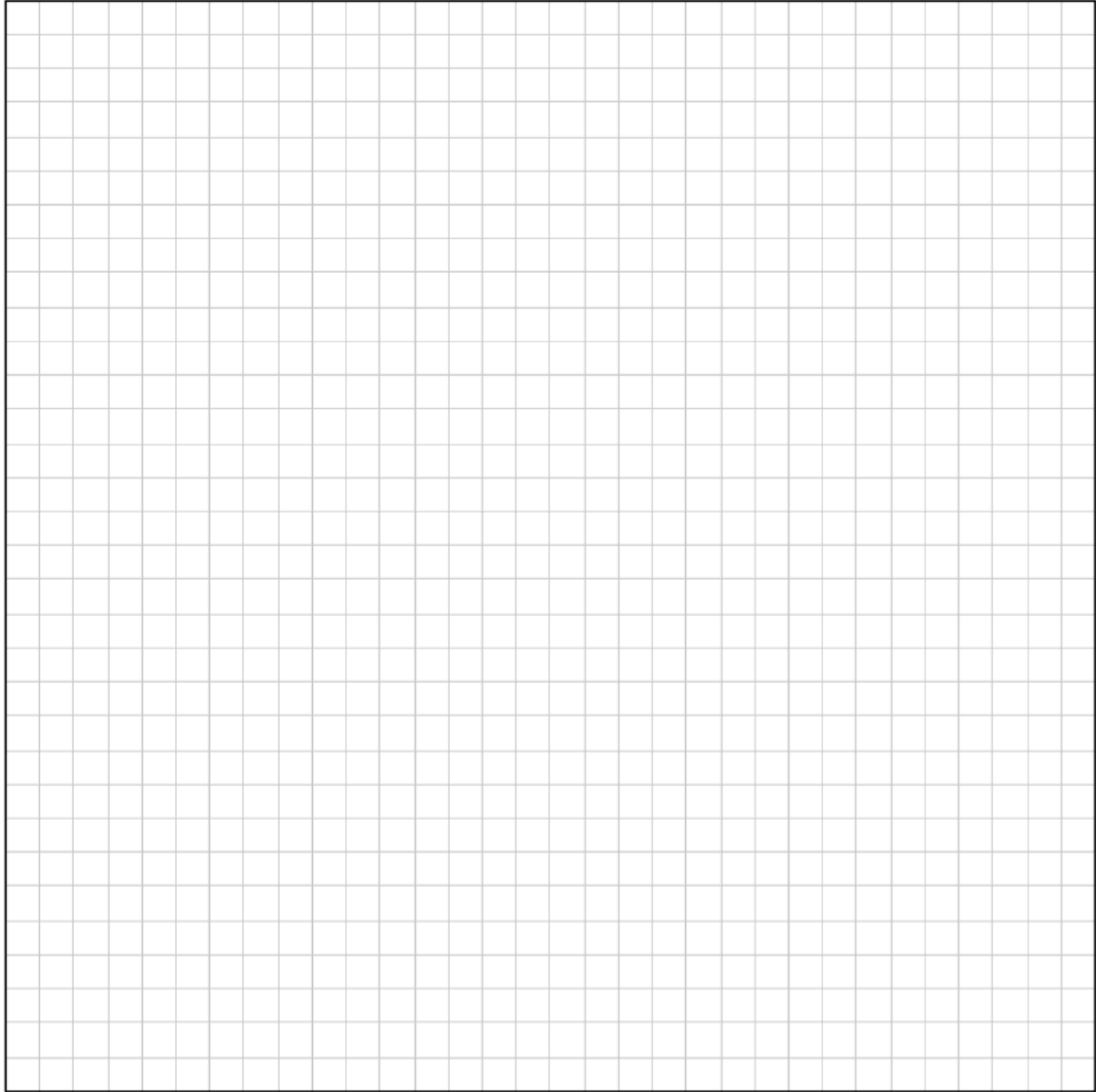
In a survey, 1000 people were selected at random and asked some questions about online shopping.

(i) Find the margin of error of the survey. Give your answer correct to one decimal place.

(ii) Of those asked, 762 said they believe it is safe to give their credit card details when shopping online. Use your answer from above to create a 95% confidence interval for the percentage of people who believe it is safe to give their credit card details when shopping online.

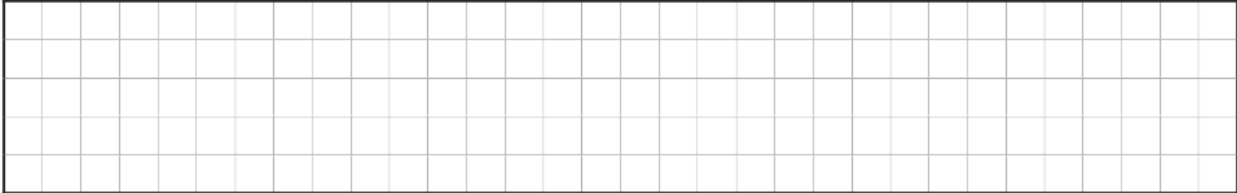
(iii) An online media company claims that 80% of people believe it is safe to give their credit card details when shopping online. Conduct a hypothesis test, at the 5% level of significance, to test the company's claim. Give your conclusion in the context of the question and give a reason for your conclusion.

The CEO of Duracell claims that 85% of its customers are happy with their batteries. Using a simple random sampling, a newspaper surveyed 300 customers. 215 customers stated that they were satisfied with the batteries. Carry out a hypothesis test at the 95% confidence level. State your conclusion clearly.

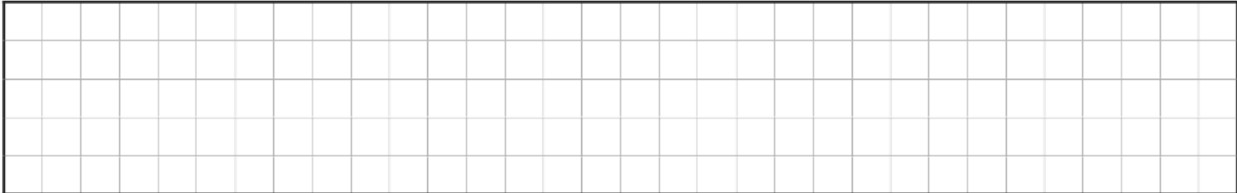


A marketing company has been contracted to investigate the recreational habits of Irish adults in 2024. It was decided to conduct an online survey on a sample of adults.

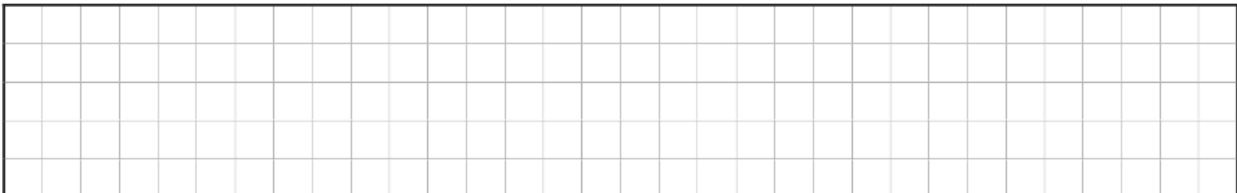
(i) A random sample of 1200 people took part in the survey. Calculate the margin of error for this sample, as a percentage, correct to one decimal place.



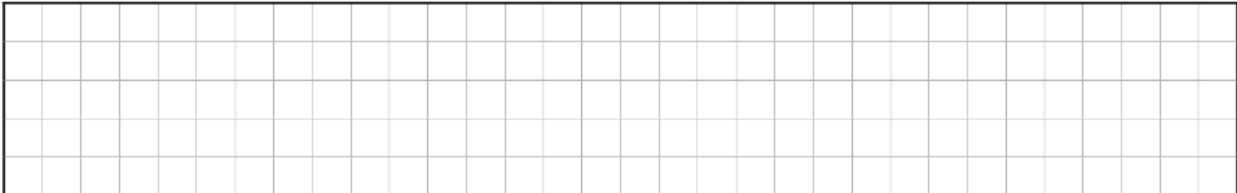
(ii) Of the 1200 surveyed, 744 indicated that they attended the cinema on a regular basis. Express this as a percentage.



(iii) Write down a 95% confidence interval for the percentage of adults who attend the cinema on a regular basis in 2024.

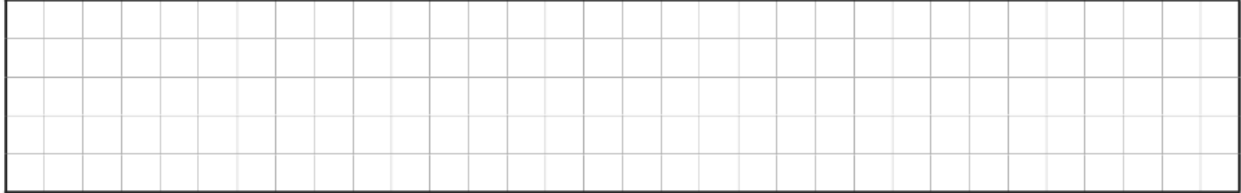


(iii) An older survey conducted in 2020, found that 53% of adults in Ireland attended the cinema on a regular basis. Complete a hypothesis test, at the 5% level of significance, to test if this figure of 53% has changed in 2024, based on the results of the marketing company's survey. State your conclusion and reason clearly.

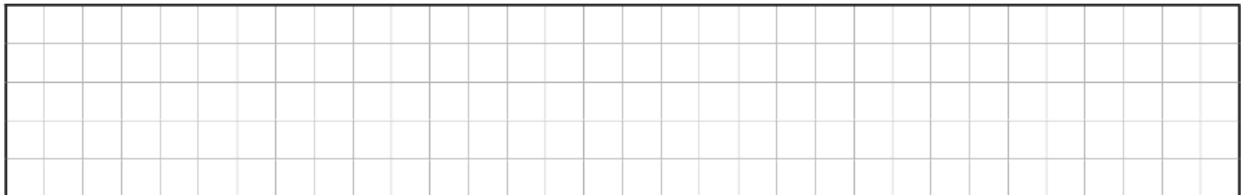


A random sample of 500 students in Cork took a statistics test. 61 of these students were given a rating of 'excellent'.

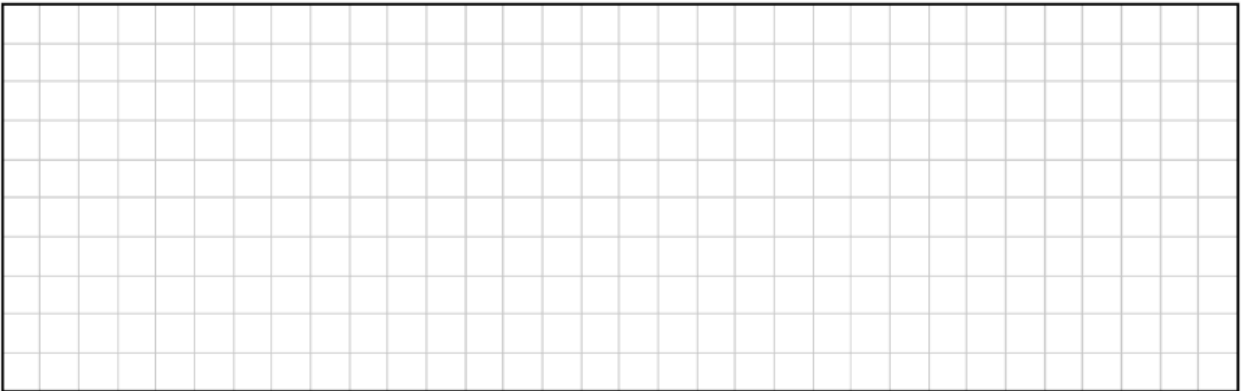
(i) Work out the percentage of students in the sample who were given a rating of 'excellent'.



(ii) Show that the margin of error for this sample is 4.5%, correct to one decimal place.



(iii) In general, 10% of students get a rating of 'excellent' on this statistics test. Use your answers from above to test the claim that the percentage of students in Cork who would get a rating of 'excellent' is different to the population percentage, at the 5% level of significance. Show relevant calculations, state your conclusion, and give a reason for your conclusion.

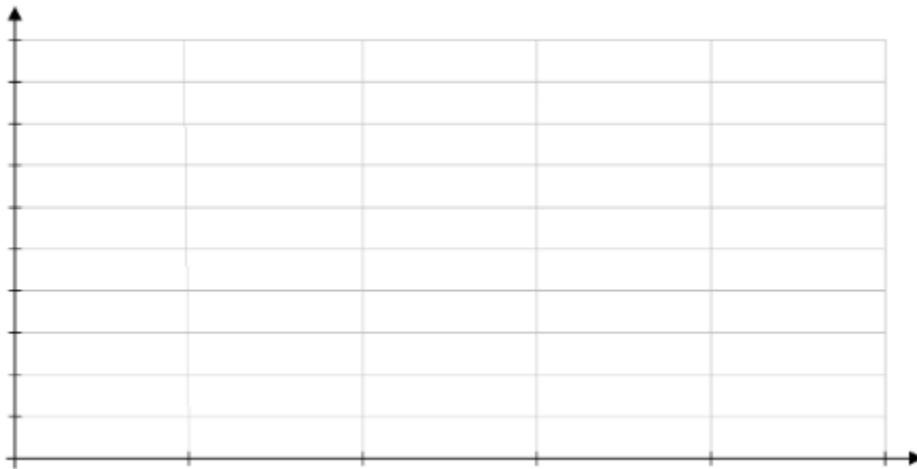


The ages of children in a creche were recorded.

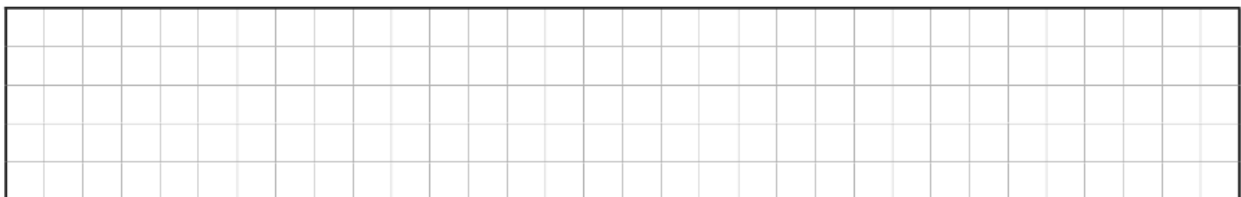
Age (years)	0 – 1	1 – 2	2 – 3	3 – 4	4 – 5
Number of children	6	10	19	3	5

[Note: 2 – 3 means “2 years or more, but less than 3 years”, etc.]

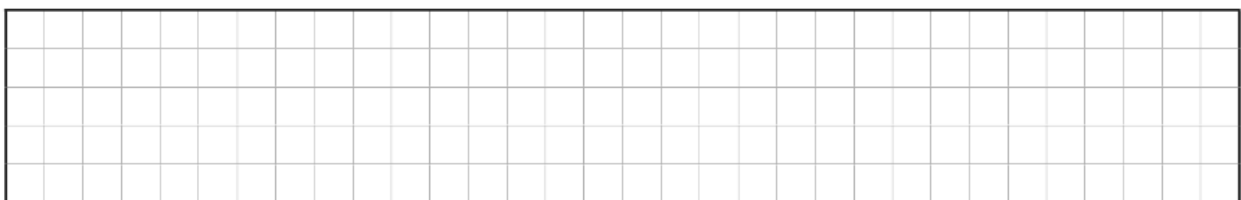
(i) Represent the data on a histogram. Label the axes clearly.



(ii) Work out the percentage of children who are aged between 1-2 years, correct to the nearest percent.



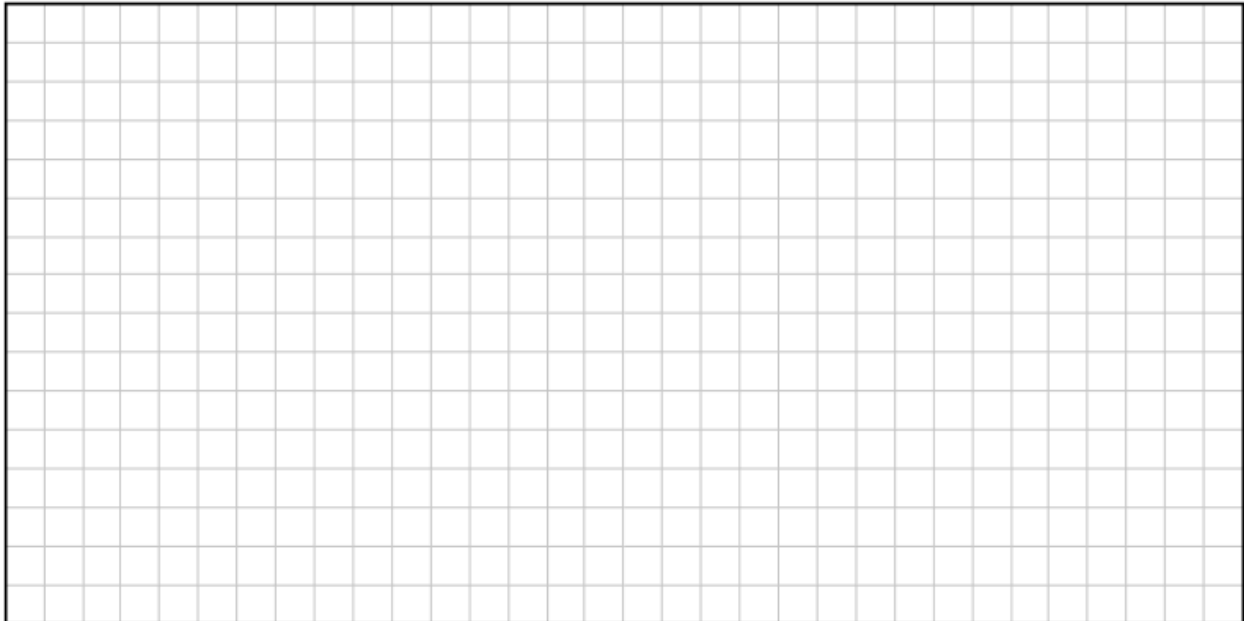
(iii) Use mid-interval values to estimate the mean age of the children in the creche.



The points scored by 5 players in a game are given below.

	James	Warren	Davis	Zander	Dean
Points scored	14	4	12	25	30

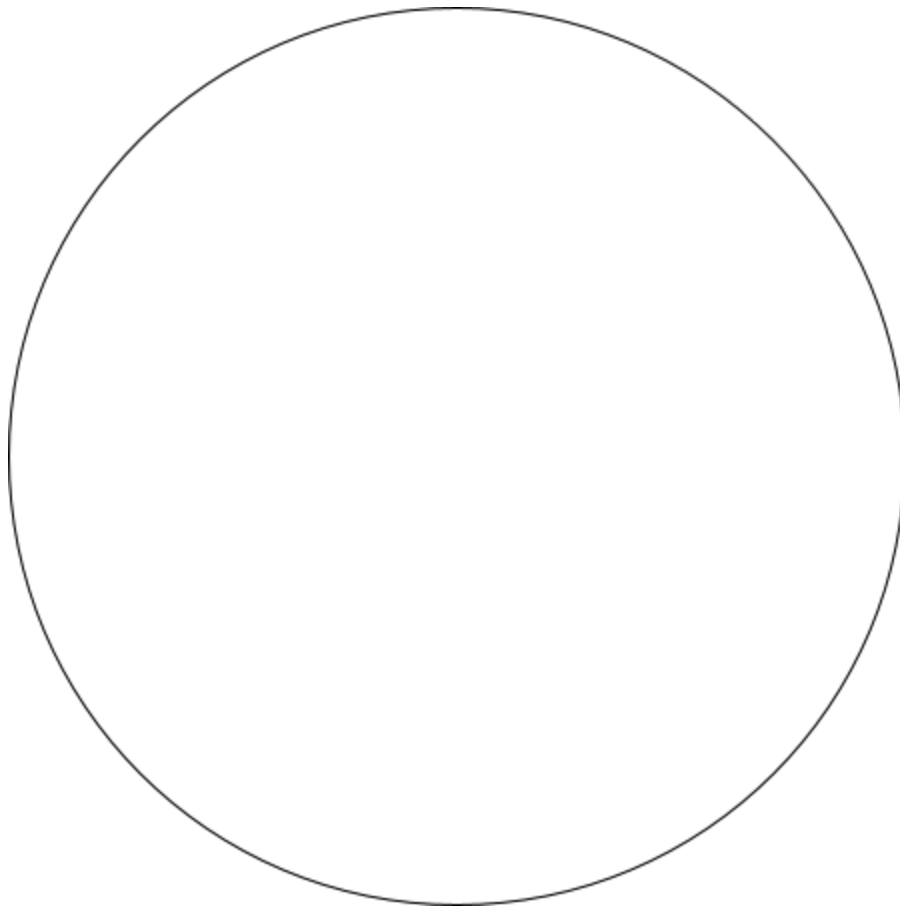
Represent these scores using a suitable graph.



Maeve's team plays 11 matches in a league. The table below shows the number of goals Maeve's team scored in each of these 11 matches.

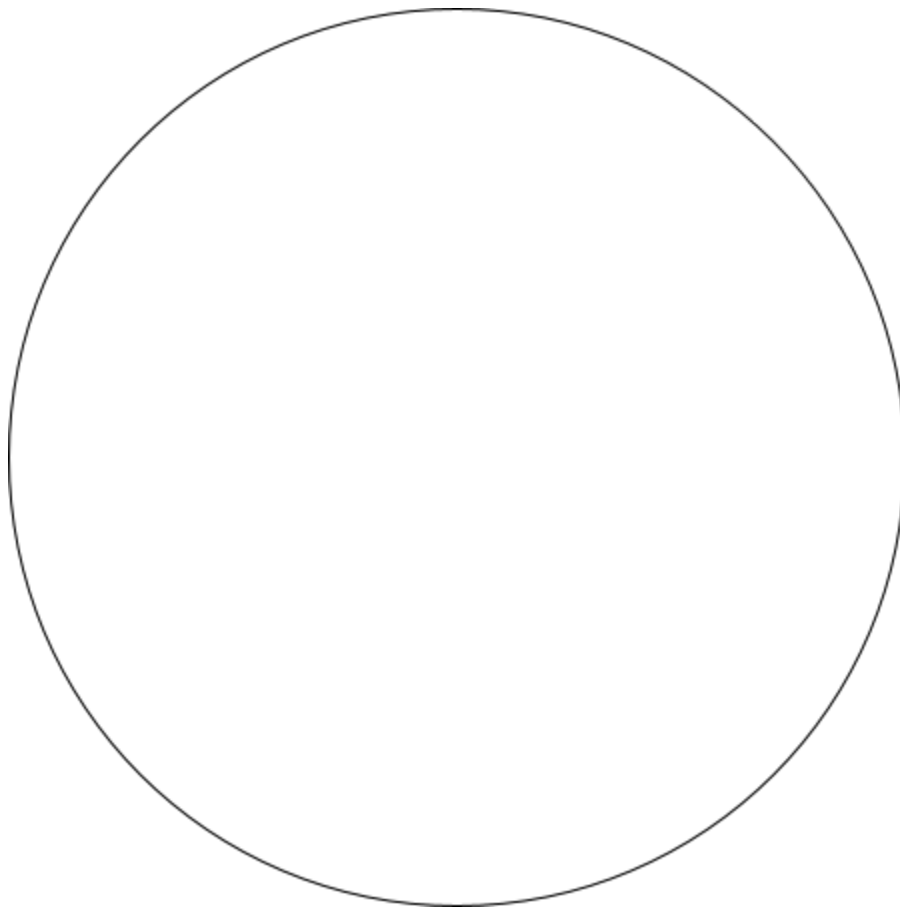
3	1	1	0	2	7	1	0	2	1	3
---	---	---	---	---	---	---	---	---	---	---

Complete the pie chart below to summarise the data above, showing the proportion of the games in which Maeve's team scored 0 goals, 1 goal and so on. Label each sector and the size of the angle clearly.



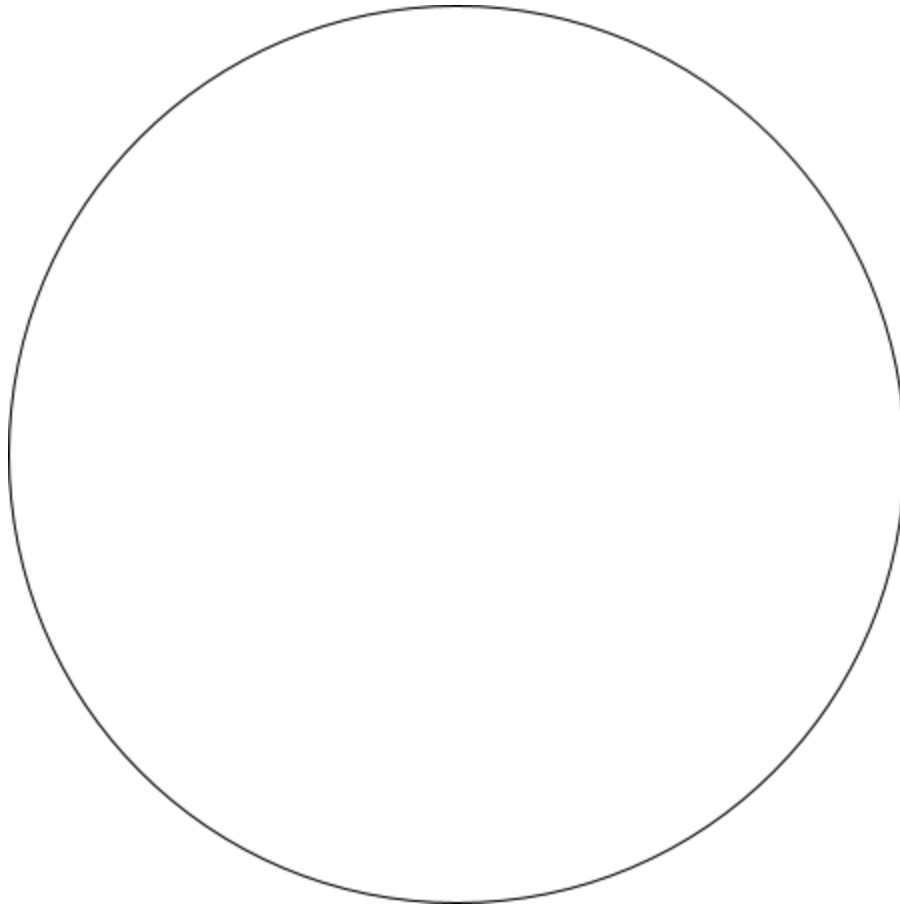
A group of tourists at an airport were asked for their final destinations. The following table shows where they were heading to. Complete the pie chart below to show this information (label each sector clearly).

Destination	Europe	North America	Asia	South America
Number of tourists	25	18	12	5



A group of students sat an exam. Each student was given a grade. The following table shows how many students got each grade. Complete the pie chart below.

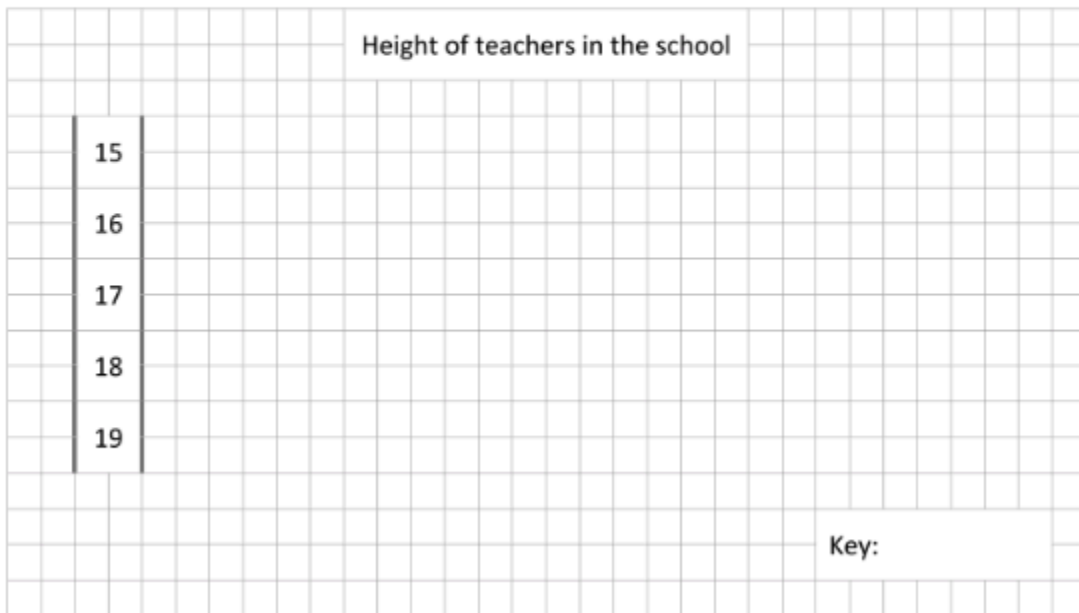
Grade	Distinction	High Merit	Merit	Achieved
Number of students	8	12	39	13



Stem & leaf diagrams

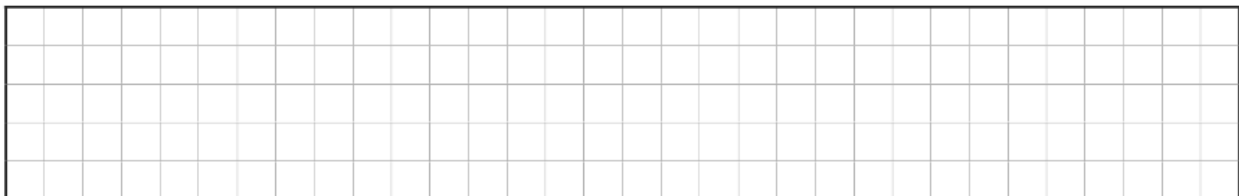
A class of students are doing a statistics project involving height. They measure the height in centimetres of the 50 teachers in their school. The results are as follows.

168	194	156	167	177	180	188	172	170	169
174	178	186	174	166	165	159	173	185	162
163	174	180	184	173	182	161	176	170	169
178	157	172	179	162	173	177	176	184	191
181	165	163	185	173	175	182	164	191	168



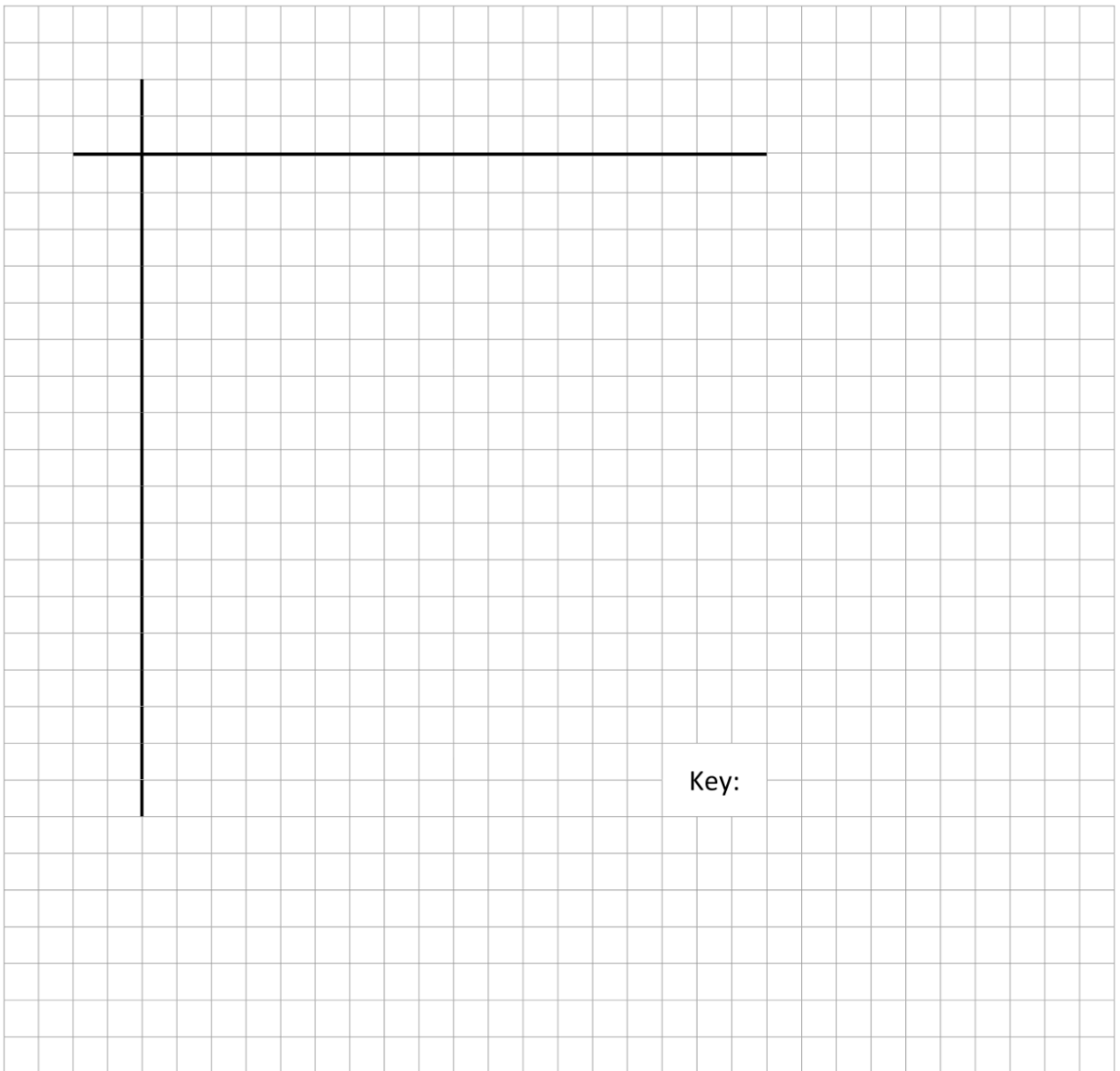
(i) Display the data on the stem & leaf diagram.

(ii) Find the modal value, median and interquartile range.



Duracell batteries are tested by a group of students to determine their mean lifespan. The number of days a sample of 30 batteries lasted is displayed below. Display the data on a stem and leaf diagram.

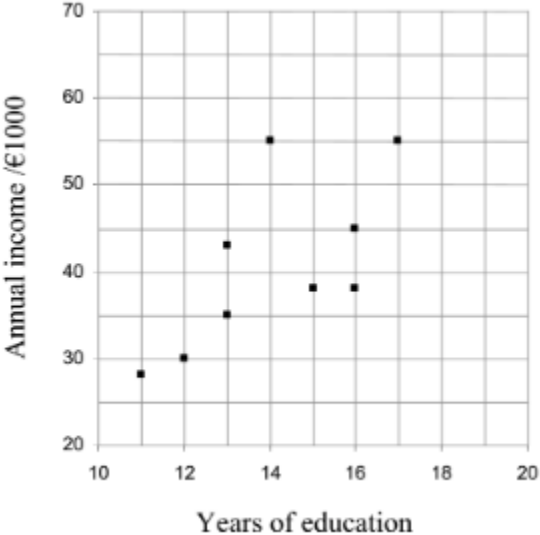
168 159 171 173 167 163 164 174 168 176
169 162 168 171 159 168 197 167 171 173
173 162 165 175 159 174 170 168 161 175



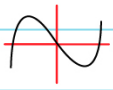
An Economics student wants to find out whether the length of time people spend in education affects how much they earn. The student carries out a small study where she asks 12 adults to state their annual income and the number of years they spent in full-time education with the data being given in the table below.

Years of education	Income /€1,000
11	28
12	30
13	35
13	43
14	55
15	38
16	45
16	38
17	55
17	60
17	30
19	58

The last three rows of data have not been included in the scatter plot. Insert them now.



Chapter 12



$\frac{dy}{dx}$

DIFFERENTIATION

$f'(x)$

Rules

Max/Min

Tangents

Rules

Bring the power to the front to multiply, and decrease the power by one

Examples:

$$2x \rightarrow 2$$

$$x \rightarrow 1$$

$$2x^3 \rightarrow 6x^2$$

$$7x^4 \rightarrow 28x^3$$

$$1 \rightarrow 0$$

$$7 \rightarrow 0$$

$$2x^4 \rightarrow 8x^3$$

$$3x^2 + 5x + 9 \rightarrow 6x + 5$$

$$x^3 + 2x^2 + 3x + 10 \rightarrow 3x^2 + 4x + 3$$

Max/Min

If a question mentions Max/Min or least/greatest:

- ① Get 1st derivative
- ② Let it = 0 and solve
- ③ Sub your answer back into original equation if required

Tip

$f'(x)$ is the same as $\frac{dy}{dx}$

- They both mean the derivative
- $f''(x)$ and $\frac{d^2y}{dx^2}$ both means get the 2nd derivative

Example:

$$f(x) = 4x^3 + 3x$$

$$f'(x) = 12x^2 + 3$$

$$f''(x) = 24x$$

$$y = 5x^3 + 2x^2$$

$$\frac{dy}{dx} = 15x^2 + 4x$$

$$\frac{d^2y}{dx^2} = 30x + 4$$

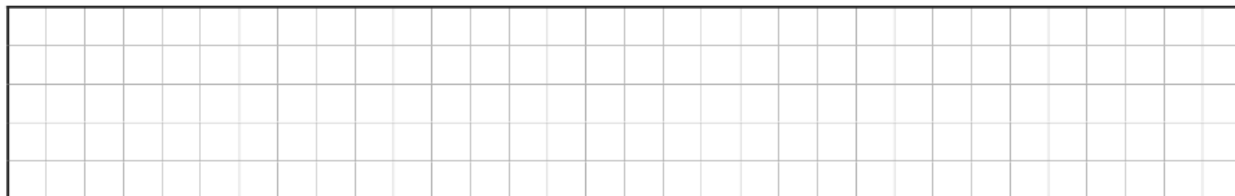
Tangents

If you sub X into the original equation, you should be outputted y

If you sub X into the derivative equation, you should be outputted the slope of a tangent @ X

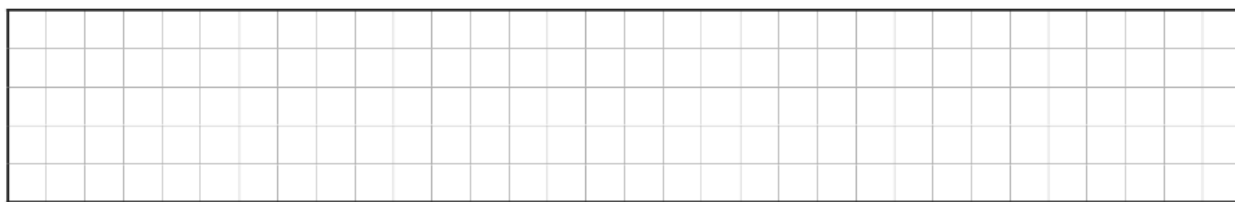
$$\text{Let } f(x) = x^2 - 7x + 12$$

Find $f'(x)$.



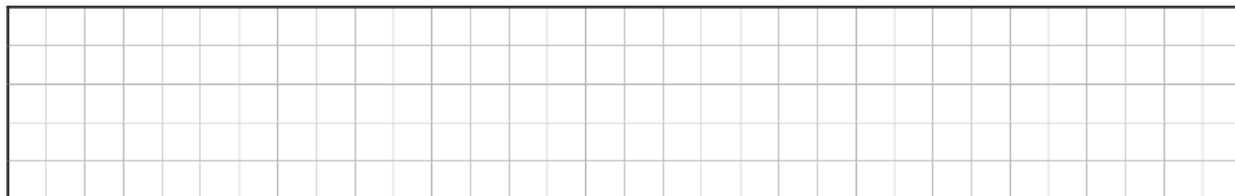
$$y = 3x^4 + 2x^2 + 7$$

Find $\frac{dy}{dx}$.

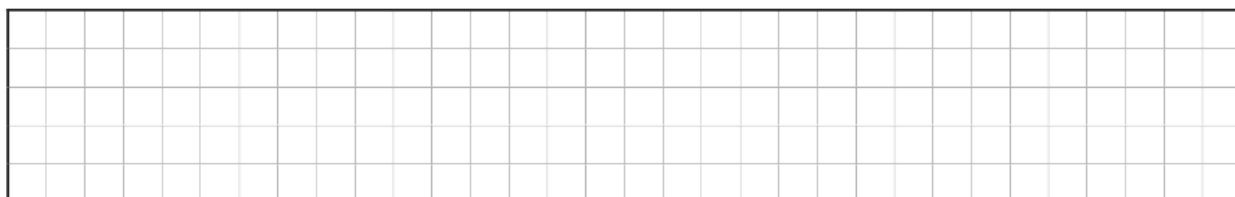


$$f(x) = -0.5x - 0.013x^3$$

Find $f'(x)$.

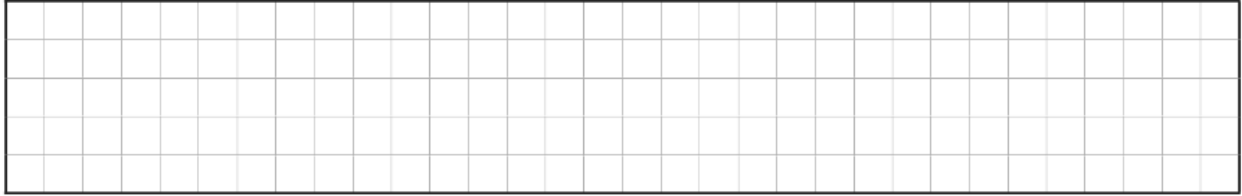


$$y = x^3 + 7x^2 + 0.4x - 10, \text{ find } \frac{d_2y}{dx^2}$$



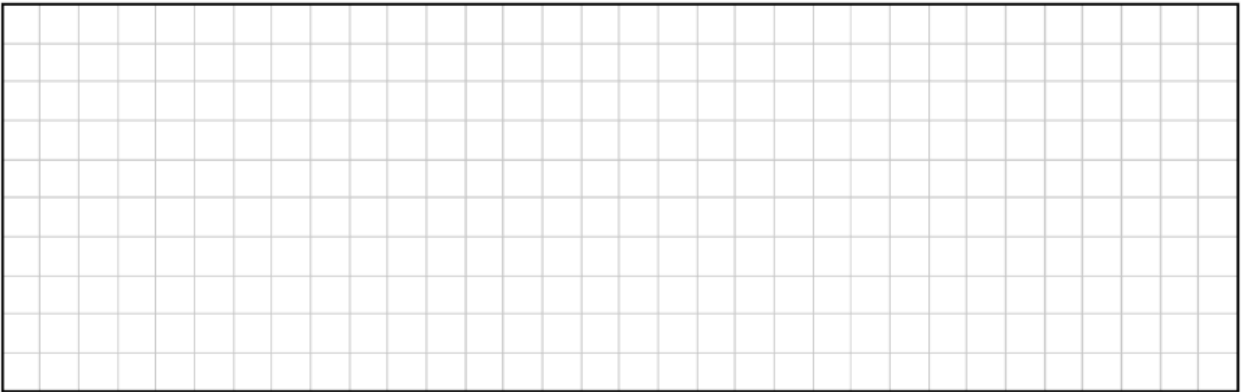
Let $f(x) = x^3 - 7x^2 + 15 + 9$

Find $f''(x)$.

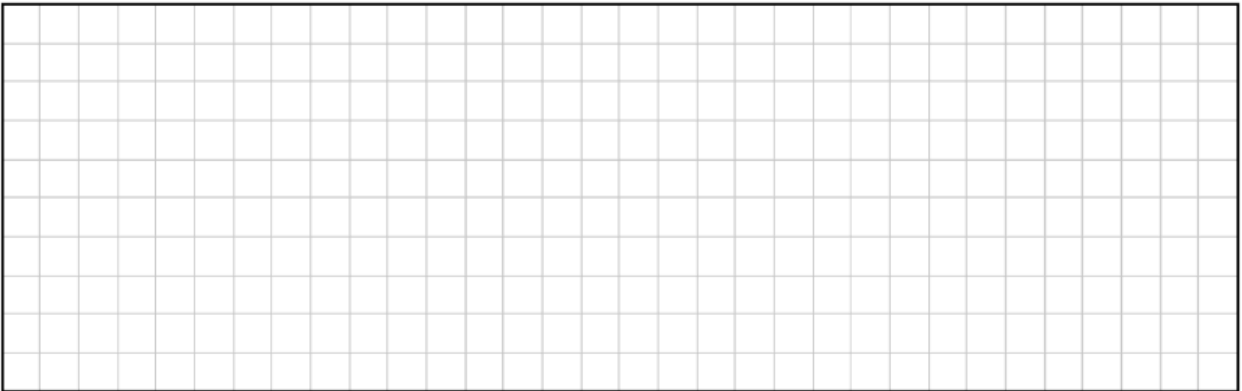


$f(x) = x^2 + 8x + 10$

Find the value of x for which $f'(x) = 2$.



Solve the equation $f'(x) = 0$ if $f(x) = 2x^2 + 16x + 10$.



The profit made by a robotic lawnmower company is modelled by the following function:

$$f(x) = 25\,000x - 10x^2.$$

Here, $f(x)$ is the profit and x is the selling price of the product in euro.

(i) The robotic lawnmower is currently retailing at €1500. Find the profit that it generates at this price point.

(ii) At what price should the company sell the product to maximise profit?

At what coordinate is $f(x) = x^2 - 5x$ at its minimum point?

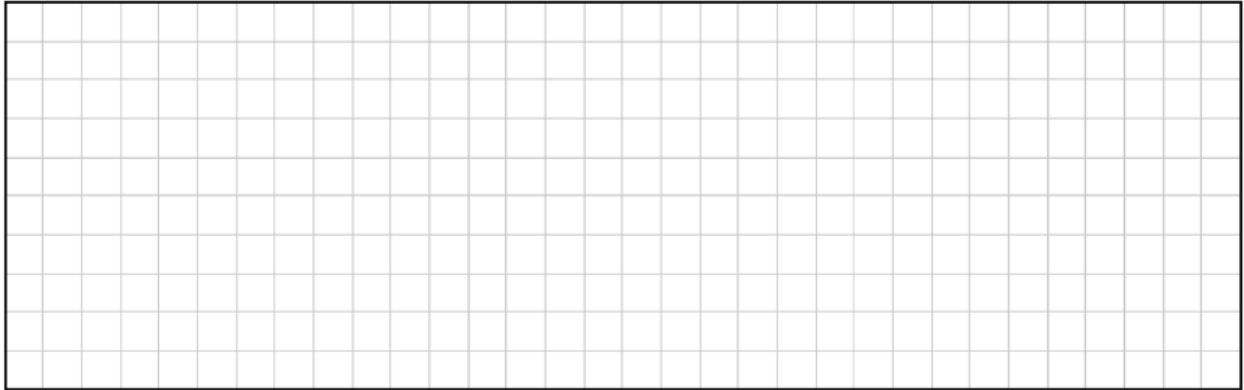
$$g(x) = x^2 + 8x + 6$$

(i) Find the value of $g(5)$

(ii) Use calculus to find the value of x which gives the minimum value of $g(x)$.

The area of algae growing on a lake can be given by $A = 15t^2 - 0.8t^3$, where t is time in months after the algae starts growing. Find $\frac{dA}{dt}$ and hence find the rate of change in the area of algae after 2 months. Find the value of t when the maximum area is covered by the algae.

The curve C has equation $y = x^3 - 11x^2 + 35x - 25$. Verify that C has a stationary point at $x = 5$.



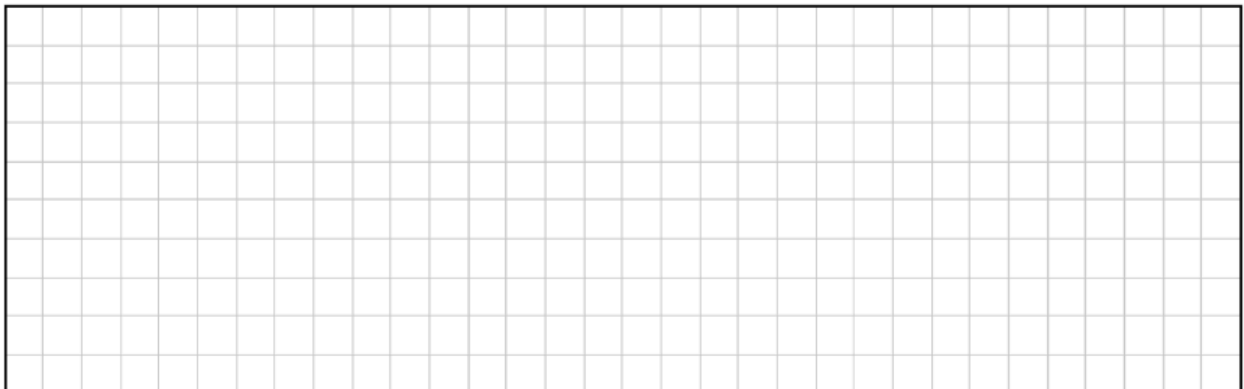
Keith plays hurling. During a match he hits the ball with his hurl. The height of the ball could be modelled by the following quadratic function:

$$h = -2t^2 + 5t + 1.2, \text{ where } t \text{ is in seconds.}$$

Find how long it took the ball to reach its greatest height, and hence find its greatest height.

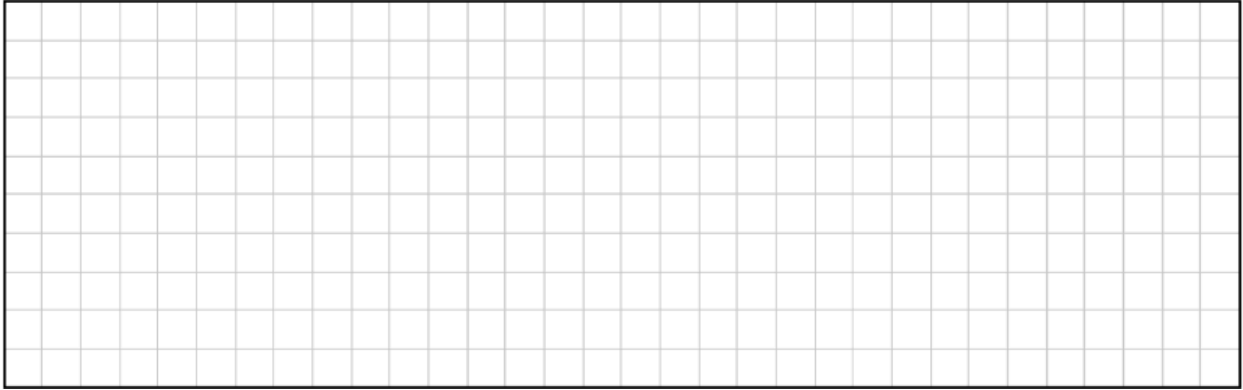


$f(x) = 3 - 5x - 2x^2$. Find the coordinates of the local maximum point of $f(x)$ in the form $(\frac{a}{4}, \frac{b}{8})$ where $a, b \in N$.



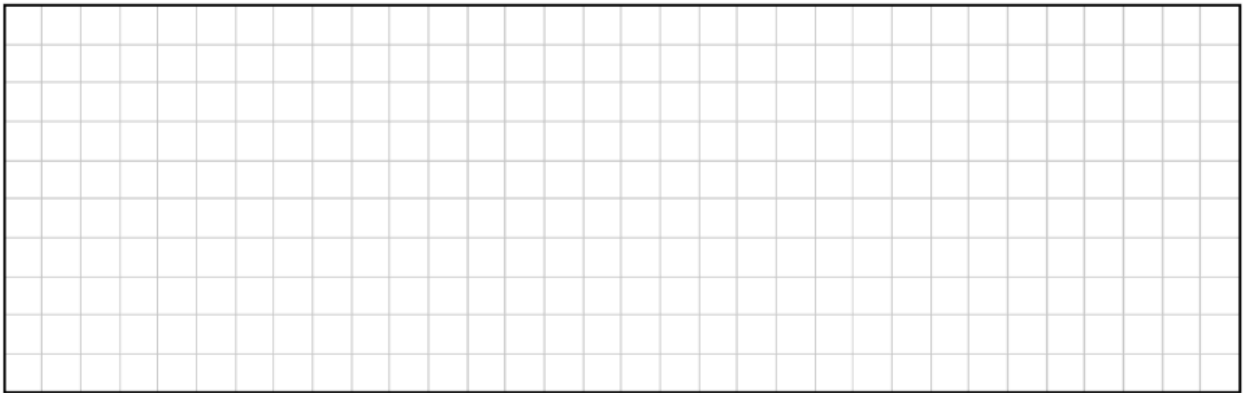
A function $h(x)$ has equation $h(x) = x^3 + 2x^2 - x - 8$.

Find the equation of the tangent to $h(x)$ at the point $(2, 6)$.



The function $f(x) = 2x^2 + kx + 2$ where $k \in \mathbb{N}$.

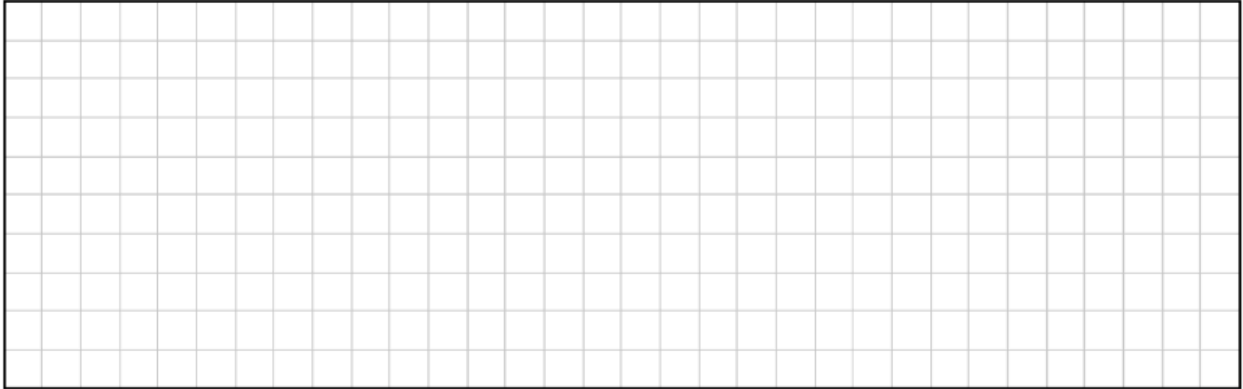
Given that $f'(1) = 7$, find the value of k , and hence find the equation of the tangent to $f(x)$ at $x = 1$.



Find the equation of the tangent to the curve $y = x^3 - 4x + 7$ at the point $(2, -3)$.



$g(x) = x^3 - 7x^2 + x - 12$. Find the equation of the tangent to y at the point where $x = 5$.



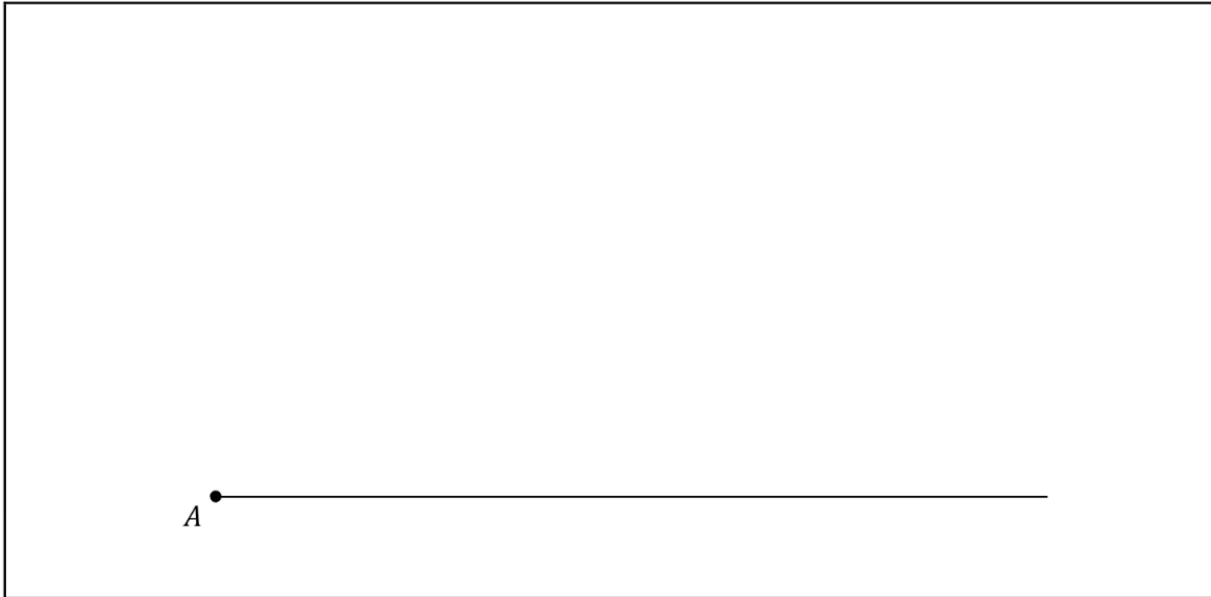
Chapter 13

CONSTRUCTIONS

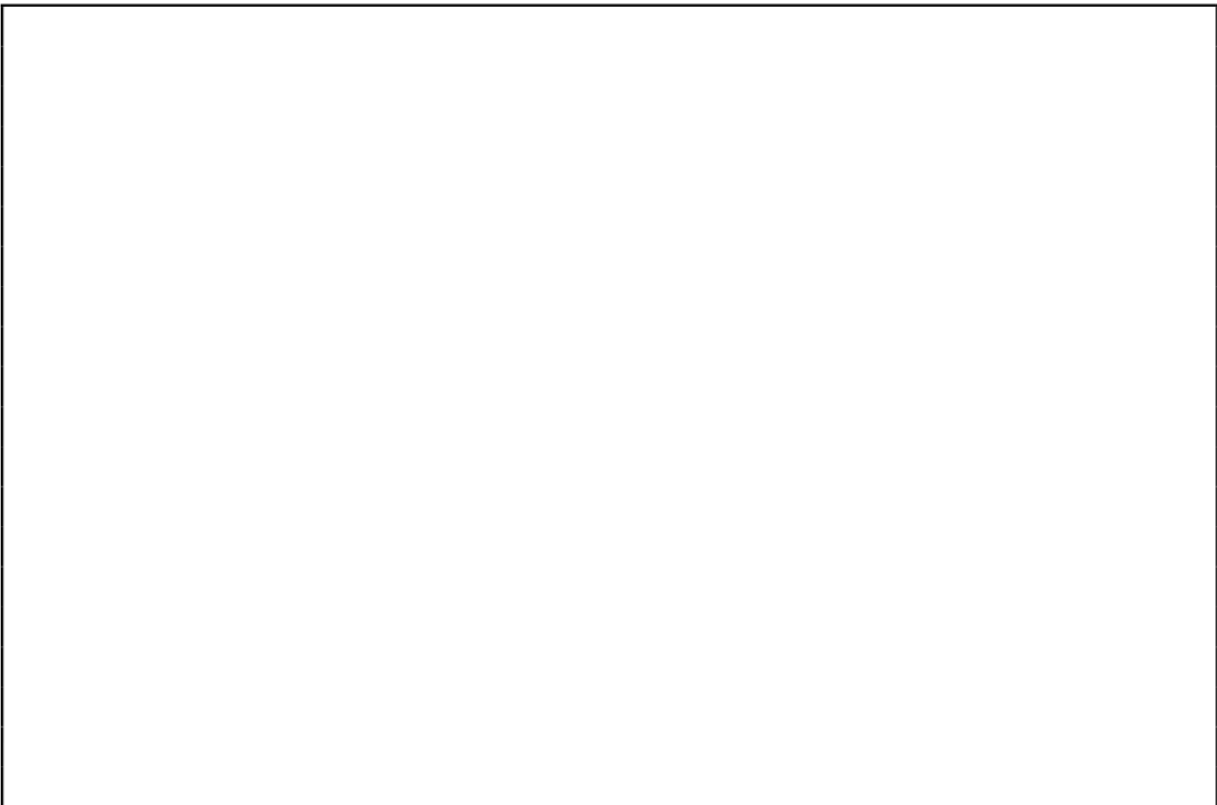
Construct a :

- ① • parallelogram
- ② • Triangle
- ③ • Enlargement
- ④ • Circumcircle
- ⑤ • Incircle

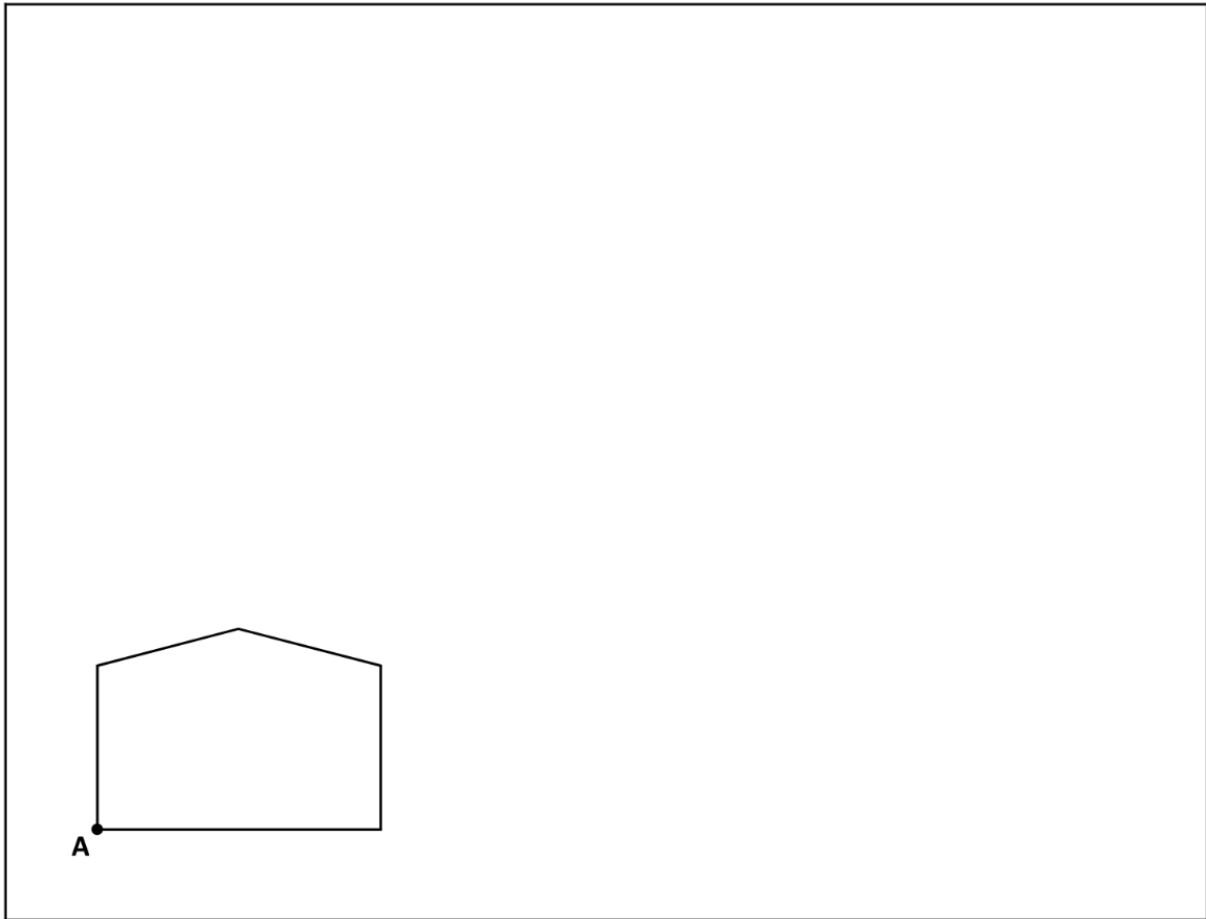
Construct the parallelogram $ABCD$ where $|AB| = 8\text{ cm}$, $|BC| = 6\text{ cm}$, $|\angle ABC| = 130^\circ$



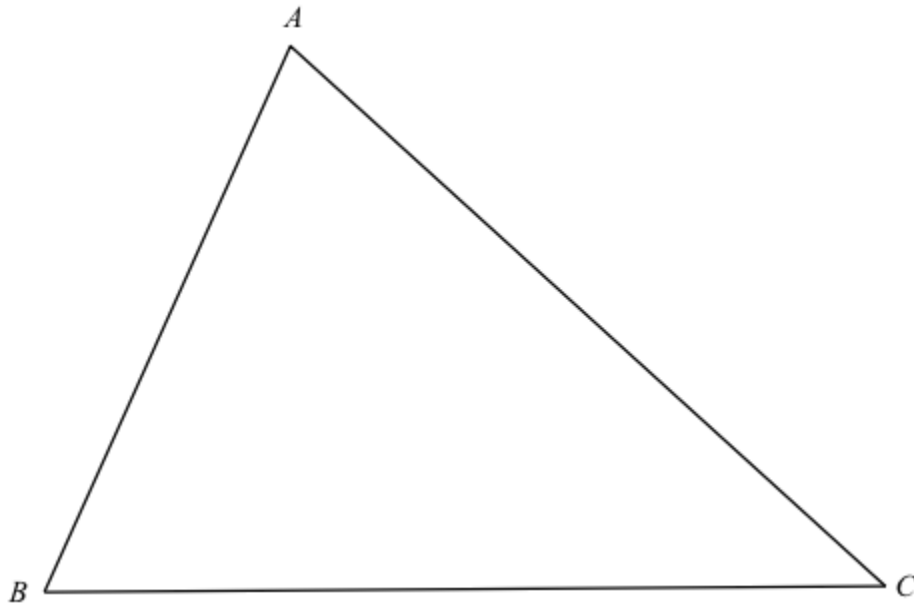
Accurately construct the triangle ABC where $|AB| = 9\text{ cm}$, $|AC| = 7\text{ cm}$ and $|BC| = 8\text{ cm}$



On the diagram below, construct an enlargement with centre **A** and a scale factor of 3.



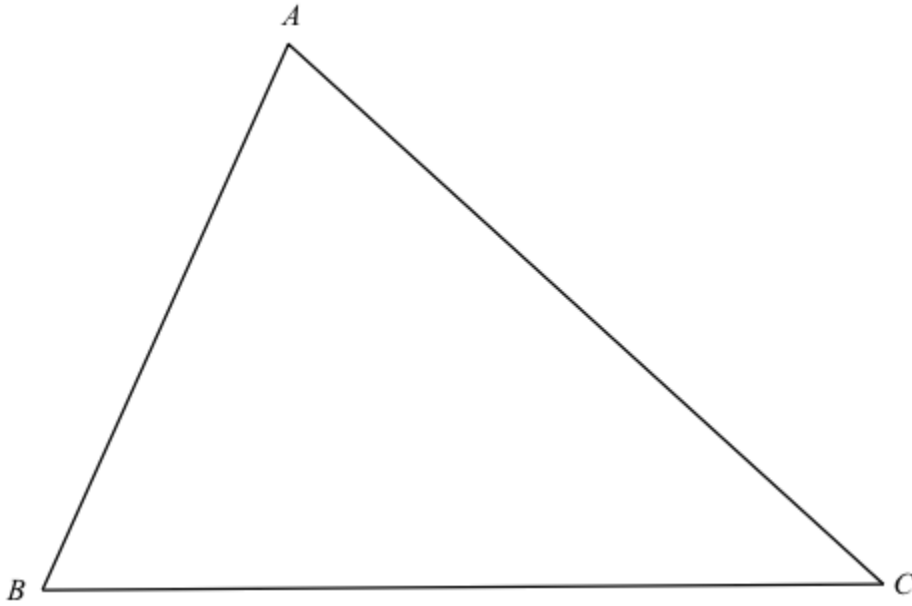
Construct the circumcircle of triangle ABC below.



Construct the triangle ABC where $|AB| = 10\text{ cm}$, $|\angle CAB| = 60^\circ$, $|\angle ABC| = 40^\circ$



Construct the incircle of triangle ABC below.



Chapter 14

6:1 MISCELLANEOUS



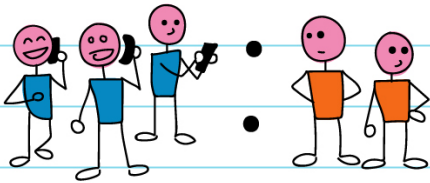
• Ratios

• Scientific notation

• Speed, Distance, Time

• Ratios

A ratio is a comparison of two quantities. For example you might use a ratio to compare the number of students who have a mobile phone to the number of students who don't.



The ratio 3 to 2 can be written as (3:2)

• Scientific Notation

$$5,000,000 \rightarrow 5.0 \times 10^6$$

$$0.00042 \rightarrow 4.2 \times 10^{-4}$$

$$123,000 \rightarrow 1.23 \times 10^5$$

$$0.0078 \rightarrow 7.8 \times 10^{-3}$$

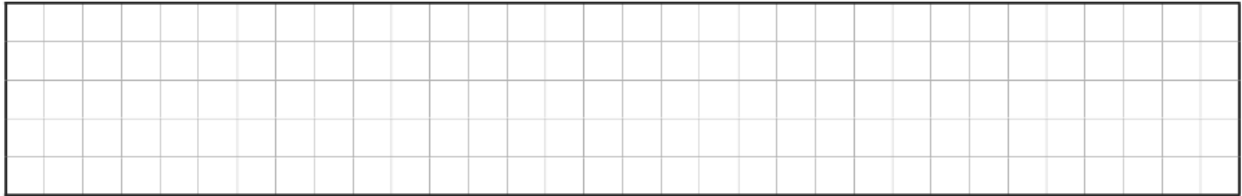
• Speed, distance, time

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

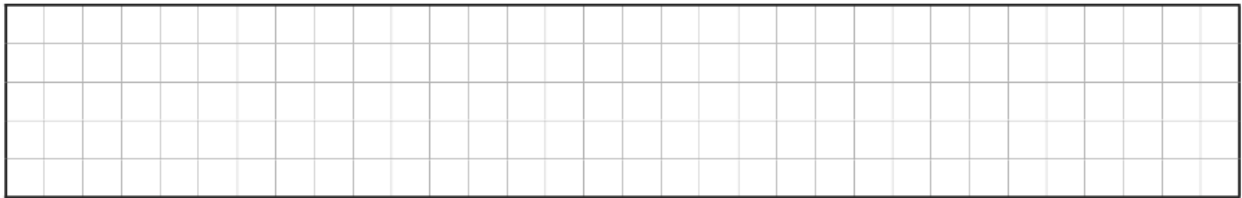
$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

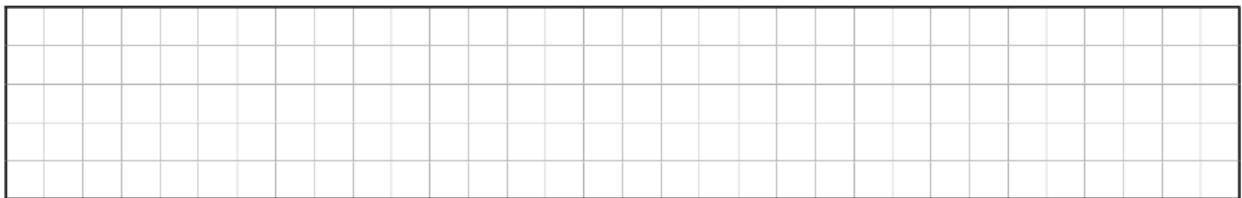
A 1 litre carton is filled with milk. The milk is poured into two cups, as follows. 25% of the litre is poured into cup A. $\frac{1}{6}$ of the litre is poured into cup B. Calculate the ratio of the volume of milk in cup A to the volume of milk in cup B.



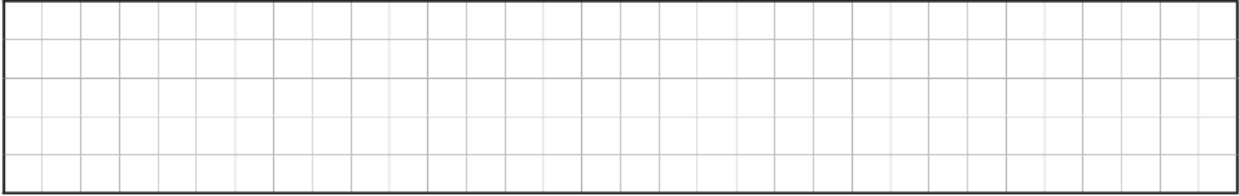
Rochelle, Eimear and Deirdre share the profits from their mini-company in the ratio 4: 2: 1. If Eimear received €56. What were the total profits?



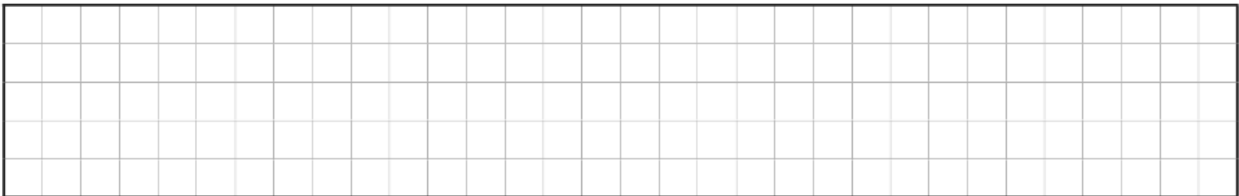
Millie bakes cakes and sells them at the local market. She makes a filling for the cakes. The ratio of butter to sugar that she should use in the filling is 5:7. If she makes 2.4 kg of filling, how many grams of sugar did she use?



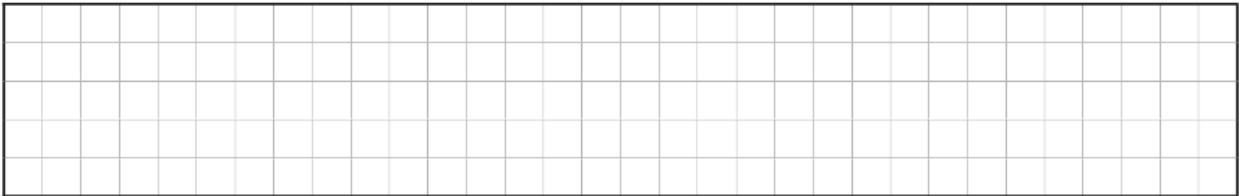
Write 868 million in the form $a \times 10^n$ where $a \in N$.



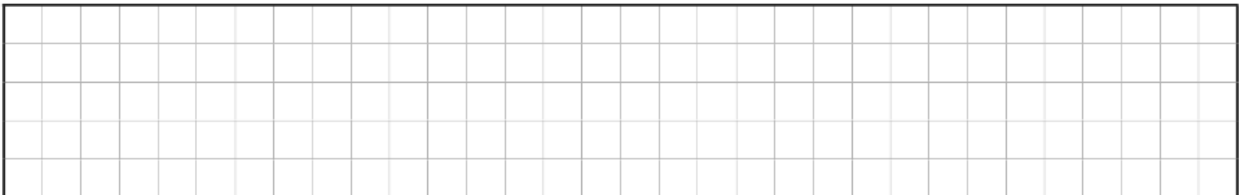
The diameter of a human hair is roughly 0.0075 cm. Write this diameter in the form $a \times 10^n$ where $n \in Z$ and $1 \leq a \leq 10$.



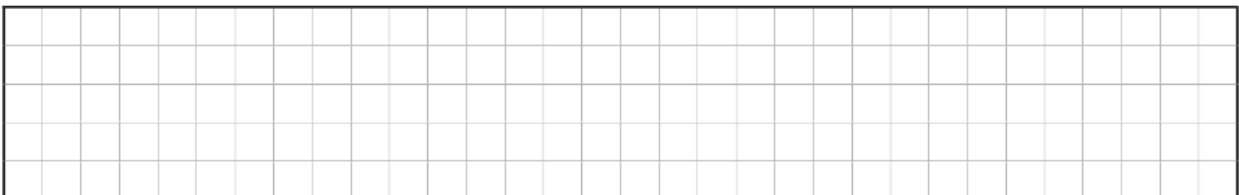
Mercury orbits the sun on a circular path, of radius length 24 440 km. Find the distance travelled by Mercury when it has completed one orbit of the sun. leave your answer in metres in the form $a \times 10^n$ where $n \in N$ and $1 \leq a \leq 10$.



The population of Africa is 1.22 billion. Express this in the form $a \times 10^n$ where $n \in N$ and $1 \leq a \leq 10$.

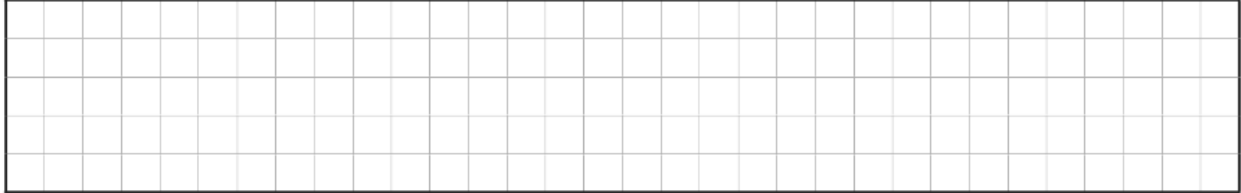


Express 3.76×10^{10} as a full number

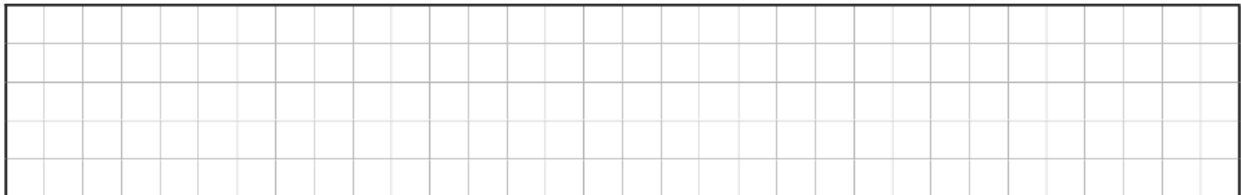


Speed, distance and time

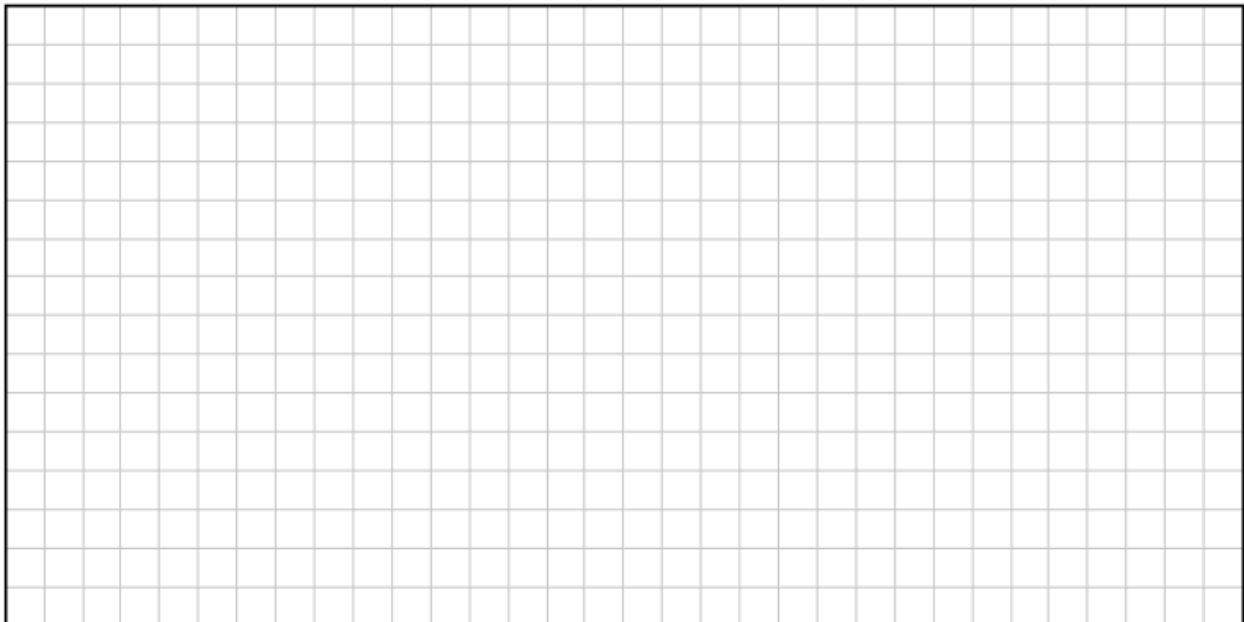
A train travels 264 km between Cork and Dublin, at an average speed of 96 km/hr. If the train leaves Cork at 14:05, what time does it arrive in Dublin?



A fox travels a distance of 5 km from its den before returning. If their total journey took 1 hour and 48 minutes, what was their average speed in km/hr?



On a given day, David travels from his house to school. After school, David goes on to his part time job. The ratio of the distance from his house to the school and from his school to the job is 5:6. David rides his scooter at a speed of 15 km/hr and it took him 10 mins to arrive at his school. Find the distance from the school to his part time job.



Lewis was driving at 90 km/hr when he sneezed. During the sneeze, his eyes were closed for half a second. How many metres did he travel during this time.

